DISEQUILIBRIUM MACRO-ECONOMICS IN A CLOSED ECONOMY

Some Extensions

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1. INTRODUCTION

1.1 The Integration of Microeconomic and Macroeconomic Theory

Many contributions to economic theory may be broadly classified as microeconomic or macroeconomic, by which it is meant that they are concerned with questions about the behaviour of rational individual agents in the case of microeconomics or large aggregates in the case of macroeconomics. It would seem reasonable to expect that the results obtained by studying the aggregates should in some sense be compatible with adding up of the behaviour of the individuals which comprise those aggregates. That micro theory and macro theory should at least not be contradictory. On a simple level this may not appear to be the case.

Consider the General Equilibrium model originating from the work of Arrow-Debreu and the standard 'elasticity pessimism' IS/LM representation of Keynesian economics to be the paradigms of micro and macro respectively. It may appear that these two models are in conflict, the general equilibrium model assumes that prices clear markets, such phenomena as unemployment can only arise by the individual choice of the agents concerned, all unemployment is voluntary. One of the crucial features of Keynesian macroeconomic models is the presence of involuntary unemployment. To present these models in this light is to deliberately misinterpret the questions which they are addressing.

The basic question to which general equilibrium addresses itself is in the words of Weintraub (1979, p. 75):

> "how (might) it (be) possible for a decentralised individualistic system, operating on principles of self-interest, to produce coherent or co-ordinated outcomes? (of the economic system)."

PREFACE

Despite many notable contributions over the last two decades economic theory has not yet succeeded in effectively integrating microeconomics and macroeconomics. Macroeconomic analysis and its propositions are not derivable from any rigorous microeconomic basis, yet microeconomics in the form of Walrasian general equilibrium theory cannot explain equilibria with inefficient resource utilization, most particularly unemployment equilibria. However recent contributions in the area known variously as Neo-Keynesian economics, Disequilibrium Theory or Non-Walrasian Economics, have made considerable progress. The aim of this thesis is to examine these recent developments and to make some contributions in the areas of the role of expectations in disequilibrium analysis and the importance of union employer wage bargaining as an explanation of imperfect price adjustment upon the labour market.

Many people have given me their comments and encouragement whilst writing this thesis, and I regret that I cannot mention here the names of all those people to whom I owe a debt of thanks. I am most deeply indebted to Paul Stoneman, Avinash Dixit and Norman Ireland for supervising this thesis and also wish to acknowledge Marcus Miller, Ben Lockwood, John Fender, Bob Rothschild, Philip Michel, Torsten Persson, Nick Snowdon and Jennifer Ellis for invaluable stimulation and insightful comments.

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NOTATION

The following notation will be adopted throughout this thesis, any variations will be indicated at the appropriate point in the text.

x	Consumption good sales to consumers
y	Output
L	Labour
p	price of output
w	wage rate
M	money balances
e	Endowments
T	Total Time
π	Profit
I	Inventories
z	net demands or trades
$\bar{A}$	single bars over a variable denote an actual constraint
$\bar{\bar{A}}$	double bars over a variable denote an expected constraint.

Whilst Keynesian macroeconomics poses itself such questions as, why do phenomena such as unemployment and inflation persist, and how might policy makers manipulate economic aggregate to alleviate such problems?
General Equilibrium Theory examines the individual choice calculus of agents within a market economy and establishes the detailed conditions required for the existence of an equilibrium brought about by price adjustment. Macroeconomics implicitly takes as a fait accomplie that such conditions are violated and asks how economic aggregates interreact in reality and seeks to model these in a way that appears consistant with observation, with the aim of providing solutions to real problems.

Why bother attempting to integrate microeconomic and macroeconomic theory if macroeconomics fits the facts? indeed, why study microeconomics other than on grounds of intellectual curiosity? The reasons are two-fold. Firstly a macroeconomic model with a firm microeconomic underpinning should have greater predictive potential than one based upon the observation of aggregate behaviour. If the economic system finds itself in a new and previously unexperienced state, how will it respond and what policies are required to improve its position? If the standard aggregate analytical techniques are applied it may be the case that they are invalid, the aggregates under examination are a summation of individuals, and individuals in an entirely new state may respond in a different manner than previously observed. Only if the macroeconomics model is correctly underpinned by microeconomic analysis of exactly what it is rational for individual agents to do will such pitfalls be avoided. This is easy to state but not easy to do, but the integration of these two branches of economic theory is clearly a worthy endeavour. The second reason for regarding the integration of micro and macro theory as desirable is that it should allow a more accurate assessment of the value and implications of any aggregate economic change. Neither of these two arguments is novel but they are particularly pertinent.

The growing literature about the integration of micro and macro economic theory is known collectively as the micro-foundations of macroeconomics. This literature may be regarded as taking 'Keynesian realities' in the form of restrictions upon individual agents information sets, performing the constrained choice calculus and examining the individual and aggregate outcomes.

The approach of taking a general equilibrium model and imposing information restrictions upon agents first appears in Hicks 'Value and Capital' (1939). The behaviour of a general equilibrium system with incomplete futures markets is modelled. Agents cannot contract for all contingent commodities, the missing information is the price of commodities in the next period or 'Hicksian week'. On the Monday of the Hicksian week agents express transaction demands based upon future price expectations and thus planned future trades. Current period markets then clear by price adjustment, some agents a priori price expectations will be falsified, but they do not adjust expectation of next period prices until the beginning of the next Hicksian week. (The time lag in revising expectations could be interpreted as defining when the next week starts). Thus each periods transactions are based on expectations which may be false. By this process a series of short run 'Temporary Competitive Equilibria' are defined, in which prices clear markets but in which some expectations are incorrect. Hicks then continues to define such an equilibria as dynamically stable if price expectations are all confirmed. This approach has generated a considerable body of research, and for comprehensive and extremely well articulated surveys the reader is referred to Radner (1974) and Grandmont (1977). Hicks's concept of Temporary Competitive Equilibria allows agents to make mistakes in their expectations, but the absence of information disseminating futures prices does not cause any of the typical macroeconomic problems. Prices clear all current period markets. There is no involuntary unemployment, only voluntary unemployment based upon mistaken expectations. Perfect auction markets function at any moment t , the effect of the non-existence of future markets is only to generate inter-temporal inefficiency.

General Equilibrium and Temporary Competitive Equilibrium theory both utilize the same basic assumption, that trading takes place at equilibrium prices. Thus in both models the only information required by agents to co-ordinate economic activity is the price vector, consisting of all current and future prices in the case of General Equilibrium and omitting some future prices in the case of Temporary competitive Equilibrium. We shall refer to both these price vectors hereafter as market clearing prices.

If agents facing Walrasian prices do not display behaviour the aggregation of which is consistant with macroeconomic experience and theory, then consider what happens if the price vector in non-Walrasian. This represents a further restriction upon the information available to agents, they may no longer take as given that market supplies and demands will balance and thus they cannot assume that all planned transactions will be feasible. Hicks was aware of the problem of 'false prices' as he termed them, but regarded the volume of trading which took place at non-Walrasian prices to be of a small order of magnitude, which consequently could be ignored.

The significance of trading at false prices was first analysed by Patinkin (1965) and Clower (1965). Patinkin's contribution was the explicit recognition of the role of quantity rationing. In his analysis firms perceive that they will face constraints on their future sales, they then reduce their labour purchases causing consumer-workers to face involuntary unemployment. Implicit in this analysis is a non-Walrasian wage rate such that more consumers desire to work than are required by firms reacting to sales expectations. Clowers contribution represents the true conceptual breakthrough since he explicitly realizes the important information disseminating role of the Walrasian price system, and the implications for a general equilibrium type of analysis if the price vector is non-Walrasian. Clower examines consumer behaviour, arguing that if relative prices are wrong labour supply will exceed employment offered by firms, the shortfall in 'planned' income will thus cause a revision of workers goods purchases to be consistant with realized income. Conceptually the argument reaches far beyond the simple analysis of household behaviour. It is argued that Keynesian macroeconomics requires a reformulation of the underlying general equilibrium theory, recognizing the distinction between notional and effective demands. Notional demands being those levels of transactions that agents would wish to carry out at given prices. Effective demands being those transactions agents express, given prices and quantity rationing encountered on some markets. Notional and effective demands will be equal only if prices are Walrasian. If prices are wrong rationing such as involuntary unemployment will occur. The seeds of the integration of micro and macro economic theory appear to be sown here.

But why do non-Walrasian prices obtain? Clowers' answer is to suggest, quite correctly, that excess demand functions may only be validly specified using effective demands. Walrasian prices are typically obtained from the solution of excess demand functions specified by notional demands which cannot be expressed away from equilibrium. Thus he argues that price adjustment based upon effective excess demand functions may be incomplete. This argument is persuasive but has not yet been satisfactorily formalized. Two remarks should be made here, firstly Clower does not present an equilibrium concept in his 1965 paper, he is essentially looking at disequilibrium. Secondly prices are not assumed to be fixed, rather it is argued that they may not move sufficiently to clear all markets, and thus quantity constraints may persist.

Leijonhufvud (1968) independently replicated many of the arguments presented by Hicks, Patinkin and Clower. His work is an exegesical examination of Keynes's 'General Theory' (1936) and will not be examined in any great detail here despite its great interest in many contexts.

Barro and Grossman (1971, 76) study a representative consumer producer model of market non-clearance under parametric prices. They examine the characteristics of the equilibria established via quantity adjustments. Attention is paid to studying how the different levels of fixed prices and wages will determine on which markets quantity constraints arise.

An extensive literature has developed from the early contributions discussed above and is commonly called 'Keynesian Temporary Equilibrium Theory' or 'Disequilibrium Theory'. Research has been undertaken in two basic directions, firstly refinements to the micro-foundations of macroeconomics generally termed the 'Theory of Effective Demand' and secondly examination of the implications for conventional macroeconomic problems, termed 'Neo-Keynesian macroeconomics'. Hereafter the abbreviations T.E.D. and N-K.M. will be adopted for these two areas.

1.2 The Problems Raised by Disequilibrium Theory

It is in no sense disparaging to say that the early contributions towards the integration of micro and macro Theory raised more questions than they provided answers. A basic question raised was 'does a non-Walrasian equilibrium exist, and if so what does it look like?' It is argued in the previous section that Clowers' analysis deals analytically with the persistance of disequilibrium. What further is required for the existence of an equilibrium at non-Walrasian prices? An answer is presented by Benassy (1975) and Dreze (1975). Both consider models of sufficiently short duration that the price vector may be taken as fixed, a tatonnement process is assumed to take place on quantities, and an equilibrium is shown to exist by proving there is a fixed point in the quantity tatonnement. The two approaches differ in the specification of effective demands. Benassy, generalizing Clowers approach, assumes that agents calculate their effective demands on each market independently, taking into account constraints on <u>all other</u> markets. We may thus write a Benassy-Clower effective demand for agent i on market h as:

$$Z_{ih} = Z_{ih}\,(p, \bar{Z}, \underline{Z})$$

where

$p = p_1, \ldots, p_n \quad h = 1, \ldots, n$ prices on the n markets

$\bar{Z} = \bar{Z}_{i,1}, \ldots, \bar{Z}_{i,h-1}, \bar{Z}_{i,h+1}, \ldots, \bar{Z}_{i,n}$ upper bounds on trades on each of the other n-1 markets

$\underline{Z} = \underline{Z}_{i,1}, \ldots, \underline{Z}_{i,h-1}, \underline{Z}_{i,h+1}, \ldots, \underline{Z}_{i,n}$ lower bounds on trades on each of the other n-1 markets.

A Benassy K equilibrium is a set of effective demands, Z_{ih}, which replicate themselves under the given rationing scheme.

Drèze assumes that each agent calculates his effective demand on each market, taking into account the quantity constraints on <u>all</u> markets. We may thus write the Drèze effective demand for agent i on market h as:

$$Z_{ih} = Z_{ih}\,(p, \bar{Z}, \underline{Z})$$

where $p = p_1, \ldots, p_n$

$\bar{Z} = \bar{Z}_{i,1}, \ldots, \bar{Z}_{i,n}$ Upper bounds on trade on all markets

$\underline{Z} = \underline{Z}_{i,1}, \ldots, \underline{Z}_{i,n}$ lower bounds on trade on all markets

A Drèze equilibrium is then a set of such effective demands (trades), which replicate themselves. Drèze proves the existence of such a non-Walrasian equilibrium under a proportional rationing scheme.

These two approaches have been central in a significant proportion of all subsequent work on the T.E.D. and N-K.M., and are responsible for raising many questions. The differences between these two basic models are much more significant than it superficially appears. In the Benassy equilibrium each demand z_{ih} is calculated independently without reference to any constraints that may be in force on the market in which the demand is expressed, agents in a Benassy K equilibrium may express demands which violate their constraints. The equilibrium is a set of effective demands which reproduce themselves. In the Drèze case all constraints are perceived, thus the constraint on any market cannot be violated by the effective demand. Thus a Drèze equilibrium is a set of effective demands which replicate themselves, where the effective demands are the actual transactions. This difference has two important implications, firstly the Benassy approach has a natural measure of excess demand, the difference between the effective demand and the constraint. The Drèze approach has no equivalent. Secondly the form of the rationing scheme will be more important in the Benassy case since individuals may be able to manipulate their ration through their effective demands. This is not to suggest that all rationing schemes are non-manipulable in a Drèze equilibrium, but it is difficult to see how an agent in Drèze model would learn how to manipulate his ration. Finally it should be noted, as has been pointed out by numerous contributors, e.g. Svensson (1977), that the sum of the Benassy effective demands may violate the budget constraint, but since effective demands are not transactions it is not obvious that this implies some underlying transaction irrationality as some have suggested.

The non-Walrasian equilibrium concepts of Benassy and Drèze do not examine two important questions. How are prices determined, and by what process is a non-Walrasian equilibrium generated? In both approaches prices are parametric and no trading takes place away from the non-Walrasian equilibrium. A fully articulated model would need to explain which agents set prices and how individual decision making is undertaken over both prices and quantities. It would also have to examine the actual transaction structure, explaining the sequence in which agents visit markets and how this may or may not lead to a stable equilibria.

Another question not fully examined in the Benassy and Drèze papers of 1975, is the specification of an appropriate rationing scheme. It is noted that the choice of rationing scheme selects the non-Walrasian equilibria of the system. But what would be an appropriate rationing scheme? It would seem sensible that such a scheme should have the property that it is generated by the transactions structure of the economy under study, rather than being given by an arbitrary rule.

All of these questions have been subsequently analysed by more recent contributors to the T.E.D. whose contributions will be examined in greater detail in later chapters.

The area of N-K.M. also raises many new questions by examining well known macroeconomic problems in a new context. Many disequilibrium models are based upon either the Drèze or Benassy equilibrium concepts and adopt the methods developed by Malinvaud (1977) and Muellbauer and Portes (1978). In these models a representative consumer and producer are adopted to model the behaviour of the aggregates from which they are drawn. The representative agents perform their choice calculus at non-Walrasian prices, supplies and demands do not match and quantity constraints arise. There are two markets, labour and consumption good, and thus four possible regimes may be defined according to which agent is constrained on which market. The case where consumers are constrained on both markets is called classical, where firms are constrained on both markets, under-consumption, where firms are constrained on the goods market and households on the labour market, Keynesian, and where firms are constrained for labour and households for goods, repressed inflation. Malinvaud points out that more markets mean more regimes.

This division of the aggregate macroeconomic model raises serious questions in the area of public finance as policies effective on one regime may actually be damaging upon another. Further, successive application of a particular policy instrument may at first alleviate a problem, for example, an increase in government expenditure will reduce involuntary unemployment on the Keynesian regime, but will then generate problems due to a regime switch. A switch from the Keynesian to the repressed inflation regime will occur in our example, with the government denying consumption goods to consumers. Similar complexities will be encountered with monetary or taxation instruments. Muellbauer and Portes, and Dixit (1976) provide good discussions of such problems.

A second important area where N-K.M. raise many questions is in the consideration of the formation and effects of expectations. Current transaction decisions will be based upon expectations of both the prices and the associated vector of quantity constraints. These expectations may well have a bootstrap effect as considered by Neary and Stiglitz (1979).

The policy implications of N-K.M. and the role of expectations in such models will be examined in greater detail in the chapters which follow. The reader who is interested in obtaining a more extensive overview of the questions raised above is referred to Drazen (1980).

1.3 Main Themes of the Thesis

The preceding sections 1.1 and 1.2 have given a brief discussion of why an integration of micro and macro theory should be considered desirable, and also presented a simple discussion of some of the early literature in the area, the questions the literature has asked and the concepts developed. In the rest of this thesis some of these questions will be developed further, and where appropriate the more recent literature will be examined in greater detail.

Two basic themes run through this thesis, first the role of expectations in Neo-Keynesian economics, particularly how expectations effect equilibria achieved through a quantity tatonnament process, and second, the various explanations for price rigidity or sluggish adjustment will be considered, with special emphasis placed upon the role of trade unions and bargaining as determinants of the wage rate.

Chapter 2 examines the various roles that expectations have performed in contemporary analysis. It is noticed that essentially there are two types of disequilibrium or non-Walrasian equilibrium models in the literature, those where the equilibrium is conceptually based upon expectations being validated (or self-validating), and approaches when rigidities rather than expectations, are the central feature. A simple stylized model of the Malinvaud (1977) type is developed which has the characteristics that initial transactions demands are based upon constraint expectations held with subjective certainty, and that these offers once made may only be revised downwards in the short-run. The rationale for considering such a model, its potential long run characteristics and its policy implications are examined and discussed. In an appendix to chapter 2 it is demonstrated that if agents in this class of model hold expectations then the trade vectors which are typically termed Walrasian equilibria are in fact Hicksian Temporary competitive equilibria, and not Walrasian in the sense of Debreu (1959).

Chapter 3 examines the introduction of uncertainty and adjustment costs into Disequilibrium models. The methods and costs of obtaining information and the way in which adjustment may take place in a quantity tatonnement process are considered. A model is developed in which workers form subjective probability distributions of expected labour demand and firms form subjective probability distributions of expected goods demand, where initial transaction demands are generated from Von-Neumann Morgenstern objective functions. Final market outcomes are derived in the face of adjustment costs which agents encounter in revising their demands.

Chapter 4 examines the treatment of expectations in tatonnement and non-tatonnement models with particular emphasis upon sequential trading. Dynamic analysis is performed on a representative firm-representative consumer sequential trading model. The model's stability properties are examined and it is found that although government policy may be used to stabilise the economy, this may be at a 'low level' equilibrium with high unemployment.

Chapters 5, 6 and 7 are concerned with the second theme of the thesis, the examination of why prices and most particularly wages do not adjust to clear markets. Chapter 5 considers the various approaches that have or perhaps may be adopted to endogenize prices in Neo-Keynesian models. These approaches are evaluated in the context of what might be considered some desirable properties for a price adjustment mechanism. Special attention is given to implicit and explicit contracts in the labour market.

In chapter 6 a simple model of labour market bargaining is developed and a number of solutions and their comparative static properties are derived. The effects of rationing, in both the goods market and labour markets, upon the bargain struck between a union and employer is considered.

Finally in chapter 7 several simple models are developed to examine the effects of introducing bargaining into a Neo-Keynesian type economy. Three basic scenarios are studied. (i) The whole labour-force is assumed unionised and an efficient bargain is continuously struck, in one case with a fixed product price, and in another where the product price adjusts to clear the market for output. (ii) The labour force and hence the economy is divided into a unionised and non-unionised sector producing different output. The prices of outputs and non-unionised labour adjust to clear their respective markets. (iii) Is the same as (ii) but makes the further assumption that all prices are fixed in the short run. Attention is focused upon the effects of government policy in these models and the impact of bargaining upon their comparative static properties.

In ,an appendix to the main thesis, there is a brief review of the literature on open disequilibrium models.

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2. THE ROLE OF EXPECTATION IN NEO-KEYNESIAN MACROECONOMIC MODELS

2.1 The Treatment of Expectations in the Literature

In an Arrow-Debreu world, expectations have no role, prices for all contingent commodities exist, and clear their respective markets. Agents form expectations when information is incomplete. In a Temporary Competitive Equilibrium of the Hicksian sort discussed earlier, information is missing about the market - clearing price in some future markets. The absence of a complete intertemporal price vector requires agents, in carrying out transactions today, to form price expectations for the future. This sort of expectations formation occurs in all macroeconomic models of market economics that have an intertemporal aspect.

If however, those prices that do exist today do not clear current period markets then expectations take on an even more important role, since associated with any expected price vector is also a set of feasible trades. Price and quantity expectations are formed. In the literature on Neo-Keynesian Macroeconomics models, there are two distinct forms of models analysed and one of the key distinguishing features between the two is their treatment of expectations. These two types may be called exogenous and endogenous price models. Consider first exogenous price models, such as Malinvaud (1977), Muellbauer and Portes (1978), Neary and Stiglitz (1979) and Honkapohja and Ito (1979). These are fix-price quantity rationing models based upon the Benassy (1975) equilibrium formulation. A representative consumer - representative producer form is adopted, and the economy operates as follows.

At the start of the market period the fix-price vector is announced and the representative agents then compute and announce their supplies and demands. If the plans are not mutually consistent, an auctioneer announces a vector of constraints. Agents then re-evaluate their demands subject to the constraint vector and announce their new set of plans. This process continues until both agents achieve transactions consistant with the constraints they face. Expectations impinge on such a process in two ways. Firstly expectations of price and quantity constraints held in the previous period about the current period affect the current period equilibrium via the endowments carried forwards. Secondly expectations of next periods prices and quantity constraints determine the amounts of goods and money agents wish to carry forwards, and thus effect the current period equilibrium.

Malinvaud presents a model in which expectations are implicit. He assumes consumers maximize indirect utility functions expressed over an aggregate consumption good, leisure and end of period money balances. The money balance term being included 'because it will permit future consumption' (Malinvaud (1977, p.21). The expectations upon which the money holding decision is based are not specified. The importance of Malinvauds implicit assumptions about expectations is demonstrated by Hildenbrand and Hildenbrand (1978)[1] who show that consumers are in fact maximising a two period Cobb-Douglas utility function of the following form:

$$\text{Max } u(x_1,X_2,L_1,L_2) = \alpha_1 \ln x_1+\alpha_2 \ln x_2+\beta_1 \ln(T-L_1)+\beta_2 \ln(T-L_2) \quad (2.1.1)$$

$$\text{s.t} \qquad p_1x_1 + M_1 = w_1L_1 + e_1 \quad (2.1.2)$$

$$p_2x_2 + M_2 = w_2L_2 + e_2 \quad (2.1.3)$$

For this problem to collapse to the one examined by Malinvaud the following assumptions are required:

$\alpha_1 = 2$, $\alpha_2 = \beta_1 = 1$, $e_2 = 0$, $M_2 = 0$, $p_1 = p_2$, $w_1 = w_2$

and $L_2 = 0$. Such that (2.1.1) can be collapsed to (2.1.4)

$$V(x_1, L_1, M_1, p, w) = x_1^2(T-L_1)(M_1/p) \tag{2.1.4}$$

The role of expectations has now been made explicit, at the start of the first period consumers expect prices and wages to remain constant throughout both periods, and they expect to be unemployed in the second period. There is no link between current period market experience and expectations, indeed even if the worker sells his chosen labour supply today he still anticipates being totally unemployed tomorrow. Muellbauer and Portes (1978)[2] make expectations one of the central themes of their work. In their analysis of households both deterministic and probabalistic approaches are examined.

A two period utility function is defined over consumption and leisure.

$$U^H = U(x_1, T_1-L_1, x_2, T_2-L_2) \tag{2.1.5}$$

Given that the household has no use for money balances at the end of the second period, the second period budget constraint is:

$$M_1 + e_2 + w_2L_2 \geq p_2x_2 \tag{2.1.6}$$

There are two further possible constraints in the second period, the consumer may be rationed for labour $L_2 = \bar{L}_2$ and/or consumption goods $x_2 = \bar{x}_2$

$$L_2 = \bar{L}_2 \tag{2.1.7}$$

$$x_2 = \bar{x}_2 \tag{2.1.8}$$

The household in period 1 must have expectations on prices w_2, p_2, quantity constraints $\bar{\bar{L}}_2, \bar{\bar{x}}_2$ and endowments e_2. Each expectation is held with subjective certainty.

In general the consumer may expect one of four outcomes in the second period, either, neither, one or both of (2.1.7) and (2.1.8) to bind. The case of both constraints in the second period being expected to bind does not make sense in a two period model, but would do so if a third period was added.[3]

The households maximization problem for period 2 is then solved conditional upon x_1 and L_1, to yield the three following conditional indirect utility functions.

(i) $L_2^s < \bar{\bar{L}}_2$ and $x_2^d < \bar{\bar{x}}_2$ gives

$$U^H = V(x_1, T_1-L_1, x_2^d\left[(M_1+e_2)/p_2, w_2/p_2, x_1, T_1-L_1\right],$$
$$T_2-L_2^s\left[(M_1+e_2)/p_2, w_2/p_2, x_1, T_1-L_1\right]) \qquad (2.1.9)$$

(ii) $L_2^s > \bar{\bar{L}}_2$ and $x_2^d < \bar{\bar{x}}_2$ yields

$$U^H = V(x_1, T_1-L_1, (M_1+e_2+w_2\bar{\bar{L}}_2)/p_2, T_2-\bar{\bar{L}}_2) \qquad (2.1.10)$$

(iii) $L_2^s < \bar{\bar{L}}_2$ and $x_2^d > \bar{\bar{x}}_2$ gives

$$U^H = V(x_1, T_1-L_1, \bar{\bar{x}}_2, T_2 - (p_2\bar{\bar{x}}_2 - M_1 - e_2)/w_2) \qquad (2.1.11)$$

There are four possible constraint combinations in the first period, and three alternative utility functions for each dependent upon expectations, yielding 12 possible behaviour regimes for period 1. An n period model would have $3(4^{n-2})$ behaviour regimes for the first period. As Muellbauer and Portes point out this approach to expectations is not very tractable. Problematic also is the way period 2 expectations are formed as there is no link between the wages, prices and trades experienced in period 1, and those expected for the second period.

Muellbauer and Portes suggest that a probabalistic Von Neumann-Morgenstern approach is more attractive. The method adopted is as follows: a transformation of the utility function (2.1.5) is chosen, the expected value of which will be the new utility indicator to be maximized. Its arguments will be x_1, T_1-L_1, M_1, T_2 and the parameters which generate the probabalistic expectations of $e_2, p_2, w_2, \bar{x}_2$ and $\bar{L}_2$. The effects of x_1 and L_1 on expectations are absorbed into the utility function, and the other parameters which govern expectations p_1, w_1, e_1 are represented by the vector θ.

Thus the expected indirect utility function may be written:

$$U^H = V\,(x_1, T_1-L_1, M_1, T_2, \theta) \qquad (2.1.12)$$

The current period budget constraint may be written:

$$M_1 = d_1+M_o+w_1L_1-p_1x_1 \qquad (2.1.13)$$

Substituting (2.1.13) into (2.1.12) yields:

$$U^H = V(x_1, T_1-L_1, d_1+M_o+w_1L_1-p_1x_1T_2, \theta) \qquad (2.1.14)$$

d_1 is dividends distributed at the end of the period. The indifference curves generated by (2.1.14) are ellipsoid in (x_1, L_1) space as figure (2.1.1)

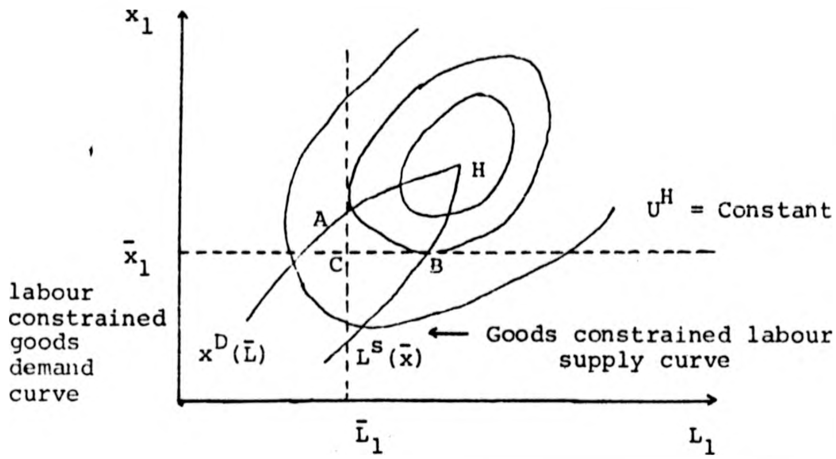


Figure (2.1.1)

This is half of the well known Muellbauer and Portes 'Double wedge' diagram, where point

A is the consumers goods demand if constrained to sell $\bar{L}_1$ units of labour

B labour supply if rationed for goods at $\bar{x}_1$

C position of 'forced saving' when constrained on both markets.[4]

H is the consumers 'bliss' point where he is unconstrained on either market.

The role of expectations here is fundamental. The indifference curves have an ellipsoid shape because future consumption is represented in the utility function (2.1.12) by end of period money balances, and because expectations of future price and quantity constraints are represented by θ and also absorbed into the x_1 and L_1 terms. The ellipsoid shaped indifference curves based upon probabalistic expectations generate the 'consumers wedge'. A 'producers wedge' is then constructed in a similar fashion. An additively separable two period probabalistic profit function of the following form is examined.

$$\text{Max } U^F = f_1[p_1x_1 - w_1L(y_1)] + f_2^*[I_1, L(y_1), x_1, \psi] \qquad (2.1.15)$$

$$\text{[illegible]} \quad I_1 = h_1(I_0) + y_1 - x_1 \quad I_1 \geq 0, y_1 \geq 0, x_1 \geq 0 \qquad (2.1.16)$$

Substituting (2.1.16) into (2.1.15) yields

$$U^F = f_1[p_1x_1 - w_1L_1] + f_2^*[h_1(I_0) + y(L_1) - x_1, L_1, x_1, \psi] \qquad (2.1.17)$$

Equation (2.1.17) expresses the firms current profit f_1, and expected profit f_2^*, as a function of current endogenous variables employment L_1 and sales x_1, and exogeneous variables I_O and the exogenous vector of [illegible] that component of expectations on L_2, x_2, [illegible] which does not depend on x_1 and L_1. The interperiod link for the firm is inventories, where end of period inventory holdings are determined by current market experience and expectations. Equation (2.1.17) generates ellipsoidal firms indifference (multi-period discounted expected profit) curves and thus a producer's wedge in a manner similar to the consumers problem, and as represented below in figure (2.1.2).

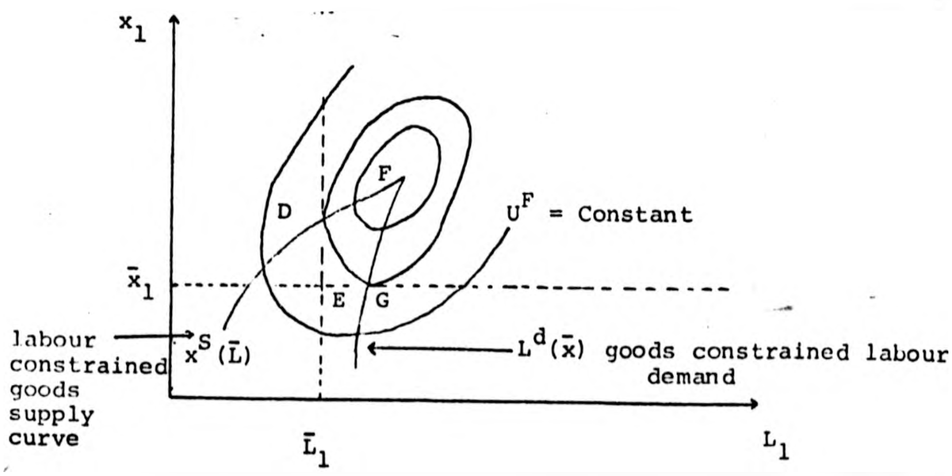


Figure (2.1.2)

A point such as

D is the firms goods supply if constrained by $\bar{L}$ units of available labour

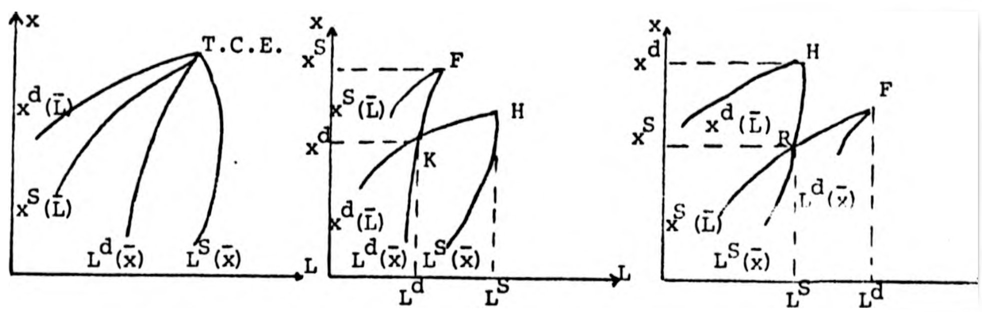
G Labour demand if constrained for sales at $\bar{x}_1$

E Position of 'forced inventory accumulation', when constrained both for current labour and sales.[5]

F The producers bliss point where he is unconstrained on either market.

Combining the two analyses allows a representation of five different temporary equilibria in sales employment space.[6]

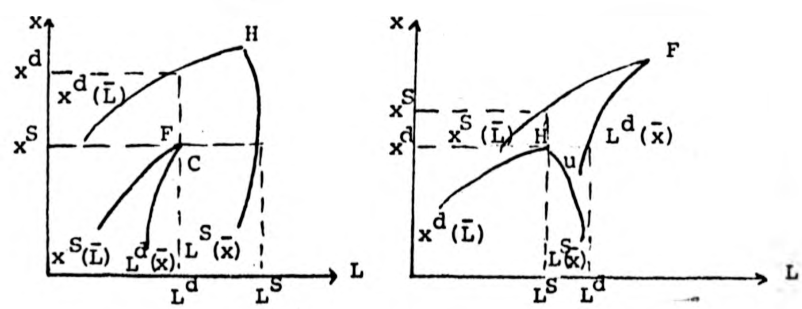
Figure (2.1.3)



(a) Temporary Competative Equilibrium in quantity space

(d) Keynesian Unemployment

(c) Repressed Inflation



(d) Classical Unemployment

(e) Underconsumption

A proof that the equilibrium illustrated in figure (2.1.3)(a) is a Temporary Competitive Equilibrium and not Walrasian equilibrium as claimed in Muellbauer and Portes (1978) is provided in an appendix to this chapter.

In figure (2.1.3) it is clear that the type of equilibrium which obtains, its exact position in quantity space and its comparative static properties, will depend upon the position and shape of the two wedges. The role of expectations in this model is central to generating these results, expectations provide the rationale for the inter-temporal transfer of resources, and it is this transfer possibility which provides firms, for example, with the option of not relating sales directly to employment via the production function.

It should be noted that the equilibria represented in figure (2.1.3) are defined for a given set of expectations based upon the equilibrium conditions. If the comparative static exercise of shifting either the F or H system is carried out, then expectations will be modified and the wedges need to be redrawn. Muellbauer and Portes also suggest that this expectations effect may modify or reverse the comparative static effects of changes in government policy. In principle the Muellbauer Portes treatments of expectations may be extended to an n agent m period model similar to that of Benassy (1975).

Neary and Stiglitz (1981) examine the role of expectations in a two period model, two types of expectations are examined: arbitrary and rational constraint expectations. A model similar to Barro and Grossman (1971) is extended to two periods, and is used to examine how expectations of quantity constraints in the second period effect the type of equilibrium that will obtain in the first. When constraint expectations are arbitrary, in the sense that they are exogeneously given, Neary and Stiglitz demonstrate three interesting results. Firstly that the vector of current and expected future prices which clears current period markets is not unique, rather a different vector will be required with each different given configuration of constraint expectations. Secondly equilibria in effective demands are not uniquely determined by current and expected prices, a particular fixed price vector may be consistant with either 'Keynesian' or 'Classical' unemployment regimes in the current period, given different constraint expectations in future periods. Thirdly, exogeneous constraint expectations may have a 'bootstrap' effect, in the sense that the expectation of, say, a Keynesian regime in the second period will cause households to increase saving and firms to lower employment in the first period, thus making a Keynesian regime more likely in the first period.

Neary and Stiglitz also study what they term rational constraint expectations. Constraint expectations are said to be rational if in the first period, household and firms expectations of the constraints that they will face in the second period, are consistant with the transactions that the other side of the market plans to carry out. It is demonstrated that rational constraint expectations have the same 'bootstrap' characteristics as discussed above. The obvious question to ask is; if agents have sufficient information of other agents plans that constraint expectations are rational, in the above sense, why do not workers and firms communicate and generate Walrasian equilibrium? Neary and Stiglitz argue that agents only know the aggregate plans that the other side of any particular markets wishes to carry out next period. Thus there may not be sufficient detailed information available to facilitate price adjustment bargaining.

In the Neary and Stiglitz approach expectations are assumed to be held with certainty. Honkapohja and Ito (1979) examine a model based on the recent literature of models with trading uncertainty or stocastic rationing schemes.[8] Consumers and producers attach probabilities to their succeeding or failing to achieve their initial trade offers. The agents calculate these probabilities by observing disequilibrium signals defined by (2.1.18) and (2.1.19)

$$u = L^s/L^d \qquad (2.1.18)$$

$$v = x^s/x^d \qquad (2.1.19)$$

Honkapohja and Ito hypothesise that each agent is a signal taker as well as a price taker, and describe how agents cope with trading uncertainty as below:

Goods Market

demand side: $\text{actual trade} = \begin{cases} (s)x(\text{offer}), \text{ with probability } \psi \\ \text{offer, with probability } 1-\psi \end{cases}$

Supply side: $\text{actual trade} = \begin{cases} (z)x(\text{offer}), \text{ with probability } \Lambda \\ \text{offer, with probability } 1-\Lambda \end{cases}$

Labour Market

demand side: $\text{actual trade} \begin{cases} (t)x(\text{offer}), \text{ with probability } \theta \\ \text{offer, with probability } 1-\theta \end{cases}$

supply side: $\text{actual trade} \begin{cases} (q)x(\text{offer}), \text{ with probability } \eta \\ \text{offer, with probability } 1-\eta \end{cases}$

where $s=s(v)$, $z=z(v)$, $r=r(u)$, $q=q(u)$ and $\psi=\psi(v)$, $\Lambda=\Lambda(v)$

$\theta=\theta(u)$, $\eta=\eta(u)$

and $r(u)=1$ for $u\gtrless 1$, $q(u)=1$ for $u\geq 1$, $s(v)=1$ for $v\geq 1$

and $z(v)=1$ for $v\leq 1$

Each agent bases his trade offers on the maximization of expected utility, subject to the probabalistic expectation ψ,Λ,θ,η . A stocastic rationing equilibrium is defined by Honkapohja and Ito as 'a pair of disequilibrium signals u,v which induce agents to express effective demands and supplies which reproduce the signals'. In this model expectations play a different role than in those previously examined as here expectations are based upon agents observing aggregate disequilibrium signals and attaching probabilities to the different outcomes they may achieve. This model requires that aggregate effective demands and quantity constraints (signals) replicate themselves. Prices are exogenous and presumably, although this is not examined in the paper, when prices do change a new period begins and a new equilibrium is established. Since households carry money stocks forwards, these must be based upon expectations in a manner similar to the models examined previously.

In exogenous price models expectations are formed on the basis of current and previous market experience, and are used by economic agents to decide what stocks to carry forwards. A Temporary equilibrium in exogenous price models requires that effective demands and quantity constraints replicate themselves.[9] Expectations do not need to be 'exact' in the sense of Michel (1980).[10] The confirmation of expectations plays no role in defining the equilibrium.

In endogenous price models the validation of expectations is central to defining the equilibria.[11] A simple but interesting illustration of this approach in the following model due to Varian (1977).

Let price setting firms have point expectations of the demand for their output $\bar{\bar{x}}$ which will constrain their demand for labour, by an inverse production function, to $f^{-1}(\bar{\bar{x}})$ if $w/p<\partial L/\partial x$ at $\bar{\bar{x}}$, if not their labour demand will be of Walrasian form $L^d(w/p)$.

Thus firms constrained labour demand functions may be written

$$L^d(w/p,\bar{\bar{x}}) = \text{Min} \begin{cases} f^{-1}(\bar{\bar{x}}) \text{ if } w/p<\partial L/\partial x \text{ at } \bar{\bar{x}} \\ L^d(w/p) \text{ otherwise} \end{cases}$$

Let households have a consumption function out of profit income $x^\pi(w/p,\bar{\bar{x}})$ and a Walrasian labour supply $L^s(w/p)$. It is assumed there are no savings and thus their Walrasian demand for consumption may be written $(w/p)L^s(w/p)+x^\pi(w/p,\bar{\bar{x}})$.
The actual amount of labour that households may sell is

$$L(w/p,\bar{\bar{x}}) = \min \begin{cases} L^s(w/p) \\ L^d(w/p,\bar{x}) \end{cases}$$

Therefore effective demand for consumption will be

$$x(w/p,\bar{\bar{x}}) = w/p\ L(w/p,\bar{\bar{x}}) + x^\pi(w/p,\bar{\bar{x}})$$

The economy may now be described by the following dynamical system.

If demand for labour is not equal to supply then the nominal wage rate adjusts as:

$$\dot{w} = L^d(w/p,\bar{\bar{x}}) - L^s(w/p) \qquad (2.1.20)$$

If expected demand for output is not equal to actual effective demand expectations will be revised.

$$\dot{\bar{\bar{x}}}' = x(w/p, \bar{\bar{x}}) - \bar{\bar{x}} \qquad (2.1.21)$$

If actual demand for output is less than effective supply then prices will change.

$$\dot{p} = x(w/p, \bar{\bar{x}}) - f(L(w/p, \bar{\bar{x}})) \qquad (2.1.22)$$

An equilibrium in Varians model is a real wage, output expectation pair $(w/p, \bar{\bar{x}})$ which equates (2.1.20), (2.1.21) and (2.1.22) to zero, such that (a) expected demand is equal to actual demand so that expectations are static; (b) actual consumption demand equals its supply, so that the price level [illegible] the actual [illegible] conditional demand so that nominal wage rate is constant. Illustration of one such equilibrium is given below in figure (2.1.4).

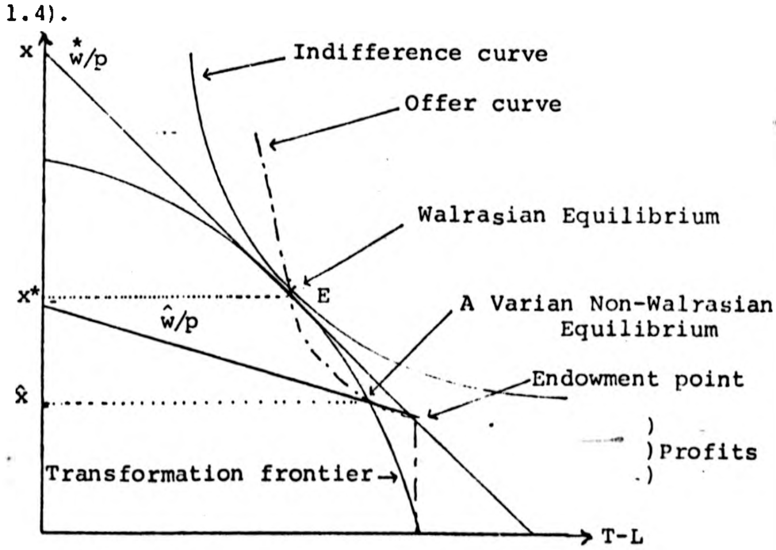


Figure (2.1.4)

Define the profit consumption function $x^{\pi}(w/p, \bar{\bar{x}}) = \overset{*}{x}{}^{\pi}$ to be compatible with Walrasian equilibrium. Then the offer curve must pass through the endowment point and through the transformation frontier at two points, one of which is the Walrasian equilibrium denoted $(\overset{*}{w}/p, \overset{*}{x})$, the other is the non-Walrasian equilibrium $(\hat{w}/p, \hat{x})$.[12] At the non-Walrasian equilibrium firms pessimistic expectations of demand lead them to employ very little labour, there is a low real wage $\hat{w}/p$, thus households supply little labour and have a low demand for goods. There is then an equilibrium in the effective demand system.[13]

This is clearly a 'bootstrap' type of model in which expectations are essential to the definition of equilibrium. Notice however, that the bootstrap effect is different from that presented by Neary and Stiglitz. In that model expectations of a particular regime tomorrow made that outcome more likely to obtain today, but in this case expectations of demand today have a bootstrap effect on todays equilibrium. There are, as Drazen (1980)[14] points out, some problems with Varian's model. Essentially the problem is that firms believe that cutting their prices, when price is greater than marginal cost, will not increase demand for their output. This is because they expect the following to happen, if one firm cuts price all others will follow suit, real wages will rise and households will increase their labour supplies and the nominal wage will be bid down. Nominal wages and prices will be lower but no real variables will be effected, thus the firms will not bother. The problem is that there are no money balances in the model, and no real balance effect is associated with a price cut. If these were present producers expectations would be that a price cut will increase their sales.

Heller and Starr (1978) consider a model which is in many respects a multigood version of Varians. Their treatment of expectations is however slightly different. The unemployment equilibrium they examine is a Nash equilibrium, thus once each agent has achieved a constrained maximization plan no attempt to break constraints is attempted, because all other agents are observed to be in equilibrium, and each agent then takes all others behaviour as given. Therefore, expectations in the Heller Starr model perform differently than in the Varian treatment. In Varian it is an expectation of a particular transaction level which if achieved generates the equilibrium concept, in Heller and Starr it is the expectation that others will not change their behaviour which generates a Nash type equilibrium.

Neglishi (1974, 1978, 1979) presents an endogeneous price model in which firms and consumers perceive kinked demand curves for goods and labour respectively. Firms perceive a kinked demand for their good since they believe that if they raise the price of their good all consumers will immediately purchase from a different supplier, but if they lower price imperfect information will mean that not all consumers will attempt to switch their purchases to the supplier in question. A worker perceives a kinked demand curve for his services since a rise in the wage he charges will cause firms to employ another, but a cut in his reservation wage will not ensure employment, because firms will be reluctant to hire at a lower wage rate as this will cause dissatisfaction and a lowering of productivity amongst other workers. Prices are endogenous but sticky with consequent quantity rationing. Expectations, here the perceived demand curves, are central to the definition of a non-Walrasian equilibrium. Given a starting point which must be on the kink of the demand curve, workers and firms attach probabilities to achieving sales and employment at lower respective prices. Both sides of the market base their behaviour, price quoting and quantity transactions, upon the maximization of their expected objective functions and when actual transactions validate expectations an equilibrium is achieved. This implies that the requirement for an equilibrium in Negishi's model is only that the actual and perceived demand curves intersect, away from the equilibrium trade on a market all expectations may be incorrect. Consider figure (2.1.4) which illustrates equilibrium on the labour market.

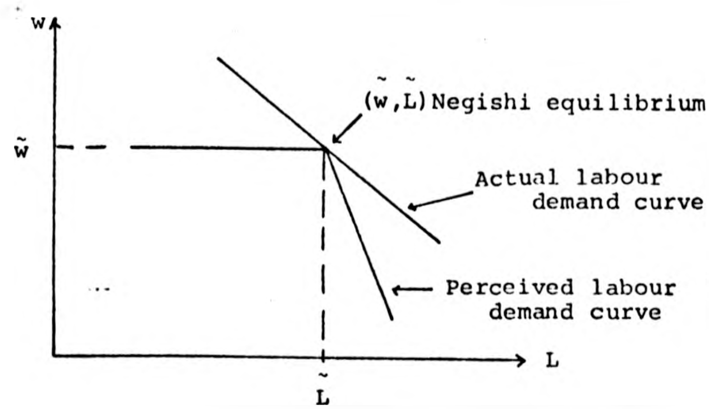


Figure (2.1.4)

As Drazen (1980) points out, the equilibrium does not require actual demand curves to have kinks and it is not entirely clear why economic agents should perceive kinked demand curves.[15]

Futia (1977) presents an approach in some ways similar to Negishi's but which appears more intuitively reasonable. Firms are assumed to find it too costly to search the whole labour force to find the workers quoting the lowest reservation wages. They therefore, sample a random subset of the labour force and employ members of the subset up to point where the marginal product of labour is equal to the expected average real wage. Given firms behaviour households quote a whole distribution of reservation wages (not one common wage as in Negishi), because they perceive a trade off between the probability of employment and the wage rate they quote. Quoting a very high reservation wage will not reduce the probability of employment to zero since the worker may be lucky enough to find himself in the labour pool of a firm which has randomly selected many workers also quoting high reservation wages. A 'Keynesian' unemployment equilibrium in Futia's model is defined by a distribution of reservation wages, an output price level, and a level of exogenous demand together with an aggregate average real wage which validates firms expectations. Such an equilibrium is shown to exist at different levels of exogenous demand.

In the two types of models examined, expectations play fundamentally different roles. In the exogenous price quantity tatonnement models there is no requirement that expectations be correct for an equilibrium to exist. Expectations held in previous periods determine current period endowments, and expectations of future periods determine the amount of goods or income agents wish to transfer for consumption in future periods. The economy will be stationary over the period even if expectations are invalidated. In the second class of Neo-Keynesian macroeconomic models examined correct expectations are the essential feature of the equilibria. When expectations are realised agents tend to refrain from further market experimentation even if mutually beneficial transactions are possible. Therefore equilibria with an under usage of resources arise.

2.2 Expectations and Limited Quantity Adjustment

In the previous section 2.1 a discussion was presented of how expectations are treated in the literature. In this section I present an exogenous price model which has the novel characteristic that expectations held about the rations agents will face in the current period affect the actual current period equilibria.

The economy to be studied behaves as follows. At the end of any market period agents hold expectations of the quantities they will be able to trade in the next period. The money balances they carry forwards are therefore based upon expected supply and demand curves. At the start of the next market period fixed prices are announced and from their expected demand and supply curves agents calculate their expected upper transaction bounds, they base their initial transaction offers upon these expected constraints. In a standard treatment such as Muellbauer and Portes these initial non-Walrasian transaction offers are unimportant since it is assumed that there is sufficient flexibility within each market period for agents to adjust their behaviour on each market to be consistant with the quantity constraints experienced on others. Thus in the conventional treatment the starting point of the tatonnement is unimportant. In the economy we study below it is argued that such flexibility is less than perfect particularly in the upward direction. It is argued that it is easier for an agent to revise transactions downwards, since this requires only that he abstains from some trades, than it is to revise transactions upwards, since this will require that he locates an agent on the other side of the market who also wishes to revise his trades in the appropriate way.[16]

To simplify the analysis we shall work with a representative consumer, representative producer model, such as in Muellbauer and Portes (1978), Ellis (1980) and Ito (1980). Two strong simplifying assumptions will be made:

(A1) Expectations are held with subjective certainty.

(A2) In a given market period transaction demands may only be revised downwards.

These assumptions are justified since they allow an easy first approach to several interesting and complex questions.

The Microeconomic Model

(a) Consumers

Assume the representative consumer maximizes a single period Cobb-Douglas utility function.

$$V = x^{\alpha}(T-L)^{\beta}\ (M_1/p)^{\gamma} \qquad \alpha+\beta+\gamma=1 \qquad (2.2.1)$$

subject to a budget constraint

$$M_o + wL = px + M_1 \qquad (2.2.2)$$

Solution of this problem without quantity constraints yields the notional supplies and demands

$$\tilde{x} = \alpha\ \left(\frac{M_o+wT}{p}\right) \qquad (2.2.3)$$

$$\tilde{L} = (\alpha+\gamma)T - \beta\left(\frac{M_o}{w}\right) \qquad (2.2.4)$$

$$\tilde{M}_1 = \gamma(M_o+wT) \qquad (2.2.5)$$

Here subscripts o and 1 refer to beginning and end of period variables.

Next consider consumers expectations as described by his expected labour demand and goods supply functions (2.2.6) and (2.2.7):

$$\bar{\bar{L}} = L(w/p) + A \qquad (2.2.6)$$

$$\bar{\bar{x}} = x(w/p) + B \qquad (2.2.7)$$

where A and B are treated as constants in the current period. Their adjustment over time will be examined in a later section.

Utilising the technique of Benassy (1975), discussed in the introduction, we may now calculate the consumers <u>initial</u> labour supply and <u>initial</u> goods demand given (2.2.6) and (2.2.7) and fixed prices w and p.

To calculate the consumers goods demand given the expected labour supply curve we solve the following programme.

$$\left.\begin{array}{ll} \text{Max} & V = x^{\alpha}(T-L)^{\beta}\ (M_1/p)^{\gamma} \\ \text{S.T} & M_o + wL \geq p.x + M_1 \\ & L \leq \bar{\bar{L}} = L(w/p) + A \end{array}\right] \qquad (2.2.8)$$

From the first order conditions of (2.2.8) the choice of x and M_1 when $L = \bar{\bar{L}} = L(w/p) + A$ we get:

$$x = \frac{\alpha}{\alpha+\gamma} \left(\frac{M_o + w\bar{\bar{L}}}{p} \right) \qquad (2.2.9)$$

$$M_1 = \frac{\gamma}{\alpha+\gamma} (w\bar{\bar{L}} + M_o) \qquad (2.2.10)$$

due to a manipulation first noticed by Ito (1980) we rewrite (2.2.9) as (2.2.11)

$$x = \tilde{x} + \left(\frac{\alpha}{\alpha+\gamma} \right) \frac{w}{p} \left[\bar{\bar{L}} - \tilde{L}\right] \qquad (2.2.11)$$

(2.2.11) is the representative consumers initial goods demand if he expects to be constrained on the labour market. As the expression illustrates this is comprised of two components, the notional goods demand, and the expected spillover effect.

From the Kuhn-Tucker condition (for a formal derivation see Ellis (1980)) we may write:

$$x = \begin{cases} \tilde{x} & \text{if} \quad w < \dfrac{\beta M_o}{(1-\beta)T-\bar{\bar{L}}} \\ \tilde{x} + \left(\dfrac{\alpha}{\alpha+\gamma}\right)\dfrac{w}{p}\left[\bar{\bar{L}}-\tilde{L}\right] & \text{otherwise} \end{cases} \qquad (2.2.12)$$

This may be interpreted as that the consumer always bases his initial goods purchase offer on the minimum of his notional and constrained demands.

Expressing (2.2.3) and (2.2.11) together in (p/w,x) space holding p fixed clarifies the proceding analysis.

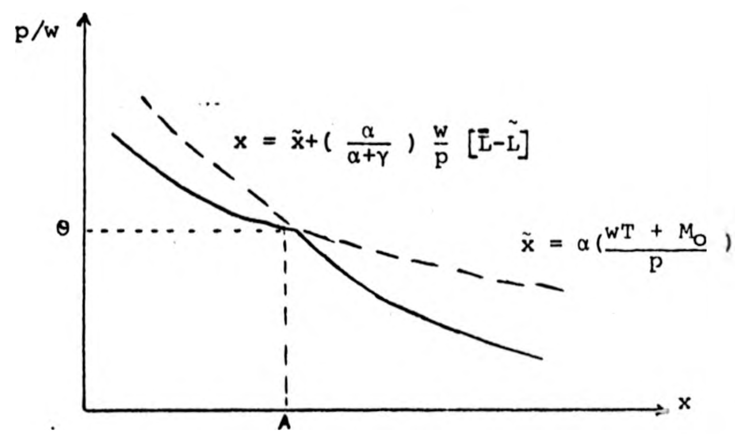


Figure (2.2.1)

At the point (θ,A) in figure (2.2.1) both the notional and constrained goods demand curves yield the same initial goods demand offer, where $\theta = \frac{\beta M_o}{p(1-\beta)T-L}$. Above (θ,A) offers are made according to the notional curve defined by (2.2.3) and below (θ, A) offers are made according to the expectations constrained curve as defined by (2.2.11). Hence the heavy line in figure (2.2.1) is the consumers initial goods demand curve. This is termed the 'expectational' goods demand function and may be written.

$$x = E_1(p, w, M_o, \bar{\bar{L}}) \qquad (2.2.13)$$

The consumers behaviour on the labour market given the expected goods supply function may be analysed in a similar manner. The programme being:

$$\left.\begin{aligned} &\text{Max} \quad V = x^{\alpha}(T-L)^{\beta}\,(M_1/p)^{\gamma} \\ &\text{S.T} \quad M_o + wL \geq px + M_1 \\ &\qquad\quad x \leq x(w/p) + B = \bar{\bar{x}} \end{aligned}\right] \qquad (2.2.14)$$

From the first order conditions of (2.2.14) the choice of L_1 and M_1 when $x = x(w/p)+B = \bar{\bar{x}}$ is:

$$L = \left(\frac{\gamma}{\beta+\gamma}\right)T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_o}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p\bar{\bar{x}}}{w} \qquad (2.2.15)$$

$$M_1 = \left(\frac{\gamma}{\beta+\gamma}\right)(M_o + wT - p\bar{\bar{x}}) \qquad (2.2.16)$$

where again using Ito (1980) we may rewrite (2.2.15) to emphasise the expected spillover effect as (2.2.17):

$$L = \tilde{L} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\,[\bar{\bar{x}} - \tilde{x}] \qquad (2.2.17)$$

(2.2.17) is the representative consumers initial labour supply if he expects to be constrained on the goods market.

From the Kuhn-Tucker condition of (2.2.14) we may write:

and below (θ, A) offers are made according to the expectations constrained curve as defined by (2.2.11). Hence the heavy line in figure (2.2.1) is the consumers initial goods demand curve. This is termed the 'expectational' goods demand function and may be written.

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$$\left.\begin{aligned} &\text{Max} \quad V = x^{\alpha}(T-L)^{\beta} \, (M_1/p)^{\gamma} \\ &\text{S.T} \quad M_o + wL \geq px + M_1 \\ &\qquad x \leq x(w/p) + B = \bar{\bar{x}} \end{aligned}\right] \qquad (2.2.14)$$

From the first order conditions of (2.2.14) the choice of L_1 and M_1 when $x = x(w/p)+B = \bar{\bar{x}}$ is:

$$L = \left(\frac{\gamma}{\beta+\gamma}\right)T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_o}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p\bar{\bar{x}}}{w} \qquad (2.2.15)$$

$$M_1 = \left(\frac{\gamma}{\beta+\gamma}\right)(M_o + wT - p\bar{\bar{x}}) \qquad (2.2.16)$$

where again using Ito (1980) we may rewrite (2.2.15) to emphasise the expected spillover effect as (2.2.17):

$$L = \tilde{L} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\left[\bar{\bar{x}} - \tilde{x}\right] \qquad (2.2.17)$$

(2.2.17) is the representative consumers initial labour supply if he expects to be constrained on the goods market.

From the Kuhn-Tucker condition of (2.2.14) we may write:

$$L = \begin{cases} \tilde{L} \text{ if } & w > (px - \alpha M_o)/\alpha T \\ \tilde{L} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\left[\bar{\bar{x}} - \tilde{x}\right] & \text{otherwise} \end{cases} \qquad (2.2.18)$$

This condition (2.2.18) states that the consumer bases his initial labour supply offer upon the minimum of his notional and constrained supplies. To illustrate this (2.2.4) and (2.2.17) are drawn together in (w/p,L) space, again represented for fixed p.

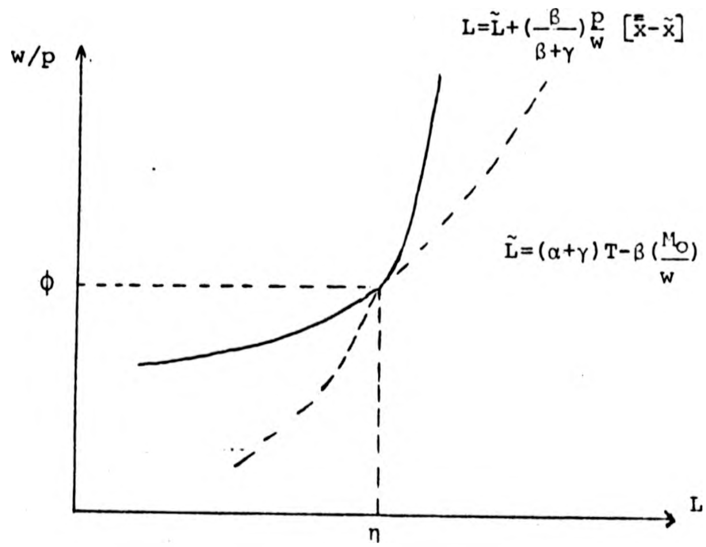


Figure (2.2.2)

At the point (ϕ,η) in figure (2.2.2) both the notional and constrained labour supply curves yield the same initial labour supply offer, where $\phi = (px - \alpha M_o)/p\alpha T$. Above ϕ offers are made according to the expectations constrained curve defined by (2.2.17) whilst below ϕ offers are made along the notional curve. The heavy line in figure (2.2.2) is termed the 'expectational' labour supply curve, and may be written:

$$L = E_2(p, w, Mo, \bar{\bar{x}}) \qquad (2.2.19)$$

Expressions (2.2.13) and (2.2.19) imply that the demand for end of period money balances at the time when initial offers are made on the good and labour markets will be of the form:

$$M_1 = E_3(Mo, p, w, \bar{\bar{L}}, \bar{\bar{x}}) \qquad (2.2.20)$$

Notice that in both the expectational labours supply and goods demand functions, it is the expected spillover effect which modifies initial transaction offers. An expected constraint on one market effects the other.

(b) Producers

The producer maximizes profit subject to a short run production function:

$$\left.\begin{aligned} &\text{Max } \pi = py - wL \\ &\text{S.T} \quad y = kL^{\delta} \qquad 0<\delta<1 \end{aligned}\right] \qquad (2.2.21)$$

where $\partial y/\partial L = \delta k L^{\delta-1} > 0$ and $\partial^2 y/\partial L^2 < 0$

The profit maximizing condition of (2.2.21) is the standard

$$w/p = \delta k L^{\delta-1} \qquad (2.2.22)$$

Manipulating (2.2.22) the real wage equals to the marginal physical product of labour condition yields the producers unconstrained labour demand and goods supply functions.

$$\tilde{L} = \left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{1}{1-\delta}} \qquad (2.2.23)(a)$$

$$\tilde{y} = kL^{\delta} = k\left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{\delta}{1-\delta}} \qquad (2.2.23)(b)$$

Assuming the producers holds expectations of labour supply and goods demand as (2.2.24) and (2.2.25):

$$\bar{\bar{L}} = L(w/p) + C \qquad (2.2.24)$$

$$\bar{\bar{y}} = y(p/w) + D \qquad (2.2.25)$$

Where C and D are treated as constants in the current period. The adjustment of C and D over time will be examined in a later section.

Producers are assumed to have no means of inter-period income transfer, we could allow inventories as in Muellbauer and Portes (1978) or retained money balances, then the constrained good supply and labour demand function would form a wedge. However, this provides further complications without yielding any conceptually new results to this model.

Using the production function, (2.2.24) and (2.2.25) we may write the producers constrained labour demand and goods supply functions as:

$$L = \left[\frac{y(p/w)+D}{k}\right]^{1/\delta} = \left[\frac{\bar{\bar{y}}}{k}\right]^{1/\delta} \qquad (2.2.26)$$

$$y = k\left[L(w/p) + C\right]^{\delta} = k\bar{\bar{L}}^{\delta} \qquad (2.2.27)$$

Notice that if the producer expects a constraint on one market this determines his behaviour upon both, since he must be on his production function. Thus his initial goods supply is:

$$y = \min\left[\bar{\bar{y}},\ k\bar{\bar{L}}^{\delta},\ k\left[\frac{w}{p}\cdot\frac{1}{\delta k}\right]^{-\frac{\delta}{1-\delta}}\right] \qquad (2.2.28)$$

rewrite (2.2.28) as:

$$y = E_4(p,w,k,\bar{\bar{y}},\bar{\bar{L}}) \qquad (2.2.29)$$

which may be represented in (p/w,y) space as figure (2.2.3)

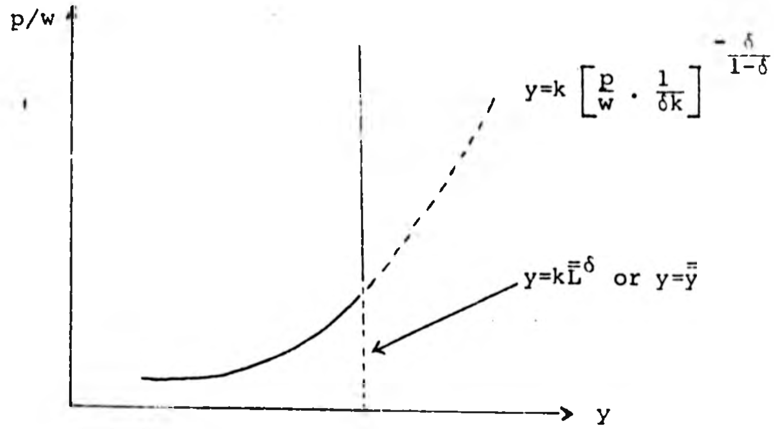


Figure (2.2.3)

in a similar manner the producers labour demand may be written:

$$L = \mathrm{Min}\left[\ \bar{\bar{L}}\ ,\ \left(\frac{\bar{\bar{y}}}{k}\right)^{1/\delta}\ ,\ \left(\frac{w}{p}\cdot\frac{1}{\delta k}\right)^{-\frac{1}{1-\delta}}\right] \qquad (2.2.30)$$

which we rewrite as the producers initial labour demand curve viz (2.2.31).

$$L = E_5(p,w,k,\bar{\bar{y}},\bar{\bar{L}}) \qquad (2.2.31)$$

Note that the producer cannot expect to meet binding constraints simultaneously on both markets.

Characterisation of Microeconomic Outcomes

Having examined how both representative agents calculate their initial transaction demands and supplies, all that is required to characterise market outcomes is an explanation of how initial demands and supplies are revised. The market mechanism is as described previously, once made market offers may only be varied downwards. Consequently two types of market outcome are possible, either both sets of agents achieve mutually consistant transactions or one achieves his desired trades and the other wishes to revise upwards.

It will be assumed for the moment that x=y, all output is available for consumption by households, so that the market outcome can be characterised by the following minimum conditions.

$$x^d = \begin{cases} E_1(p,w,Mo,\bar{\bar{L}},\bar{\bar{y}}) & \text{if } \bar{\bar{L}} < L^d \\ E_1^*(p,w,Mo,L^d,\bar{\bar{y}}) & \text{if } L^d \leq \bar{\bar{L}} \end{cases} \qquad (2.2.32)$$

$$x^s = \begin{cases} E_4(p,w,K,\bar{\bar{L}}) & \text{if } \bar{\bar{L}} < L^s \\ E_4^*(p,w,K,L^s) & \text{if } L^s \leq \bar{\bar{L}} \end{cases} \qquad (2.2.33)$$

$$x = \min(x^d, x^s) \qquad (2.2.34)$$

$$L^s = \begin{cases} E_2(p,w,Mo,\bar{\bar{x}}) & \text{if } \bar{\bar{x}} < x \\ E_2^*(p,w,Mo,x^s) & \text{if } x \leq \bar{\bar{x}} \end{cases} \qquad (2.2.35)$$

$$L^d = \begin{cases} E_5(p,w,k,\bar{\bar{y}},\bar{\bar{L}}) & \text{if } \bar{\bar{x}} < x^d \\ E_5^*(p,w,k,x^d,\bar{\bar{L}}) & \text{if } x^d \leq \bar{\bar{x}} \end{cases} \qquad (2.2.36)$$

$$L = \min(L^d, L^s) \qquad (2.2.37)$$

The pairs of expressions (2.2.32), (2.2.33), (2.2.35) and (2.2.36) have the same functional forms, and differ only in that the first equation in each pair the constraint is expected, whilst in the second it is actually generated in (2.2.34) and (2.2.37).

We see from (2.2.32)-(2.2.37) that either household or firms expectations may constrain the economic system.

Expectations and Macroeconomic Outcomes

To examine the macroeconomic implications of the preceding analysis, some accounting identities are required to define the structure of the economy.

$$y_t = x_t + g_t \tag{I1}$$

$$S_t = w_t L_t - p_t x_t \tag{I2}$$

$$\pi_t = p_t y_t - w_t L_t + G_t \tag{I3}$$

$$M_t = (M_{t-1} + \pi_{t-1})(1-\phi) + S_t \tag{I4}$$

$$\pi_t + M_t = \pi_{t-1} + M_{t-1} + pg_t - \phi(M_{t-1} + \pi_{t-1}) + G_t \tag{I5}$$

where S_t = flow [illegible]

g_t = flow of government expenditure in period t: which has a prior claim on output.

ϕ = Government monetary transfer as a proportion of initial household money holdings.

G_t = government tax/subsidy on profits.

It is assumed that all firms profits earned during a period are distributed to households at the beginning of the next period.

To express the macroeconomic potentialities of this model, the familiar wedge diagram developed by Muellbauer and Portes (1978) is adopted, this allows consumers constrained labour supply and commodity demand, and the production function to be expressed together in (x,L) space. The interpretation of the consumers wedge here is different in one significant respect, for the consumers two constrained curves represent initial offer curves if the constraint is an expected constraint, and represent loci of temporary equilibria if the constraint is effective. It will later be shown that the expected constraints ($\bar{\bar{L}}$ or $\bar{\bar{x}}$) truncate the wedge at the level of the initial market offers they generate. No underconsumption case is possible in the analysis since the producer has to be on his production function, therefore only points on or inside the consumers wedge are of interest.

The system displays the standard quantity constrained equilibria as demonstrated by figure (2.2.4)

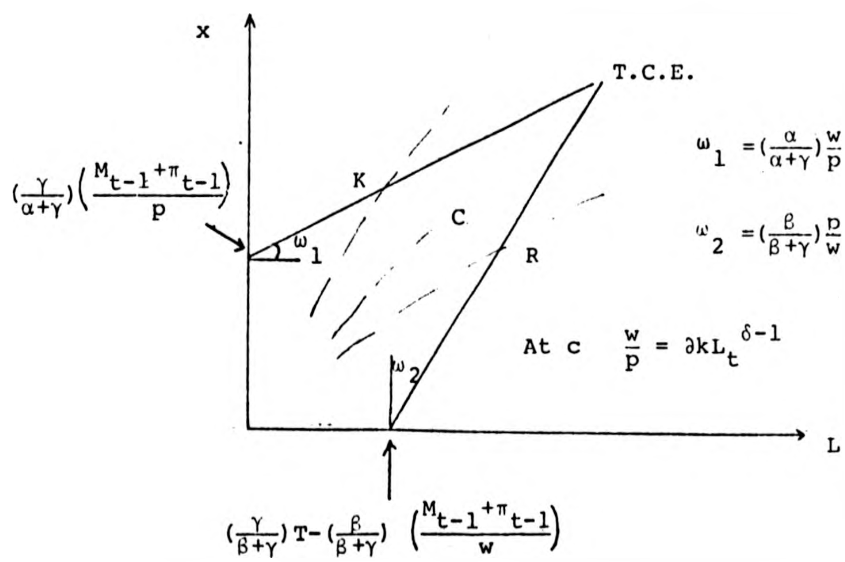


Figure (2.2.4)

The broken lines in figure (2.2.4) are firms production functions net of government goods purchases. Points K, C and R are standard Keynesian, Classical and Repressed Inflation temporary equilibria with quantity rationing.[17]

To examine the effects of expectations, first assume only firms hold constraint expectations, then, given that the firms expected constrained input/output combination lies within the consumers wedge and below the unconstrained choice point, an expectational classical temporary equilibrium will result as indicated in figure (2.2.5). Notice that at the equilibrium point C' the real wage is below the marginal product of labour.

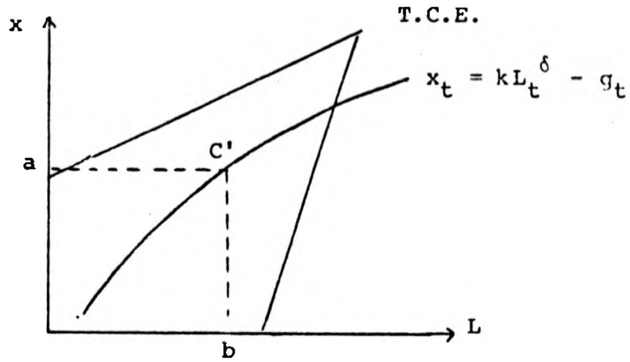


Figure (2.2.5)

Where at C' $(a,b) = (\bar{\bar{L}}_t, k\bar{\bar{L}}_t^{\delta})$ if $k\bar{\bar{L}}_t^{\delta} < \bar{\bar{x}}_t \leq k\left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{\delta}{1-\delta}} - g_t$

$$(a,b) = \left(\frac{\bar{\bar{x}}}{k}^{1/\delta}, \bar{\bar{x}}\right) \quad \text{if} \quad \bar{\bar{x}} < k\bar{\bar{L}}^{\delta} \leq \left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{1}{1-\delta}}$$

At point C' in figure (2.2.5) the firm expecting to encounter a constraint on trade in one market adjusts offers to be mutually consistent, the household wishes to trade more on both markets. Hence the economy has achieved an expectations constrained, or <u>expectational</u> classical outcome.

The case of firms expectations is not stressed in the following analysis, as the firms behaviour described in this manner, without inventories, is rather restricted. Consequently the next step is the introduction of household expectations into the analysis as major emphasis will be placed upon household expectations and hereafter firms expectations will be mainly surpressed. The rule that firms must be on their production functions will of course be enforced. The case where households expect a constraint on labour sales is described by figure (2.2.6)

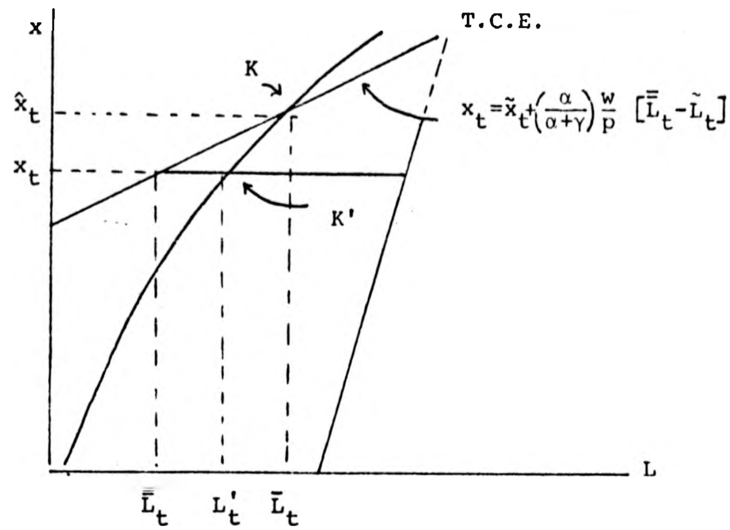


Figure (2.2.6)

Households in figure (2.2.6) solve a constrained utility maximization problem subject to the expectation $\bar{\bar{L}}_t = L(w/p)+A$, hence they offer x_t on the goods market. Once the initial transactions offer x_t is made this truncates the wedge of the consumer as no upward adjustment is allowed, the area above x_t is now unattainable, and the truncated wedge now describes actual levels of x_t as a function of actual labour sales. Firms are constrained by x_t and employ L'_t. At the point t', the expectational Keynesian unemployment equilibrium, households achieve L'_t on the labour market and their expected labour constrained goods demand. Unanticipated savings of $w(L'_t - \bar{\bar{L}}_t)$ arise[18] since consumers obtain less consumption good.

The important point of figure (2.2.6) is that it is the <u>spillover effect</u> of the expected constraint $\bar{\bar{L}}_t$ generating a low initial trade offer on the goods market which constrains the system.

The case where the household expects to be constrained on the goods market is described by figure (2.2.7).

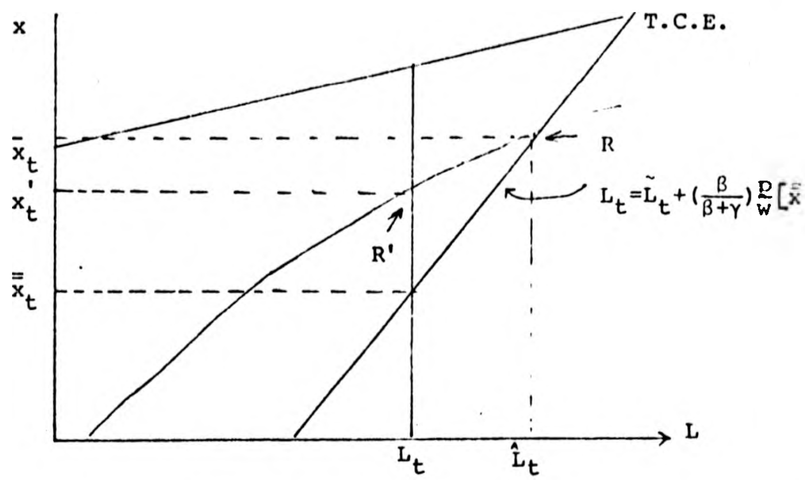


Figure (2.2.7)

In figure (2.2.7) households subjectively certain of being constrained on the goods market make an initial transaction offer on the labour market of L_t, thus truncating the wedge at this level of employment. Firms constrained by the households labour offer provide x'_t to the goods market. At point R' households achieve their expected goods constrained labour sales, and purchase x'_t, giving unanticipated dissaving $p(x'_t-\bar{\bar{x}}_t)$ which arises since the consumer cannot realise more labour income. The market outcome has repressed inflation characteristics, household expectations cause firms to be constrained for labour, and households are constrained on both markets relative to the usual repressed inflation equilibrium R.

Figures (2.2.6) and (2.2.7) demonstrate situations where consumers only hold constraint expectations about one market, but since we are using a Benassy formulation to calculate agents demands there is no reason why consumers should not anticipate an upper bound on trades on both markets simultaneously. Indeed there is no reason why agents should not make offers on markets above the constraints they anticipate to meet on those markets, since only the spillover effects matter. This aspect of the Benassy formulation has been criticised by, for example, Svensson (1977). However, it does have the advantage here of allowing agents to find out if their constraint expectations are correct. Figure (2.2.8) thus describes the consumers wedge when expectations of upper transaction bounds on both markets are introduced.

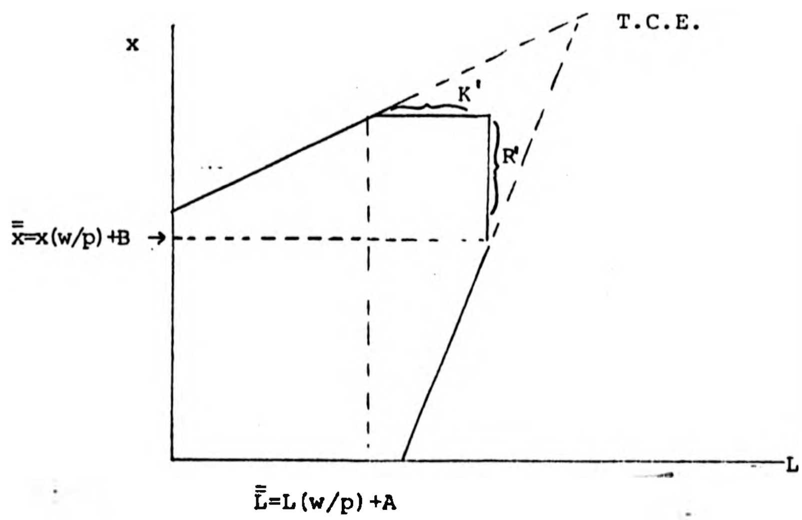


Figure (2.2.8)

In figure (2.2.8) intersection of a production function with the boundary of the truncated wedge along the K' line will yield expectational Keynesian Temporary equilibria, intersection with the R' line yields expectational repressed inflation temporary equilibria. The mechanisms being as in figures (2.2.6) and (2.2.7).

Notice that although termed expectational Keynesian and Repressed inflation the outcome K' and R' in figures (2.2.6) and (2.2.7) are quite different from their standard Keynesian and Repressed inflation counterparts K and R in figure (2.2.4) At the K and R equilibria both consumers and producers are achieving mutually consistant trades on the two markets. At the K' and R' equilibria the consumers level of money balances are changing as a residual. The consumer is achieving transactions consistant with his expectations, even though those expectations may not be fulfilled on the market they were formed about. It is therefore argued that consumers own expectations will place them in a semi-classical situation. The position of the equilibria will depend upon relative prices, the production technology and the form of the expectations functions. Finally it should be noted that these equilibria are not 'bootstrap' in either the sense of Varian (1977) or Neary and Stiglitz (1979) since it is the market offers generated by expectations which are always achieved, the expectations themselves will typically be incorrect.

In the next section the way agents may adjust their behaviour over successive periods is examined.

EXPECTATIONS ADJUSTMENT AND MARKET OUTCOMES OVER SEVERAL PERIODS

In the preceding analysis the impact of expectations on a single period model with partial quantity adjustment was examined. In this section we shall examine how the economic system under study may adjust over several market periods. Two basic questions will be considered. Firstly, if relative prices are stable over successive periods will the system automatically adjust to higher levels of output and employment? Secondly, what effects will relative price changes have? In considering these two questions it will be found that an intuitively reasonable interpretation may be placed on the limited quantity adjustment assumption made in the preceeding sections.

From this juncture onwards Keynesian cases will be examined in detail. Symmetric cases can be constructed for repressed inflation outcomes the results of which will be stated but not analysed here.

Consider the case where only consumers hold expectations. Let relative prices be fixed over several successive periods and expectations be formed as follows:

$$\bar{\bar{L}}_t = \bar{\bar{L}}_{t-1} + \sigma(L'_{t-1} - \bar{\bar{L}}_{t-1}) \tag{2.2.38}$$

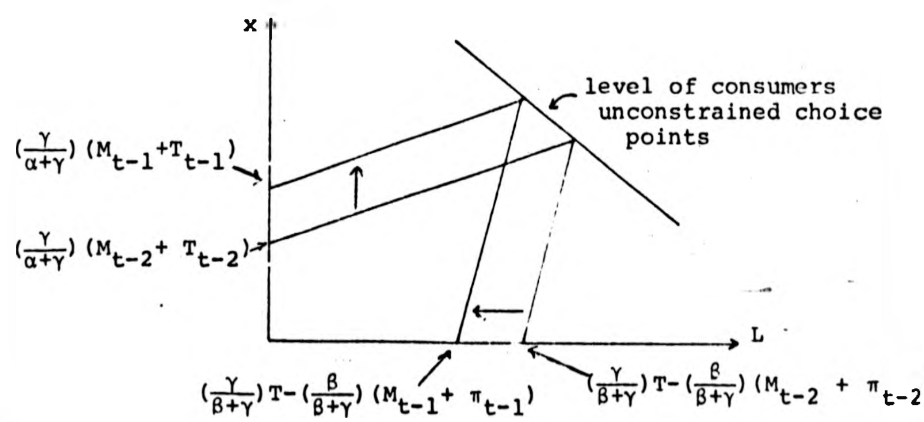
which using (2.2.6) may be written:

$$\bar{\bar{L}}_t = L(w/p) + \sigma A'_{t-1} + (1-\sigma)A_{t-1} \tag{2.2.39}$$

(2.2.39) is interpreted as follows; when in period t-1 agents discover that the constraint they expected to face on the labour market is incorrect they do not revise their belief about the shape of the expected labour demand curve. Consumers assume that they have located the demand curve incorrectly. This seems a plausible hypothesis for agents to hold, as in a given period fixed-prices mean that they do not observe any movement along a demand curve.[19]

The adjustment of consumers behaviour can be decomposed into two effects. Firstly, the accumulation of money balances in period t-1 will shift the intercepts of the consumers wedge, since the solution of the consumer utility maximization problem in period t will be based upon a different initial money endowment. Secondly the expected quantity constraint on the labour market will be revised according to expression [illegible]

Figure (2.2.9) demonstrates the effect of money balance accumulation upon the consumers wedge. Note that the movement of the wedge here is due to involuntary money balance accumulation as opposed to a chosen accumulation as in Muellbauer and Portes (1978).



The locus of consumers unconstrained choice points in figure (2.2.9) is given by (2.2.40) (solving 2.2.9) and (2.2.15) for $L=\bar{\bar{L}}$ and $x=\bar{\bar{x}}$).

$$L = \left[\frac{1}{\beta(\beta+\gamma)}\right]\left[\left(\frac{\alpha}{\beta+\gamma}\right) T + \frac{p}{w}\left(\frac{\beta}{\beta+\gamma}\right) - \frac{\gamma x}{\alpha}\right] \tag{2.2.40}$$

which has slope

$$\frac{dL}{dx} = - \frac{\gamma}{\alpha\beta(\beta+\gamma)} < 0 \tag{2.2.41}$$

Hence a negatively sloped straight line.

Using (IS), (2.2.10) and the expression $w(L'_{t-1} - \bar{\bar{L}}_{t-1})$ we note that the change in <u>beginning of period</u> money balances must obey the following equation (2.2.42), written in our end of period terms.

$$\Delta M\Big|_{t-2 \rightarrow t-1} = G_{t-1} + pg_{t-1} - \phi(M_{t-2} + \pi_{t-2}) = \left(\frac{\gamma}{\alpha+\gamma}\right)(w\bar{\bar{L}}_{t-1} + M_{t-2} + \pi_{t-2})$$

$$+ w(L'_{t-1} - \bar{\bar{L}}_{t-1}) + \pi_{t-1} - M_{t-2} \tag{2.2.42}$$

Expression (2.2.42) is simply an adding up condition, which says the total change in the money stock must equal the government budget deficit.

Having examined how the wedge shifts with changes in consumers money balances, consider how expectations adjust over successive periods of constant prices. Figure (2.2.10) examines expectation adjustment in a static monetary environment, i.e. where the government balances its budget.

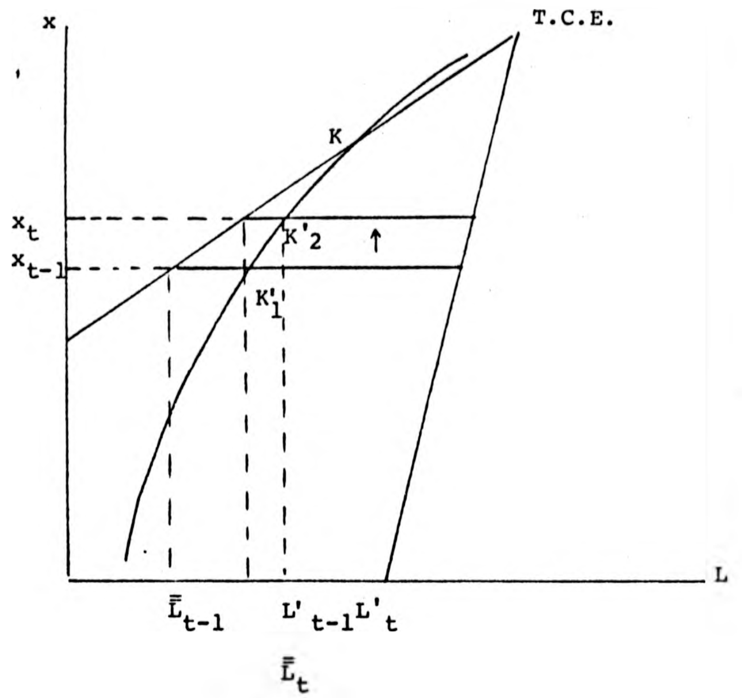


Figure (2.2.10)

Figure (2.2.10) is drawn for $\sigma=1$ in equation (2.2.39) the expectations adjustment function. As expectations adjust consumers initial goods market purchase offers rise from x_{t-1} to x_t over time period t-1. This has the effect of pushing up the ceiling of the truncated wedge as indicated by the arrow. Thus over successive time periods the system will move out along the production function, through a series of expectational Keynesian equilibria, until the point K is achieved. At K expectations are correct and there is no tendency for the system to move further. A similar result holds for R' equilibria tending to R equilibria.

Figure (2.2.10) furnishes part of the answer to the first question posed at the beginning of this section. When the monetary environment is static, the government is balancing its budget, and when prices are fixed over successive periods, then the economy does display automatic adjustment to higher levels of output and employment. The adjustment, of course, is to a standard Keynesian unemployment equilibrium, not to the consumers temporary competitive equilibrium point.

To examine the adjustment of consumer behaviour when both initial money balances and expectations adjust, figures (2.2.9) and (2.2.10) are combined to give (2.2.11).

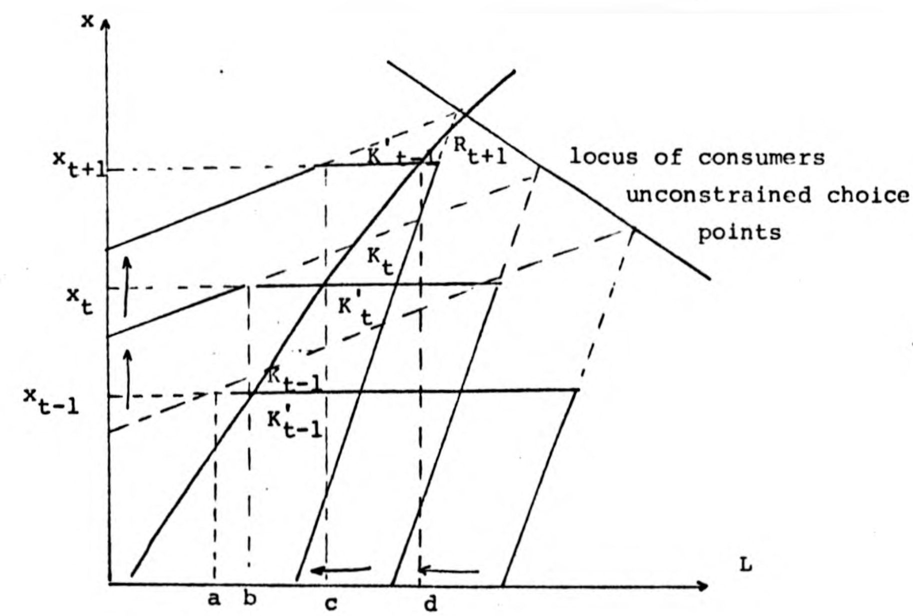


Figure (2.2.11)

$a=\bar{\bar{L}}_{t-1}$ $b=L'_{t-1}, \bar{L}_t$ $c=L'_t, \bar{\bar{L}}_{t+1}$ $d=L'_{t+1}$

In figure (2.2.11) as the money stock grows the consumers wedge shifts upwards and to the left. Expectations adjust by the partial adjustment mechanism (2.2.38), and initial goods purchase offers correspondingly rise. Thus expectations push the system out along the production function in the same manner as was described in figure (2.2.10). The significance of the shifts in the consumers wedge is also apparent; if consumers were at the K'_{t-1} equilibrium we saw that given no change in the money stock the system would be driven by expectations to the K_{t-1} equilibrium. As the money stock grows expectations drive the system towards successively higher Keynesian unemployment equilibria $K_{t-1} \to K_t$, and as we observe on the diagram, may also cause a regime switch as indicated by the point R_{t+1}. If on this (Keynesian) regime the money supply were to be contracting then the wedge would shift in the opposite direction to that indicated, tracing out a series of orthodox Keynesian temporary equilibrium points. On the repressed inflation regime the same results hold but in each case the direction of the money supply change is reversed, i.e. a cut in the money supply is required to shift the system out to higher levels of output and employment. Expectations and money stock adjustments may push the system out to an equilibrium at the intersection of the production function and the locus of consumers unconstrained choice points. The levels of output and employment which will thus be achieved are described by (2.2.43) and (2.2.44).

$$L+L^{\delta} \left(\frac{\gamma}{\beta+\gamma}\right) \left(\frac{\alpha+\gamma}{\alpha}\right)\left(\frac{\beta}{\beta+\gamma}\right) kT - \left(\frac{\alpha}{\alpha+\gamma}\right) \frac{\bar{\bar{L}}}{k} + \left(\frac{\alpha}{\alpha+\gamma}\right) \frac{p}{w} \frac{\bar{\bar{x}}}{k} \qquad (2.2.43)$$

$$x+x^{1/\delta} = \left[\left(\frac{\gamma}{\beta+\gamma}\right)\left(\frac{\alpha+\gamma}{\alpha}\right)^2\left(\frac{\beta}{\beta+\gamma}\right)^2\right] \frac{p^2 T}{w^2 k^{1/\delta}} - \frac{1}{\beta(\beta+\gamma)k^{1/\delta}}\left[\bar{\bar{L}}+\frac{\bar{\bar{x}}}{w}\right] \qquad (2.2.$$

where $\bar{\bar{x}}=x$ and $\bar{\bar{L}}=L$

If producers expectations are reintroduced, and are also based upon a partial adjustment mechanism, then there will come a point at which their expectations will place them on the short side of the markets. Consequently the consumers adjustment process will be halted by the occurrence of an expectational classical outcome.[20] The market outcomes over several periods will thus depend upon three rates of change, consumers expectations, producers expectationsand exogenous monetary changes. Also highly significant will be the 'original' state of agents expectations. There are many permutations of rates of change and original expectations which will produce quantatively different outcomes, and the likelihood of the various possibilities may well be a question for empirical study.

The second question to which this analysis addresses itself is what impact will changes in the relative price vector have upon the model. For the representative consumer a change in relative prices will have income, substitution, and real balance effects as found in models such as Muellbauer and Portes (1978), here there will also be an expectations effect, recalling (2.2.6), $\bar{\bar{L}}=L(w/p)+A$, we see that a change in relative prices causes the consumer to anticipate his being employed at a different point on the labour demand curve.

We assume:

$$\frac{\partial \bar{\bar{L}}}{\partial (w/p)} < 0 \qquad (2.2.45)$$

$$\frac{\partial \bar{\bar{x}}}{\partial (w/p)} < 0 \qquad (2.2.46)$$

Further to maintain the Keynesian flavour of our analysis we shall assume that at the end of a period of Keynesian unemployment both the nominal prices of labour and goods fall.

Furthermore it will be assumed that the change in wages and prices raises the real wage rate, i.e. we shall examine a case where there is some resistance to a reduction in the nominal wage. Surpressing for the moment both producer and consumer expectations, it may be seen that in the case chosen for analysis the consumers wedge will move as described by figure (2.2.12)

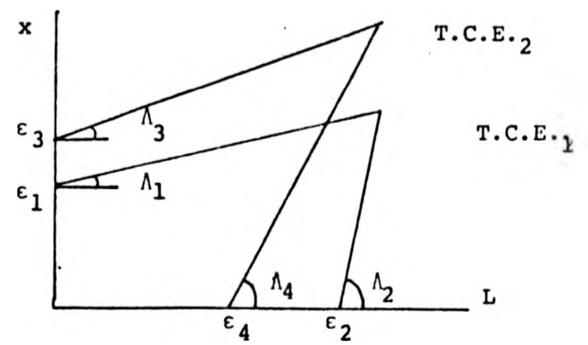


Figure (2.2.12)

In figure (2.2.12), the wedge associated with T.C.E.$_1$ has higher nominal wages and prices than the wedge associated with T.C.E.$_2$, but has a lower real wage rate.

In figure (2.2.12) $w_t > w_{t+1}$, $p_t > p_{t+1}$, $w_t/p_t < w_{t+1}/p_{t+1}$

and
$$\varepsilon_1 = \left(\frac{\gamma}{\alpha+\gamma}\right)\left(\frac{M_{t-1} + \pi_{t-1}}{p_t}\right) < \varepsilon_3 = \left(\frac{\gamma}{\alpha+\gamma}\right)\left(\frac{M_t + \pi_t}{p_{t+1}}\right)$$

$$\varepsilon_2 = \left(\frac{\gamma}{\beta+\gamma}\right)T - \left(\frac{\beta}{\beta+\gamma}\right)\left(\frac{M_{t-1} + \pi_{t-1}}{w_t}\right) > \varepsilon_4 = \left(\frac{\gamma}{\beta+\gamma}\right)T - \left(\frac{\beta}{\beta+\gamma}\right)\left(\frac{M_t + \pi_t}{w_{t+1}}\right)$$

$$\Lambda_1 = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{w_t}{p_t} < \Lambda_3 = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{w_{t+1}}{p_{t+1}}$$

$$\Lambda_2 = \left(\frac{\beta}{\beta+\gamma}\right)\frac{p_t}{w_t} > \Lambda_4 = \left(\frac{\beta}{\beta+\gamma}\right)\frac{p_{t+1}}{w_{t+1}}$$

Consumer expectations adjustment over successive periods of relative price changes will comprise two components, one, the relocation of the expected labour demand curve in $(w/p, L)$

space by the partial adjustment mechanism (2.2.39), and two, the relative price effect on consumers expected labour demand curve, which in the case under analysis here tightens the consumers expected labour sales ration according to (2.2.45). Diagramatically consumers expectations are added to figure (2.2.12) to produce figure (2.2.13).

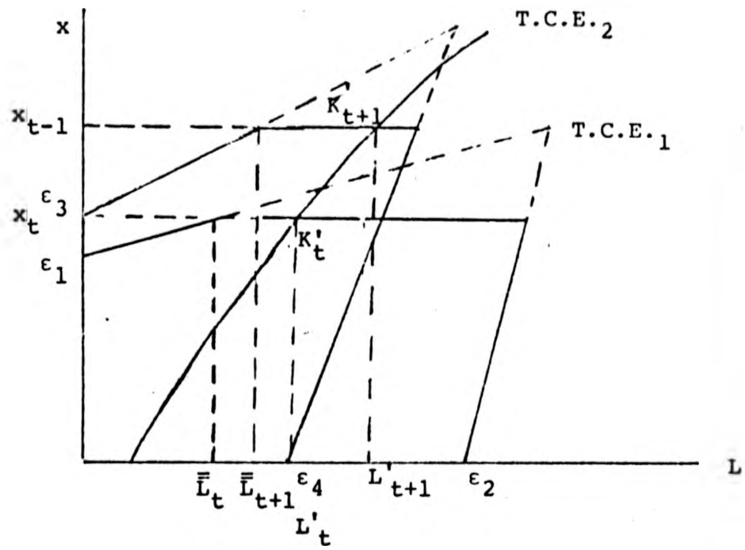


Figure (2.2.13)

In figure (2.2.13) consumers start the market period with expectations $\bar{\bar{L}}_t$ of the upper bound they will face on labour sales. They initially offer x_t on the goods market and this truncates the wedge as described previously. In period t an expectational Keynesian equilibrium arises at K'_t. Between periods t and t+1 excess supply on both markets forces both nominal prices down and raises the real wage. The consumer wedge moves as described by figure (2.2.12), and consumer expectations adjust in two ways, the relocation and relative price effects discussed above. The relocation of the consumers expected labour demand curve in period t+1 is in response to the error in expectations, $L'_t - \bar{\bar{L}}_t$ in the previous period.

This works to increase output and employment. The relative price effect lowers the level of labour sales the consumer anticipates achieving, and thus works to reduce output and employment. In figure (2.2.13) the net effect of the change in relative prices moves the economic system from the K'_t equilibrium to the K'_{t+1} equilibrium, with a consequent increase in both output and employment.[21]

For the producer with expectations formed as (2.2.24) and (2.2.25), changes in relative prices will push the system up or down the production function in both the expectational and standard classical temporary equilibria. Relative price changes will move an orthodox classical temporary equilibrium by changing the real wage equals the marginal physical product of labour condition. Relative price changes will move expectational classical equilibria along the production function due to the relative price effect upon the consumers expected goods demand function.

The preceding discussion of expectations adjustment suggests two possible interpretations. The analysis of expectations adjusting over several periods of fixed prices may be interpreted as an analysis of the very short run, where expectations adjust in a succession of sub-periods. This would be consistant with the argument presented earlier that limited quantity adjustment as represented in this model is, in the very short run, a consequence of agents being unable to locate others who also wish to increase transaction levels in a limited time period. Alternatively, the analysis presented examining expectations adjustment in tandem with price adjustments suggests a rather longer period interpretation of the model, where the length of a market period is the interval between relative price adjustments. This may well be more consistant with the hypothesis that limited quantity adjustment is based upon the cost of adjustment, rather than short run trading opportunities. (This is a point to which I shall return and consider in greater detail in chapter 3).

Government Policy

In this section the effectiveness of Government policy in the model we have developed is examined. Particular emphasis is given to the impact that government policy may have upon expectations. It will be considered whether government policy has a 'placebo' effect as in Honkapohja and Ito's (1979) analysis.

Examination of the models structural identities (I1)-(I5) reveals that the government in our economy has three policy instruments: goods purchases g_t, beginning of period monetary transfers from (to) households $-\phi(M_{t-1} + \pi_{t-1})$, and an end of period tax/subsidy on firms profits G_t.

Again concentrating on a Keynesian regime and consumer expectations: consider first an unannounced change in government expenditure at a single period expectational Keynesian equilibria. By unannounced it is meant that consumers are unaware of a change in government expenditure when they make their initial transaction demands. This is described by figure (2.2.14).

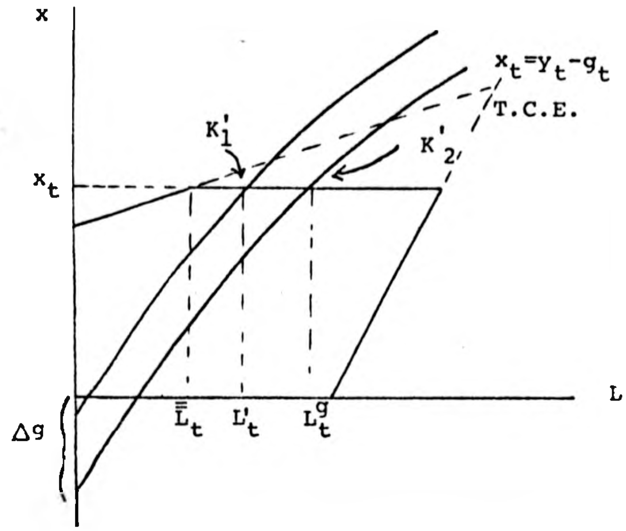
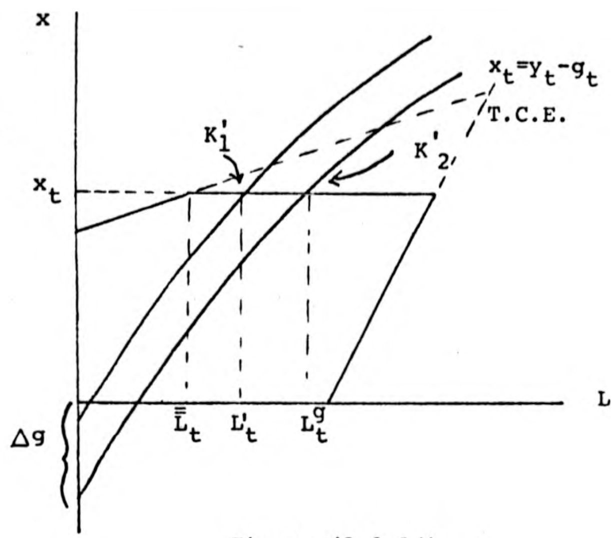


Figure (2.2.14)

The increase in government goods purchases, Δg, reduces the goods available for consumption at any level of employment, as indicated on figure (2.2.14). Consumers labour demand expectation $\bar{\bar{L}}_t$ gives an initial goods purchase offer x_t which truncates the consumers wedge. Initial government purchases would give an equilibrium at $K_1' = (L_t', x_t)$, the increased government purchases give an equilibrium at $K_2' = (L_t^g, x_t)$. Employment rises but consumption is constant. The increase in government expenditure, if not financed by increased taxation, will have an impact on consumption next period when household current period forced saving and firms current period profits become households initial money endowment. This would involve a wedge shifting effect as in figure (2.2.12). Figure (2.2.15) shows the effect of an unannounced increase in government expenditure, financed by the printing of money, over successive periods.



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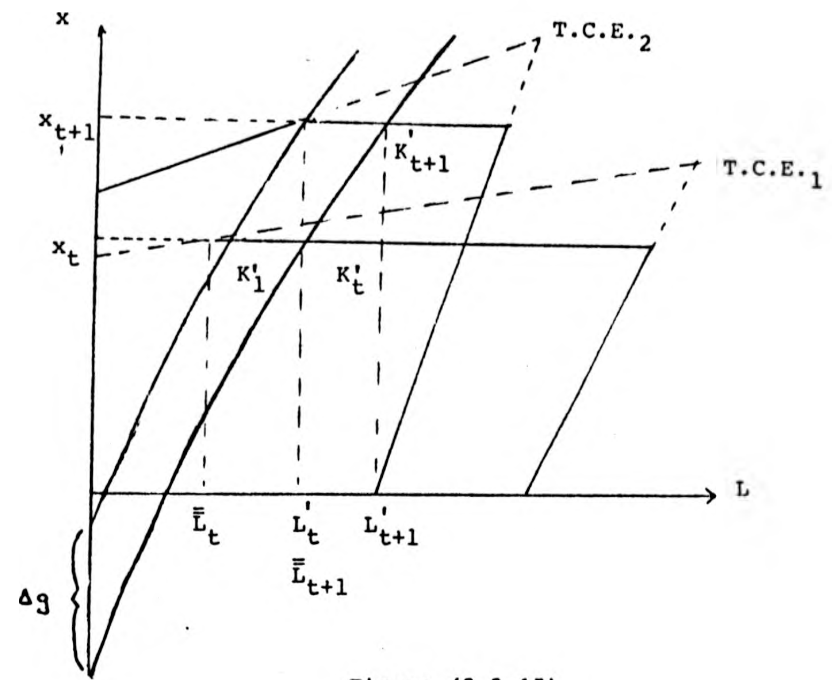

Figure (2.2.15)

Over successive periods it can be seen from figure (2.2.15), that a money financed increase in government expenditure has strong expansionary effects on output, employment and consumption on an Expectational Keynesian regime. To clarify this we trace the process through. Consumers begin the first period with money endowments which generate the wedge associated with T.C.E.$_1$, and with expectations $\bar{\bar{L}}_t$ this generates a wedge truncated at x_t. Prior to the increase in government expenditure the equilibrium is K'_1. The government increases its expenditure, the impact effect of which is just a rise in employment and savings. The equilibrium shifts to L'_t. At the beginning of period t+1 households receive previous period profits to add to the forced saving they incurred in the previous period, and also adjust their expectations to be consistant with last periods labour market experience. This generates the wedge associated with T.C.E.$_2$.

truncated at x_{t+1}, which can be seen to give an expectational equilibrium at K'_{t+1}, with higher output employment and consumption. In the figure we have also allowed the real wage to rise with the nominal wage falling less than nominal prices. Thus figure (2.2.15) represents the total effect of an unannounced increase in government expenditure, financed by the printing of money.

It has been stressed in the preceding section that the change in government expenditure is unannounced, and since it is not directly observable by consumers, it has been implicitly assumed that government behaviour has no direct effect upon consumer expectations. If the government announces, prior to agents making their initial market offers, that it intends to purchase an increased level of consumption goods, then consumers should take account of this information in forming their expectations. This we shall see has the effect of raising consumers initial goods purchase offers, and possibly making it unnecessary for governments to bother increasing their expenditure. This is the 'placebo effect' discussed in Honkopohja and Ito (1979). Figure (2.2.16) examines the case where the government announces that it will undertake sufficient expenditure to maintain a level of employment $\tilde{L}$. Given that households believe the government, and that $\tilde{L}>\bar{\bar{L}}$, they will reformulate their expectations to be $\bar{\bar{L}}=\tilde{L}$.

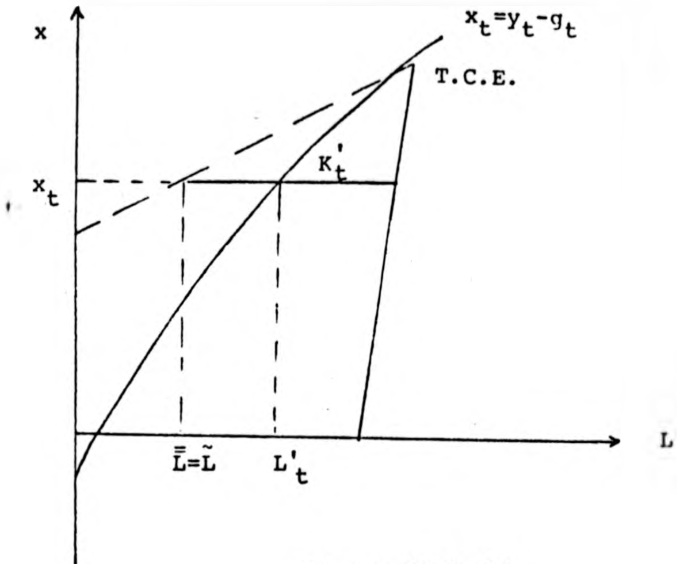

Figure (2.2.16)

We see immediately from figure (2.2.16) that the expectation of government expenditure is sufficient to generate the announced level of employment, and an actual expenditure increase is not required.

The governments alternatives to expenditure policy are taxes/transfers on household money balances and firms profits. These two policy instruments have a similar effect but differ in their timing, both are demonstrated in figure(2.2.17).

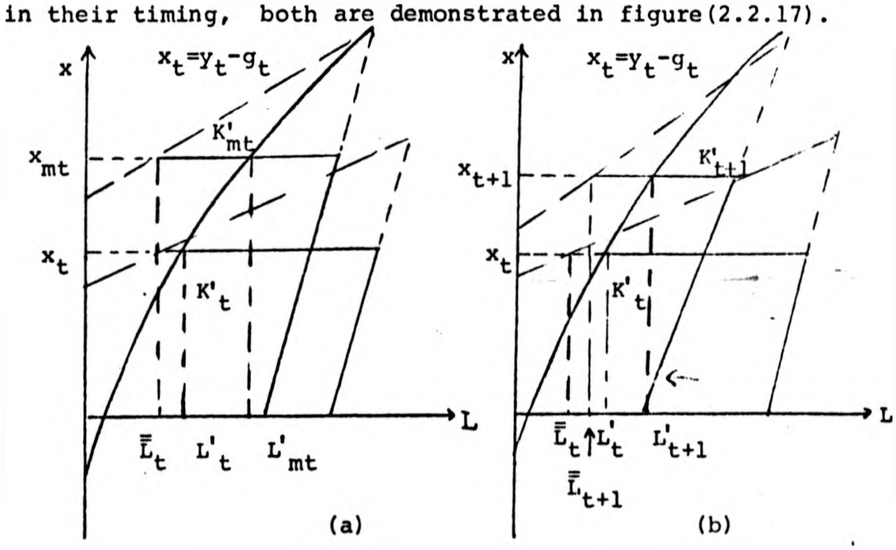

Figure (2.2.17)

In figure (2.2.17) (a) the government levies a negative tax on households initial money balances, this is done prior to any transaction demands being expressed. The consumers wedge shifts as indicated by the arrows, and the expected labour ration $\bar{\bar{L}}_t$ generates a higher goods purchase offer, x_{mt}, and consequently the consumers wedge is truncated at a higher level of goods transactions. The equilibrium becomes K'_{mt} rather than K'_t, with a consequent increase in both output employment and consumption in period t. The second part of the diagram, figure (2.2.17)(b) [illegible] effect of [illegible] period t, which does not effect the economy until the beginning of period t+1 when profits and the subsidy are distributed to consumers. This produces a wedge shifting effect as in case (a), but the picture will be complicated by relative price changes and expectation adjustment as discussed earlier. There is no guarantee that output employment and consumption will be higher at the K_{tm}' equilibrium than at the K_t' equilibrium although this seems probable. The negative impact of a real wage rise on consumers expectations is the complicating factor here.

It should be noted when examining government policy instruments, that movements of the production function in (x_t, L_t) space, and movements of the consumers wedge may lead to a regime switch, as constant increases in government expenditure to increase employment may well give rise to a classical equilibrium. At constant relative prices, and the government maintaining a balanced budget, a series of taxation financed expenditure increases will trace out a locus of classical temporary equilibria as in figure (2.2.18).

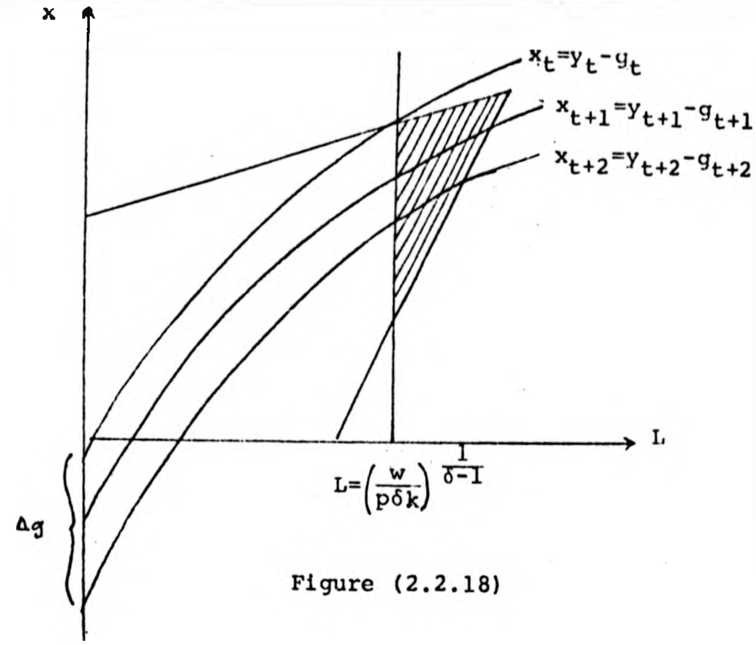

Figure (2.2.18)

Clearly the shaded area of the consumers wedge in figure (2.2.18) will not be attained, as the producer will be at a classical profit maximizing equilibrium with lower transaction levels on both markets.

Concluding Remarks

In this section (2.2) a simple Malinvaud type model has been presented in which expectations play a different role than in the literature reviewed in section (2.1). Producers and consumers in th model calculate their initial transaction offers on the basis of prices <u>and</u> expected quantity constraints. Having made initial market offers, agents, it is argued, find it more difficult to revise in the upward than the downward direction. The completion of an upward revision in trade requires of an agent that he find another who is prepared to complete the other side of the transaction. A downward revision of trade simply requires that the agent refrain from some trades. The model presented in this section takes the extreme case of this argument and examines the short-run equilibria that will result when no upward revisions of trade are possible. With this limited quantity adjustment initial market offers and consequently the expectations which generate them are most important.

The analysis presented here is not intended as a representation of any real economy, but rather a simple theoretical abstraction which allows the impact of expectations in a model with limited quantity adjustment to be examined. Clearly expectations will be important in the period about which they are held if transactions adjustment is either restricted or costly. In this section it has been assumed that agents hold expected supply and demand function with subjective certainty. This assumption was adopted for simplicity of exposition, and because otherwise the strong assumption that [illegible] quantity adjustment is free and frictionless would cause agents to make Walrasian initial transaction offers no matter what their previous market experience. In analysis to be presented in the following chapter it will be shown that a model with costly quantity adjustment and uncertainty about rationing levels generates many similar results to those presented here.

Footnotes - Chapter 2

1. Hildenbrand and Hildenbrand are primarily concerned with examining the usefulness of a representative consumer, representative producer model, and the specificity of the comparative static results of Malinvauds model to the functional forms he chooses.

2. Muellbauer and Portes have clearly had a great influence on this thesis, considerable space is devoted to a fairly full exposition of their model for this reason.

3. This would only defer the problem to the third period, but of course as the number of periods becomes large the problem becomes unimportant.

4. 'Forced saving' since money balances become the residual

$$M_1 - M_0 = d_1 + w_1\bar{L}_1 - p_1\bar{x}_1$$

5. 'Forced inventory accumulation' is described by

   $I_1 - I_0 = y_1(\bar{L}_1) - \bar{x}_1$, although a residual this still

   represents a choice by the producer to carry these goods forwards since he could simply reduce his labour demand in the current period.

6. The equilibria in figure (2.1.3) which Muellbauer and Portes term Walrasian, is a Hicksian temporary competitive equilibrium in the virtual price system, see appendix to this chapter or Ellis (1981).

7. The stability properties of this model are examined on the assumption of inelastic expectations, and if this assumption is not made instability may result. Hildenbrand and Hildenbrand (1978) discuss this possibility in relation to Malinvaud's (1977) contribution, and the point seems equally valid here.

8. For example Gale (1978), Green (1978), Honkapohja and Ito (1979), Svensson (1977).

9. This is the requirement for a Benassy K-equilibrium, a Drèze equilibrium requires that actual trades replicate themselves. The distinction arises from the definition of effective demands in the two equilibrium concepts, a discussion of which may be found in chapter 1.

10. Exact in Philip Michel's sense means that expected demand curves are accurate at equilibrium transaction levels but may be inaccurate elsewhere.

11. A conceptually stimulating approach in this area is that taken by Hahn (1977a), (1977b), who examines the concept of conjectural equilibria. A rational conjectural equilibrium is said to exist if given a set of market signals that an agent receives, and the conjectures of trading possibilities he forms, his actual trades are the best feasible trades which are consistant with the

equilibrium, and this is true for all agents. Hahn's work, despite its influence on aggregate endogeneous price models, is omitted from the main text as it has not yet been given a reasonable macroeconomic representation.

12. Clearly given different endowment points and offer curves any point on the transformation frontier to the right of Walrasian equilibrium may be a non-Walrasian equilibrium in Varians model.

13. Varian then demonstrates in a technically complex section of his paper that the Walrasian equilibrium is unstable and the non-Walrasian equilibria stable.

14. Varian himself is aware of these problems.

15. Negishi in later work (Negishi (1979)) bases his results upon demand curve elasticities rather than kinks, but again all that is required is the appropriate intersection of the perceived and actual demand curves.

16. This point is similar to Negishi's argument as to why perceived demand curves should be kinked. Those that already purchase from a supplier will all be informed of a price rise, and some will be driven away. Those who do not purchase from a supplier may not immediately receive the information that price has been cut.

17. There may be several intersections between the production function and the consumers constrained labour supply and goods demand curves. This is a well known problem but not one analysed here.

18. The worker may be prepared to supply L'_t, if the utility of money balances is greater than the disutility of labour at the margin. Figure (2.1.1) justifies this.

19. A more complete expectations adjustment function may be adopted here but will not qualitatively effect the results, unless consumers believe that on average a certain constraint will be encountered and that a 'tight' constraint in one period will increase the likelihood of a 'loose' constraint in the next.

20. Infinitely fast adjustment of producers expectations would always return the systems to a classical outcome after one period. If producers held inventories as in Muellbauer and Portes (1978) this would not necessarily happen.

21. Clearly there are numerous permutations that may be achieved here, depending mainly on whether the real wage rises or falls, and the consequent signs and relative magnitudes of the expectations adjustment effects.

APPENDIX TO CHAPTER 2

It was stated during the discussion of the role of expectations in Model's with exogenous prices, that the equilibria termed Walrasian in such models are actually Hicksian Temporary Competitive Equilibria. They only correspond to Walrasian equilibria as defined by Debreu (1959) if all agent expectations expressed in personalized virtual prices are correct.

To demonstrate this we need to establish two propositions.

Proposition 1: If fixed prices clear current markets, this is a necessary but not sufficient condition for Benassy demands to be Walrasian Demands.

Proposition 2: If fixed prices clear current markets this is a necessary and sufficient condition for Benassy demands to be Hicksian demands.

Let there be $i=1,\ldots,n$ trades $,j=1,\ldots,M$ goods and

$t=1,\ldots,T$ periods $(i,j,t \in \mathbb{Z}_+)$.

Each trader: has endowments $e_i \in \mathbb{R}_+^2$, faces fixed prices $p \in \mathbb{R}_+^2$ and upper and lower bounds $\bar{Z}_i, \underline{Z}_i \in \mathbb{R}_+^2$.

We identify demands by agent market and period

$\overset{*}{Z}_{ijt} = \overset{*}{Z}_{ijt}\,(\overset{*}{p}, e_i)$ Walrasian demand

$\hat{Z}_{ijt} = \hat{Z}_{ijt}\,(\hat{p}_t, e_i, x_i)$ Benassy demand

$\tilde{Z}_{ijt} = \tilde{Z}_{ijt}\,(\hat{p}_t, e_i, p_i^x)$ Hicksian demand

Where $\overset{*}{p}$ is the full Walrasian price matrix, $\hat{p}_t$ the vector of current market clearing prices, $x_i = (\bar{p}_i, \bar{Z}_i, \underline{Z}_i)$ is the $(M \times (T-t))$ matrix of expected signals, where the $\bar{p}_i$'s are the expected fixed prices associated with the expected upper and lower bounds $\bar{Z}_i, \underline{Z}_i$. The p_i^x is an $(M\times(T-t))$ matrix of (Hicksian) expected prices.

To establish proposition 1 we prove the following lemma.

Lemma 1

$\hat{p}_t = (\hat{p}_{1t}, \ldots, \hat{p}_{mt}) = \overset{*}{p}_t = (\overset{*}{p}_{1t}, \ldots, \overset{*}{p}_{mt})$ is a necessary but not sufficient condition for $\overset{*}{z}_{ij} = \hat{z}_{ij}\ \forall_{i,j}$ at t.

Proof

By definition the Walrasian price vector $\overset{*}{p}_t$ clears markets $1, \ldots, M$ at t if $\overset{*}{p}_{t+a}$ clears the same markets at t+a $\forall(1 \leq a \leq T-t) \in \mathbb{Z}_+$.

Thus (i) If $\bar{p}_{ijt+a} \neq \overset{*}{p}_{ijt+a}$ any $a \geq 1$, we can find an incorrect price expectation, and if we replace $\overset{*}{p}_{ijt+a}$ by $\bar{p}_{ijt+a}$ in $\overset{*}{p}$, then $\overset{*}{p}$ cannot clear all markets by Walras law.

also (ii) If $\overset{*}{z}_{ijt+a} < \underline{z}_{ijt+a}$ or $\overset{*}{z}_{ijt+a} > \bar{z}_{ijt+a}$ any $a \geq 1$, we can find an incorrect quantity constraint expectation, and if we replace $\overset{*}{z}_{ijt+a}$ by $\underline{z}_{ijt+a}$ or $\bar{z}_{ijt+a}$, then again $\overset{*}{p}$ will not clear all markets.

If (i) or (ii) holds and we choose t to be a period when $\overset{*}{p}_t$ does not clear all $1, \ldots, M$ markets.

then $\overset{*}{p}_t = \hat{p}_t \not\Rightarrow \overset{*}{z}_{ijt} = \hat{z}_{ijt}$. Thus $\overset{*}{p}_t = \hat{p}_t$ is not a sufficient condition. A sufficient condition for $\overset{*}{z}_{ijt} = \hat{z}_{ijt}$ $\forall_{ijt}$ is that (i) and (ii) do not hold, hence $\overset{*}{p}_t = \hat{p}_t\ \forall_t$. Thus $\overset{*}{p}_t = \hat{p}_t$ at t is a necessary condition. □

Clearly then proposition 1 is true by lemma 1, if fixed prices clear all current markets the Benassy demands are not the Walrasian demands and the equilibria usually termed 'Walrasian' as in figure (2.1.3)(a) and most of the literature are not as defined for example in Debreu (1959). (Unless of course (i) and (ii) above do not hold).

To establish proposition 2 we prove the following lemma.

Lemma 2

At $t \; \exists \; p_i^x$ s.t. $\sum_{i=1}^{n} \hat{z}_{ijt}=0$ each $j \Leftrightarrow \hat{z}_{ijt}(\beta,e_i,x_i)=\tilde{z}_{ijt}(\beta,e_i,p_i^x)\,\forall i$.

Proof

We need to show that we may choose a matrix of expected prices p_i^x which are identical for trader i to the corresponding triple $x_i = (\bar{p}_i,\bar{z}_i,\underline{z}_i)$.

Associated with each $x_{ijt} \in x_i$ is a planned transaction $z_{ijt} \in z_i$.

$$\text{where} \qquad z_{ijt} = \begin{cases} z_{ijt}(\bar{p}_i,\bar{z}_i,\underline{z}_i) & \text{if } \underline{z}_{ijt}<z_{ijt}(\bar{p}_i,\bar{z}_i,\underline{z}_i)<\bar{z}_{ijt} \\ \bar{z}_{ijt} & \text{if } z_{ijt}(\bar{p}_i,\bar{z}_i,\underline{z}_i)\geq \bar{z}_{ijt} \\ \underline{z}_{ijt} & \text{otherwise} \end{cases}$$

If there exist prices $q_i \in R_+^{M(T-t)}$ which support $z_{ijt}\;\forall ijt$ then these are identical to each trader i to the signal x_i. Neary and Roberts (1978) show such virtual prices exist if the preference ordering R is convex, continuous and strictly monotonic, and that the virtual prices ω_i are of the form $\omega_i=(\bar{p}_i,q_i)$ where the $\bar{p}_i$'s the expected prices support the non-quantity constrained planned transactions, and the expected virtual prices q_i support the constrained transactions.

Thus $p_i^x = \omega_i \Rightarrow \sum_{i=1}^{n} \hat{z}_{ijt} = 0$ each j at $t \Leftrightarrow \hat{z}_{ijt}(\beta,e_i,x_i)=\tilde{z}_{ijt}(\beta_i e_i,p_i^x)\,\forall ij$ at t □

Thus lemma 2 establishes proposition 2. If fixed prices clear all current markets the Benassy demands are equal to the Hicksian demands where we choose each agents price expectations to be the virtual prices which support the transactions plans associated with the price quantity constraint expectations. Thus if all Hicksian and Benassy demands are identical and β_t clears current period markets, then the Benassy unconstrained equilibria are Hicksian Temporary competitive equilibria. The equilibria is only Walrasian if it is 'dynamically stable', in Hicks terms, in our analysis this means that the expected virtual prices would clear all markets if they obtained.

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3. COSTS AND UNCERTAINTY IN NEO-KEYNESIAN MACROECONOMICS

3.1 Information and the Costs of Economic Activity

In the preceding chapter I examined the role played by expectations in Neo-Keynesian Macroeconomic Models. Attention was concentrated upon the functions performed by expectations, rather than on an examination of what characteristics of these models make expectations so important.

In this chapter it will be argued that expectations formation is the economic agents response to incomplete information and the cost of economic activity. Qualification of this statement is clearly required, what information is unavailable and precisely what is meant by the cost of economic activity.

It is difficult to discuss incomplete information and expectations without introducing some concept of cost into the discourse, especially when considering Neo-Keynesian economics. However I shall attempt to first examine independently the question of incomplete information and expectations in Neo-Keynesian models. Secondly, I shall provide a more general examination of the costs of economic activity and a discussion of how some forms of cost are implicit in Neo-Keynesian models and why the introduction of other forms of costs may be desirable.

It is useful when considering incomplete information and expectations to again consider the literature in two sections, endogeneous price models of the Varian (1977) Heller and Starr (1978) and Futia (1977) type, and exogenous price-quantity rationing models of the Benassy (1975) Muellbauer and Portes (1978) and Malinvaud (1977) variety. In endogeneous price models agents do not have, for whatever reason, complete information of their current trading possibilities. Expectations take the place of the missing data. Consider the Varian (1977) model described in section 2.1 and illustrated in figure (2.1.4), which displays both a Walrasian and non-Walrasian equilibrium. The economy rests at a non-Walrasian equilibrium essentially because firms pessimistic sales expectations are correct at this level of employment and production. Firms in this economy can manipulate their output price, and since there are no real balances in the model they may thus manipulate the real wage using price and labour demand. Firms must be unaware of this possibility or they would undertake the necessary corrective measures.[1] Firms then cannot have full information of the transformation frontier which they face, and thus replace their missing data by sales expectations. In Futia's (1977) contribution, workers are aware that firms only sample at random some subset of the total available labour force when searching for employees, thus they attach a positive but diminishing probability to being employed if they raise the wage they quote. This is interesting, firstly firms do not have full information about the wage distribution being quoted by the labour force, and secondly workers are aware of this. In the absence of full information about wage quotes Futia assumes that firms hire workers up to the point where their marginal product equals the expected average real wage. The definition of equilibrium in Futia's model has already been discussed in chapter 2, section 1.

Exogenous price - quantity rationing models allow economic agents more information about current trading possibilities than endogenous price models. In work such as Neary and

Stiglitz (1981), Benassy (1975) and Malinvaud (1977) a quantity tatonnement auctioneer informs agents of fixed prices and what trades are feasible on each market.[2] Fix-price equilibria are a short-run phenomena and despite considerable information about the current period agents will have no information about prices or feasible transactions that will shortly arrive, in the next period.[3] Expectations are held about future prices and quantity constraints, which are important since agents know the equilibrium will not persist and that the current equilibria values taken by variables do not provide accurate information about the future. The recursive or 'bootstrap' effect of expectations has been demonstrated by Neary and Stiglitz and investigated in a slightly different context by Hildenbrand and Hildenbrand (1978). However the fact that expectations about tomorrows prices and feasible transactions will make those expectations more likely to occur today, does not mean that the expectations are more likely to be correct tomorrow.

Indeed in the Neary Stiglitz paper expectations fulfilment becomes less probable.[4] Honkapohja and Ito (1979) assume agents receive only aggregate information about excess demand in markets. They are assumed to know the probability distribution of trading uncertainty, and thus from the aggregate information calculate the probability of either making their choice trade or being constrained, and also the expected level of the constraint. An equilibrium is a set of trades, based upon expected utility maximization, which reproduces the aggregate signals. The missing information crucial to the functioning of this model is that agents are not aware that if they all announce notional demands based upon only current prices, ignoring the probability of being constrained, an equilibrium with more output and employment will result. In the Honkapohja Ito treatment as in other exogenous price models agents have no knowledge of next periods prices and quantity constraints.[5]

In each of the economic models discussed above the behaviour of economic agents can be seen to be related to some form of cost minimization. However, rather than pointing out the costs implicit in each model, I shall now concentrate upon the general concept of cost, using the models discussed above as examples where appropriate.

Costs may be divided into three categories:

(i) Costs of setting up markets, establishing and calling prices and quantity constraints.

(ii) Opportunity costs associated with agents making transactions which are inter-temporally inefficient.

(iii) Transactions costs of actually arranging or revising trades.

Little attention will be paid to the first category of costs, the setting up of markets, except to note that this will require increasing returns to sale at some point. The work of Heller (1972) and Heller and Starr (1973) demonstrates the necessary techniques.

The second category of costs, opportunity costs associated with inter-temporally inefficient transactions appears particularly important in fix-price quantity rationing models. This may occur in one of two contexts. Firstly in quantity tatonnement fix-price models such as Neary and Stiglitz (1981) inter-temporal inefficiency arises in the following manner; agents current transactions are based upon information about current prices, quantity constraints and expectations of demands based upon only current prices, ignoring the probability of being constrained, an equilibrium with more output and employment will result. In the Honkapohja Ito treatment as in other exogenous price models agents have no knowledge of next periods prices and quantity constraints.[5]

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The third category of costs, transactions costs arising from the arrangement or revision of trade, are perhaps the most interesting, especially in the context of Neo-Keynesian macroeconomics. In this category are a number of different types of costs, which are listed below.

(i) Cost of actually attending a market - transportation costs, the utility cost of leisure foregone whilst engaged in trading, etc.

(ii) Costs of disseminating information about prices, stock availability and characteristics of goods.

(iii) Search costs involved in finding trading partners.

(iv) Costs of adjustment in disequilibrium situations.

Clearly (i)-(iv) are not mutually exclusive or exhaustive.[6] A considerable literature has developed on the costs of carrying out transactions at equilibrium prices. Notable contributions in this area are Kurz (1974), Hahn (1971, 1973) and Foley (1970), these and other relevant works are succinctly surveyed by Ulph and Ulph (1975). Models of this sort incorporate transaction costs in one of two ways. Firstly, individual agents directly employ the resources required to carry out transactions themselves. Each agent has a set of feasible transaction activities and a transactions technology which specifies what will be required to carry out any planned trades. Thus plans will be calculated to maximize agents objectives given the costs associated with the particular agents transaction technology set. Kurz (1974) is a good example of this sort of model. Secondly, transactions resources may be employed by some intermediary who earns a margin between his buying and selling price as payment for his services. Foley (1970) considers a model of this type.

This literature developed in the context of equilibrium price models has clearly had some impact on Neo-Keynesian economic models, where one might argue that transaction costs are in their most appropriate and influential setting. Benassy (1977) remarks that transaction costs may be associated either with demands or realized transactions, and suggests that costs should vary monotonically with transactions and be of the fixed cost variety when associated with demands. Costs C_i are associated with either demands $\tilde{z}_i$ or trades $\bar{z}_i$ such that $C_i = X[\tilde{z}_i, \bar{z}_i]$, Benassy suggests that these costs appear alongside transactions in agents utility functions, making agents maximization programmes as (3.1.1)

$$\left.\begin{array}{l} \text{Max } u_i(e_i+\bar{z}_i-C_i) \\ \bar{z}_i = \Phi_i[\tilde{z}_i|O_i(t)] \\ c_i = \chi[\tilde{z}_i,\bar{z}_i] \\ p\bar{z}_i = 0 \\ e_i + \bar{z}_i - c_i \geq 0 \end{array}\right) \quad (3.1.1)$$

where Φ_i describes the perceived rationing scheme, $O_{i(t)}$ is information available at t, p is price and e_i endowment.

Cost, here c_i, may be interpreted as transactions resources utilized by the agent himself in expressing demand or completing a transaction. Benassy continues to make the interesting observation that rationing schemes may be manipulable through transactions costs. A queueing or priority systems of rationing may not be manipulated once the ranking is known. However an agent may manipulate his ranking by for example arriving earlier in a queue, this of course implies the costs involved with queueing for a longer period.

The idea of adjustment costs associated with waiting in line in a queueing system has been investigated in a Non-Walrasian equilibrium context by Varian (1975). Varian demonstrates the effects of transactions costs in his treatment with the following simple model.

Workers maximize a utility function of the following form (3.1.2)

$$\text{Max} \quad u = a \log x + (1-a) \log (1-L) \tag{3.1.2}$$

$$\text{s.t} \quad px = L \tag{3.1.3}$$

a is a positive constant, $a<1$, total time is 1 unit and p here is the price of the consumption good x in terms of labour services.

From (3.1.2) and (3.1.3) the consumers Walrasian demand may be found to be

$$x^d = a/p \tag{3.1.4}$$

Assuming a fixed supply of the consumption good equal to unity the Walrasian excess demand may be written as (3.1.5).

$$Z = (a/p) - 1 \tag{3.1.5}$$

Varian then argues that if $Z \neq 0$, i.e., there is a positive excess demand, consumers incur disutility since they must wait in a queue to receive the good, further the magnitude of this transactions cost in foregone leisure time is monotonically increasing in both Z and x. Hence the utility function becomes

$$u = a \log x + (1-a) \log (1-L-4pZx) \tag{3.1.6}$$

and maximization subject to the budget constraint (3.1.3) yields a new goods demand (3.1.7)

$$x^d = a/(p+4pZ) \tag{3.1.7}$$

This gives an 'accumulated' excess demand (3.1.8)

$$Z' = a/p + a/(p+4a-4p) - 2 \tag{3.1.8}$$

and thus a non-Walvasian equilibrium price

$$p = 2a/3, \tag{3.1.9}$$

giving the system two equilibria, one at p=a which is the Walrasian equilibrium, and another at p=2a/3 which is the non-Walrasian equilibrium. Thus if this model is perturbed away from Walrasian equilibrium the transactions costs involved in returning, due to the excess demand, effect individuals behaviour and the system may settle at the non-Walrasian equilibrium with p=2a/3. To reconcile this with the production side of the economy we may suggest that as workers have 'spent some leisure time' in obtaining the good, then the disutility of work will rise and effective labour supply will be revised down to equate it with effective labour demand. Transactions costs in the models of Benassy (1977) and Varian (1975) may be loosely described as belonging to our type (i) costs of actually attending a market. If, as seems appropriate in neo-Keynesian models, agents do not know in advance whether a visit to a particular market will yield any trade, their expectations of the level of transactions they may successfully complete will be important in this context. Agents will not be prepared to incur the transactions costs if they attach a low probability to obtaining any supply or if they expect to be severely constrained in the transaction.

Costs of types (ii) and (iii) may well be considered as naturally going together. Agents may either send out signals about the transactions they wish to carry out at different prices, or alternatively they may go to the market themselves, and search for other agents willing to carry out the other side of their desired transactions. These are the costs of disseminating or collecting information. In reality we may think of informative advertising as a cost of disseminating information and job search as involving information collecting costs. Futia (1977) constructs a non-Walrasian equilibrium model which implicitly relies on both the cost of disseminating and collecting information about wage rates. Futia's approach is to consider a continuum of workers who quote different nominal wage rates, they attach to each wage a probability of employment and thus their quotes maximize expected utility. There is a positive probability of employment attached to each nominal wage as they know firms only sample a subset of all workers in their search for employees and thus workers perceive a chance of employment even at a high wage quote since they may 'get lucky' and end up in a labour pool where all workers are quoting high wages. Futia suggests that firms only sample some subset of potential employees because 'information is costly',clearly he means that there are search cost associated with looking for employees, although these are not explicitly modelled. The model also requires, and this Futia does not appear to notice, that there are costs to the workers in disseminating information about their wage quotes, otherwise all workers will send signals to all firms, and thus,they may believe, guarantee themselves employment. Indeed these costs must be infinitely large since Futia does not allow workers to engage in this activity at all. Firms facing search costs are then assumed to employ workers from their sample up to the point of the expected average real wage.

Cost of adjusting in disequilibrium situations may be of two types. Costs of adjusting trades and costs of adjusting demands. These are the same costs as discussed above, but they are more important in the context of disequilibrium since they are incurred more frequently. In a disequilibrium situation information rapidly loses its value. Economic agents are required to gather and transmit information often. In the Neo-Keynesian macroeconomic literature little attention has been given to disequilibrium situations, interest has been directed towards showing that non-Walrasian equilibrium exist and examining their properties. This has probably obscured the importance of transactions costs to this literature.

In the following section (3.2) a fix-price model of an aggregate macro economy will be presented in which agents face adjustment costs in revising demands. These costs are assumed to vary monotonically with demand adjustments.

3.2 Uncertainty, Adjustment Costs and Expected Keynesian Unemployment

Most macroeconomic models with quantity rationing follow the lead of Clower (1965) in assuming that when consumers and producers face a new vector of fixed prices at the beginning of a market period they calculate and express notional supplies and demands. Previous market experience may lead traders to place a low probability upon realising these supplies and demands, but since quantity adjustment is assumed both costless and frictionless, traders will always try these out first. If however there are costs involved in adjusting transactions demands then agents initial trade offers must take these into account.

In this model quantity adjustment costs are introduced in the form of resources consumed in the adjustment process.[7]

Agents are aware of adjustment costs, but have to state initial transactions demands before the state of the world is known, consequently these transactions demands will be based upon the maximization of Von Neumann-Morgenstern objective functions. On learning the true state of the world, as characterised by their trade opportunities, agents then adjust optimally away from their initial trade vector.

The model developed in this section will demonstrate that the introduction of transactions costs gives results significantly different from standard fix-price models such as Malinvaud (1977) and Muellbauer and Portes (1978). Three new characteristics arise. Firstly, agents initial transactions demands will respond discontinuously to changes in the models parameters, a marginal rise in the subjective probability attached to a state of unconstrained trade may, for example, lead to a large upward jump in consumer demand. Secondly, transaction levels are less variable in the presence of adjustment costs. Thirdly the comparative static properties of the short-run equilibria that emerge will be found to be considerably modified by this treatment.

In the analysis to follow we examine only 'Keynesian' cases, however a very similar treatment may be used to examine the 'Repressed Inflation' possibilities of the model. These are omitted for brevity.

The Consumers Problem

Consider a representative consumer whose utility

depends on the amount of labour he will be able to sell to a representative firm at the end of the quantity tatonnement.

Let $f(L)$ be the consumers subjective probability density function over states of the world.

L^c be the level of labour sales that the consumer would choose at given prices in the absence of uncertainty and adjustment costs.[8]

thus define:

$$\varepsilon = \int_{L^c}^{\infty} f(L)\,dL \qquad \text{the probability of being unrationed}$$

$$1-\varepsilon = \int_{\infty}^{L^c} f(L)\,dL \qquad \text{the probability of being rationed}$$

$$E(L) = \bar{\bar{L}} \qquad \text{the expected ration level.}$$

The mathematical expectation of the distribution truncated at $L = L^c$

Assume that the consumer behaves as if there are two possible states of the world, a good state hereafter labelled i characterised by $L \geq L^c$ to which is attached the probability ε, and a bad state labelled j characterised by $L = \bar{\bar{L}}$ with attached probability $1-\varepsilon$.

The consumer faces a two-stage maximization problem. First to maximize expected utility, and state initial transactions demand. Second having learned the true state of the world, he maximizes utility subject to the costs of adjusting away from the initial trade vector.

Stage 1: Expected Utility Maximization, Initial Transactions Demands

Assume the consumer maximizes a Von Neumann-Morgenstern utility function of the following form:[9]

$$\text{Max } E(u) = \varepsilon\{\log(X_i - c|X^* - X_i|) + \log(T-L) + \log M_i\}$$
$$+ (1-\varepsilon)\{\log(X_j - c|X^* - X_j|) + \log(T-\bar{\bar{L}}) + \log M_j\} \quad (3.2.1)$$

S.T.

$$\text{(i)} \quad M_o + wL = pX_i + M_i \quad (3.2.2)$$

$$\text{(j)} \quad M_o + w\bar{\bar{L}} = pX_j + M_j \quad (3.2.3)$$

where M_o, T, p, w are parameters representing initial money balances, total work time, good and labour price respectively. X and M are variables representing goods and final money balances. c is the adjustment cost parameter $(1 \geq c \geq 0)$,[10] and superscript * indicates initial transactions demand.

To make the programme (3.2.1)-(3.2.3) differentiable note that $X_i > X^* > X_j$ must hold,[11] and rewrite the maximand as

$$\underset{X_i, X_j, \overset{*}{X}, L}{\text{Max }} E(u) = \varepsilon\{\log(X_i - c(x_i - \overset{*}{X})) + \log(T-L) + \log(M_o + wL - pX_i)\}$$

$$+ (1-\varepsilon)\{\log(X_j - c(X^* - X_j)) + \log(T-\bar{\bar{L}}) + \log(M_o + w\bar{\bar{L}} - pX_j)\} \quad (3.2.4)$$

S.T.

$$X_i - X^* \geq 0 \quad (3.2.5)$$

$$X^* - X_j \geq 0 \quad (3.2.6)$$

The problem now satisfies the Arrow-Endhoven sufficiency conditions for quasi-concave programming.

Forming the Lagrangian from (3.2.4)-(3.2.6) and from the Kuhn-Tucker conditions it can be shown that there are three cases of interest.[12] These are:

(i) $X_i > X^* > X_j$

(ii) $X_i = X^* > X_j$

(iii) $X_i > X^* = X_j$

Examining each of these three cases in turn

(i) $X_i > X^* > X_j$ Here the expected utility maximizing initial transaction demand lies between the two transactions levels that the consumer expects to adjust to once the state of the world has been revealed.

From the Kuhn-Tucker first order conditions we obtain:

$$X^* = \frac{1}{cp}\left[(2\epsilon-1+c)M_o + w(\epsilon(1+c)\bar{L} - (1-\epsilon)(1-c)T)\right] \tag{3.2.7}$$

$$X_i = \frac{1}{2}\left[\frac{M_o + wT}{p} - \left(\frac{c}{1-c}\right)X^*\right] \tag{3.2.8}$$

$$X_j = \frac{1}{2}\left[\frac{M_o + w\bar{L}}{p} + \left(\frac{c}{1+c}\right)X^*\right] \tag{3.2.9}$$

$$L = \frac{1}{4}\left[(2+w)T + (1-2/w)M_o - p\left(\frac{c}{1-c}\right)X^*\right] \tag{3.2.10}$$

Using (3.2.7)-(3.2.9) we may define the range of parameter values over which the consumer will make the decision $X_i > X^* > X_j$, this range is as below:

$$\begin{aligned} c^2(1+2\epsilon+c)M_o + wc(1+\epsilon^2+c+\epsilon c^2)\bar{\bar{L}} &> 2(2\epsilon-1+\epsilon c+c^2)M_o + w\epsilon(2+3c+c^2)\bar{\bar{L}} \\ - wT[2-c+c^2(\epsilon-\epsilon c+c-2) - \epsilon(2+c-3c\epsilon^2)] &> 2\epsilon c^2 M_o + wc(1+c)\bar{\bar{L}} \end{aligned} \tag{3.2.11}$$

Interpretation of (3.2.7)-(3.2.10) is as follows. If (3.2.11) holds with two inequalities, then X^* is initial goods transactions demand, L initial labour supply, and X_i and X_j are the goods transactions that the consumer plans in the good and bad states of the world, given that case (i) is appropriate to the parameters of the model.

(ii) $X_i = X^* > X_j$ Here the consumers expected utility maximizing initial transaction demand is equal to the transactions he plans to undertake if the good state of the world occurs.

From the Kuhn-Tucker conditions we obtain the following quadratic in X_i.

$$X_i^2 \{p^3(1+c)(2\varepsilon + 2(1-c) + 1) - 2(1+\varepsilon)p\} + X_i\{(1+c)p^2/(M_o+w\bar{L})$$

$$(2\varepsilon-2(1-c) + 1) + (\varepsilon+1-c)(2M_o+4wT) - (1-\varepsilon)(2M_o+4wT)\}$$

$$+ \{\varepsilon-1+c\} \ (1+c) \ p(M_o+w\bar{\bar{L}}) \ (2M_o + 4wT) = 0 \qquad (3.2.12)$$

Assume (3.2.12) has only one positive real root.

The range of parameter values over which this case obtains are given when the left-hand inequality of (3.2.11) is violated.

(iii) $X_i > X^* = X_j$ Here the consumer expected utility maximizing initial transaction demand is equal to the transactions he plans to undertake if the bad state of the world occurs.

From the Kuhn-Tucker conditions we find the consumers initial transaction demand is given by the following quadratic:

$$X_j^2 \ c\left[\{((1+c) + (1-\varepsilon) \ c) \ 3p^2-3p^3+\varepsilon p\}\right] + X_j \ \{(1+c)+(1-\varepsilon) \ c\}\cdot p^2$$

$$\left[3c\{M_o+w\bar{\bar{L}}\} - t(1-c)\{M_o+wT\}\right] - p^2-6(1-c)\{M_o+ wT\}$$

$$- \ \varepsilon c \{M_o+w\bar{\bar{L}}\} + \left[[(1+c) + (1-\varepsilon)c] \ [M_o+w\bar{\bar{L}}]+[6(1-c)[M_o+wT]p]\right] = 0$$

$$(3.2.13)$$

It is assumed that (3.2.13) has only one positive real root. The range of parameter values over which this case obtains are given when the right-hand inequality of (3.2.11) is violated.

We have thus established the first of our results, since a small change in one of the parameter values may cause one of the inequalities of (3.2.11) to be reversed, this will cause our consumer to make a switch from (3.2.7) to (3.2.12) or (3.2.13) in determining initial good demand, X^*. To clarify that this is in fact the case we illustrate this result with the following diagram (3.2.1).

If we define a and b such that

$$X^* = \begin{cases} (3.2.7) & \text{if } a \geq \epsilon \geq b \\ (3.2.12) & \text{if } a < \epsilon \\ (3.2.13) & \text{if } b > \epsilon \end{cases}$$

where a and b are defined by the other parameters c, M_o, $\bar{\bar{L}}$ and T in (3.2.11).

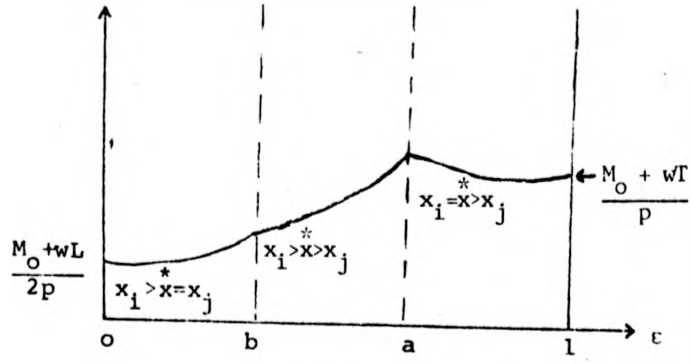


Figure (3.2.1)

As figure (3.2.1) demonstrates if ε is the neighbourhood of a or b then a small change in the probability attached to the good state of the world will cause the consumer to base initial goods demand X^* upon a different function. In general we observe that variations in the models parameters will induce switching between the different forms of the consumers initial transaction demand function.

Stage 2: Final Transactions

Having determined the consumers initial transaction demands we now examine how these will be revised once he has learned the true state of the world. We recall that $\bar{L}$ was the consumers subjective mathematical expectation of the rationing level, which need not be correct. We denote the actual rationing level, the firms labour demand, by $\bar{L}$.

To discover how the consumers initial transaction demands are revised in response to $\bar{L}$ we write this maximand as below:

$$\text{Max } U = \log(X^* - X_1 - X_2 - cX_1 - cX_2) + \log(T - \bar{L}) + \log M_1 \qquad (3.2.14$$

S.T.

$$M_o + wL = p(X^* + X_1 - X_2) + M_1 \qquad (3.2.15$$

The two new variables X_1 and X_2 represent upward and downward adjustments from X^* respectively. We require these variables to identify the range of $\bar{L}$ over which the consumer will choose to revise trades upwards downwards or not at all.

Substituting (3.2.15) into (3.2.14) differentiating and evaluating the Kuhn-Tucker conditions at $X_1=X_2=0$ we obtain:

$$\left(\frac{1-c}{X^*}\right) - \frac{p}{M_o + w\bar{L} - pX^*} \leq 0 \qquad (3.2.16)$$

$$-\left(\frac{1+c}{X^*}\right) - \frac{p}{M_o + w\bar{L} - pX^*} < 0 \qquad (3.2.17)$$

From (3.2.16) and (3.2.17) we obtain:

$$\bar{L}_L = \frac{1}{w}\left[p(X^* + (\frac{X^*}{1+c})\ ) - M_o\right] \leq \bar{L} \leq \frac{1}{w}\left[p((\frac{X^*}{1-c}) + X^*)-M_o\right]=\bar{L}_u \quad (3.2.18)$$

From expression (3.2.18) we may define the range of ration levels $\bar{L}$ over which the consumer decides to adjust his goods demand upwards, downwards, or to leave it unaltered.

Define $\bar{L}_L$ and $\bar{L}_u$ as the ration levels at which (3.2.18) holds with $\bar{L}$ as an equality with the left and right hand terms respectively.

We may make the following statements:

(i) If $\bar{L} > \bar{L}_u$ Then the consumer will revise his goods purchases upwards. The consumers maximand becomes:

$$\text{Max } U = \log(X-c(X-X^*)) + \log(T-\bar{L})+\log(M_o+w\bar{L}-pX) \qquad (3.2.19)$$

Maximization yields

$$X = \frac{1}{2}\left[\frac{M_o + w\bar{L}}{p} - (\frac{c}{1-c})X^*\right] \qquad (3.2.20)$$

(ii) If $\bar{L}_u > \bar{L} > \bar{L}_L$ Then the consumer will leave his goods purchases unaltered

hence: $X = X^*$ (3.2.21)

(iii) If $\bar{L} < \bar{L}_u$ The consumer will revise his goods purchases downwards.

The consumers maximand becomes:

$$\text{Max } U = \log(X - c\,(X^* - X)) + \log(T - \bar{L}) + \log(M_o + w\bar{L} - pX) \quad (3.2.22)$$

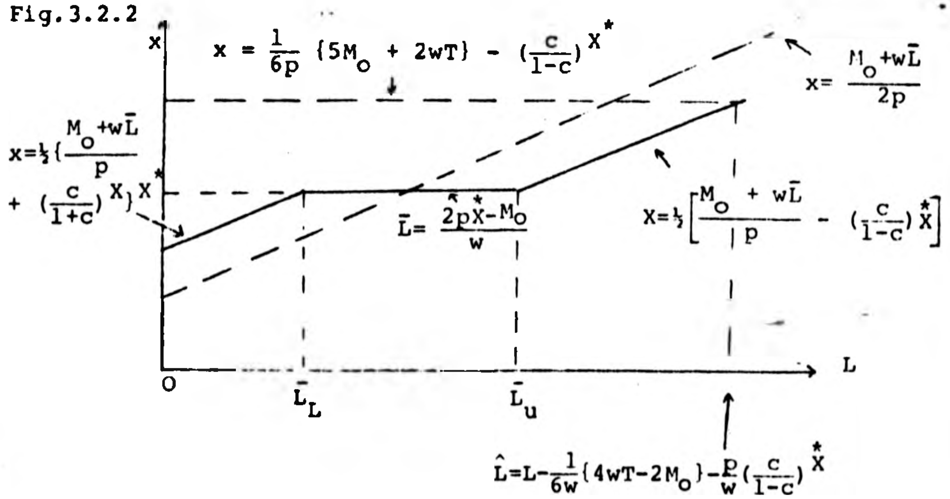
which yields:

$$X = \frac{1}{2}\left[\frac{M_o + w\bar{L}}{p} + \left(\frac{c}{1+c}\right) X^*\right] \quad (3.2.23)$$

Expressions (3.2.20), (3.2.21) and (3.2.23) describe the consumers response to all levels of employment $\bar{L}$ which he regards as constraining his behaviour. If no binding constraint occurs it can be shown that goods purchases and labour sales are as defined by (3.2.8) and (3.2.10) these trades the consumer planned to adjust to in the good state of the world.

Using (3.2.20), (3.2.21) and (3.2.23) we may now represent the consumers labour constrained goods demand curve diagramatically, as in figure (3.2.2)

Fig.3.2.2



The effects of uncertainty and adjustment costs upon the representative consumers are clearly illustrated by our diagram. The broken diagonal line represents the labour constrained goods demand curve that the consumer would express in the absence of uncertainty and adjustment costs.[13] The solid kinked line is the goods demand curve developed above. Over the range O-$\bar{L}_L$ the consumers goods demand curve is given by (3.2.23). We notice immediately that the effects of uncertainty and adjustment costs are to increase goods purchases, for any $\bar{L}$ in the range, above the level that would be demanded in their absence.

In the range $\bar{L}_u$-$\hat{L}$ consumer goods demand is given by (3.2.2[illegible]) and will be lower than in the absence of uncertainty and adjustment costs. Whilst for the range $\bar{L}_1$ - $\bar{L}_u$ we see that changes in the level of the ration have no effect upon goods demand, initial trades are left unaltered.

We now turn our attention to the representative producer anticipating a shortfall in effective demand.

3. The Producers Problem

We treat our producer in a symmetric manner to the consumer, as he too has a subjective probability distribution over possible states of the world, with states characterised by the level of effective demand. The producer behaves as though one of two possible states of the world may occur. A good state characterised by free trade and a bad state characterised by a 'known' sales ration $\bar{\bar{X}}$.[14] Probabilities ψ and $1-\psi$ are attached to the good and bad states respectively.

The producer faces a two stage maximization problem, first, to maximize expected profit and express initial transactions demand for labour, secondly, when the true state of the world is learned maximize actual profit subject to costs of adjustment.

Stage 1: Expected Profit Maximization, Initial Transaction Demand

We assume our producer maximizes the following expected profit function[15]:

$$\begin{aligned} \max_{X_1, L_k, L_m, L^*} E(\pi) &= \psi\{pX - wL_k - \phi\,(L^* - L_k)^2\} \\ &+ (1-\psi)\{p\bar{\bar{X}} - wL_m - \phi\,(L^* - L_m)^2\} \qquad (3.2.24) \end{aligned}$$

$$\text{S.T.} \qquad X \leq AL$$

All notation as in the consumers problem, except k and m, subscripts refer to the producers characterization of good and bad states. Adjustment costs are quadratic and ϕ is a constant $0 \leq \phi \leq 1$.

The producer may choose any point within the production set, he may thus hoard labour rather than incur adjustment costs. We form the Lagrangian from (3.2.24)

$$\mathcal{L} = \psi\{pX - wL_k - \phi(L^* - L_k)^2\} + (1-\psi)\{p\bar{\bar{X}} - wL_m - \phi(L^* - L_m)^2\}$$

$$+ q_m(L_m - \bar{\bar{X}}/A) + q_k(L_k - X/A) \qquad (3.2.25)$$

where q_k and q_m are the shadow prices on the production constraints in the two states of the world.

Differentiating (3.2.25) and examining the Kuhn-Tucker first order conditions we see that our producer considers the two following cases.[16]

(i) $q_k, q_m > 0$ The production constraint binds in both expected states of the world.

From the first order conditions of (3.2.25) we obtain:

$$q_m = w - \psi Ap \qquad (3.2.26)$$

$$q_k = \psi Ap \qquad (3.2.27)$$

$$L_m = \bar{\bar{X}}/A \qquad (3.2.28)$$

$$L_k = \frac{\bar{\bar{X}}}{A} + \frac{1}{2\phi\ (1-\psi)}\{A_p - w\} \qquad (3.2.29)$$

$$X = \bar{\bar{X}} + \frac{A}{2\phi\ (1-\psi)}\{Ap - w\} \qquad (3.2.30)$$

$$L^* = \frac{\bar{\bar{X}}}{A} + \frac{\psi}{2\phi\ (1-\psi)}\{Ap - w\} \qquad (3.2.31)$$

(ii) $q_k > 0,\ q_m = 0$ The production constraint binds only in the good state.

$$L_k = \frac{1}{(2-\psi)\ 2\phi}(2pA - (1+3\psi)\ w) \qquad (3.2.32)$$

$$X = \frac{1}{(2-\psi)\ 2\phi} \quad (2p - (1+3\psi)\ \frac{w}{A}) \tag{3.2.33}$$

$$L^* = \frac{1}{(2-\psi)\ 2\phi} (\psi pA - (2-1)\ w) \tag{3.2.34}$$

$$L_m = \frac{1}{(2-\psi)\ 2\phi} (\psi pA - (1+3\psi)\ w) \tag{3.2.35}$$

$$q_k = A\psi p \tag{3.2.36}$$

The producer decides whether to base initial transactions demand upon (3.2.26)-(3.2.31) or (3.2.32)-(3.2.36) by choosing the set which yields the higher expected profits at the given parameter values.

This produces initial transactions demand for labour as given by either (3.2.31) or (3.2.34). We now investigate how these initial demands will be revised once the true state of the world has been learned.

Stage 2: Final Transactions

Denote the actual ration level which characterizes the true state of the world $\bar{X}$. The producer may find himself in one of two circumstances. Consider first the case when $\bar{X}$ binds, we write the producers maximand as (3.2.37):

$$\text{Max } \pi = p\bar{X} - w\ (L + L_h) - \phi(L^* - L - L_h)^2\ L_h \tag{3.2.37}$$

where L is labour used to produce output

L_h is non-productive hoarded labour

hence

$$L = \bar{X}/A \tag{3.2.38}$$

From (3.2.37) and (3.2.38) we obtain:

$$L_h = \max \begin{cases} L^* - \bar{X}/A - w/2\phi \\ 0 \end{cases} \tag{3.2.39}$$

Expression (3.2.39) describes how the producer will respond to a shortfall in effective demand. By either employing just sufficient labour to produce the goods ration $\bar{X}$, in which case $L_h = 0$, or by withdrawing some labour from the productive process but hoarding it to avoid the adjustment costs, in this case $L_h = L^* - \bar{X}/A - w/2\phi$.

If $\bar{X}$ does not constrain the producer then his maximand may be written as (3.2.40):

$$\text{Max } \pi = pAL - wL - \phi\,(L^* - L)^2 \qquad (3.2.40)$$

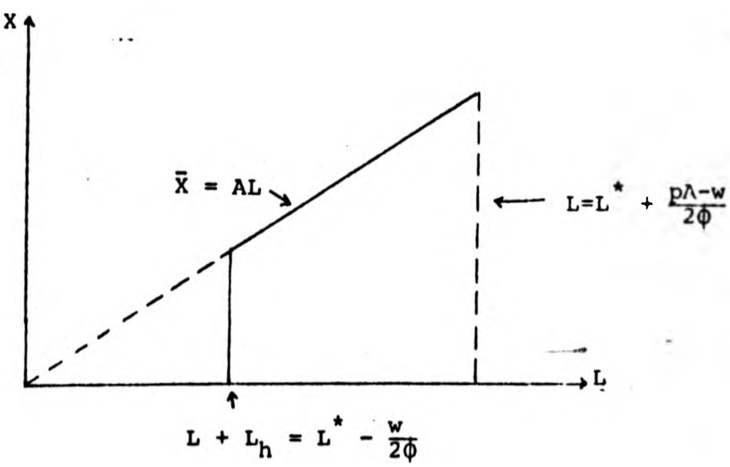
maximization yields

$$L = L^* + \frac{pA - w}{2\phi} \qquad (3.2.41)$$

Expression (3.2.41) defines the maximum employment level, the producers optimal upward adjustment point if trades are unconstrained. Notice however that the producers initial subjective probability distribution over states of the world effects his final transactions via L^*.

We may now represent the producers goods constrained labour demand curve as figure (3.2. 3)

Figure (3.2.3)

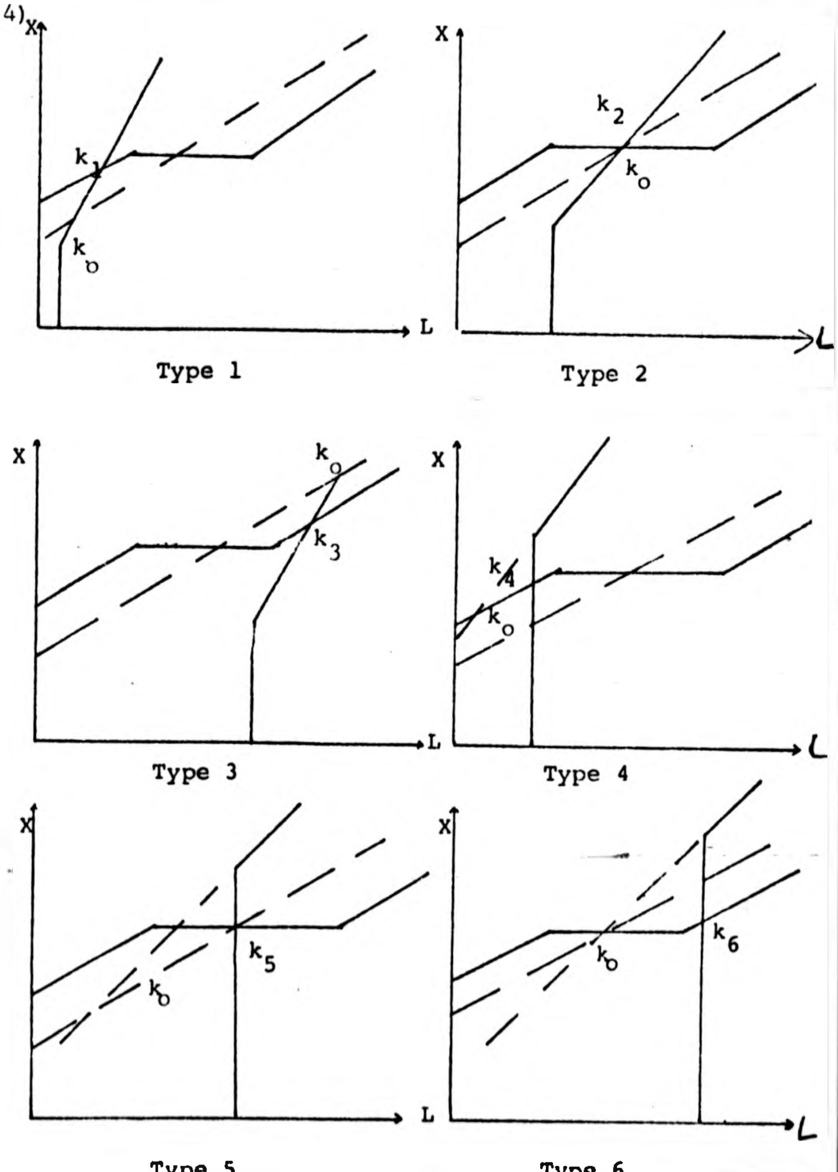


Having now established consumer and producer behaviour given that each anticipated a Keynesian unemployment regime in the forthcoming market period we may now turn our attention to the equilibria this behaviour generates.

4. MARKET PERIOD EQUILIBRIA

We define a market period equilibria as the pair (X,L) which satisfy both the consumers labour constrained goods demand function and the producers goods constrained labour demand function. Combining figures (3.2.2) and (3.2.5) we may now define and illustrate six different Keynesian outcomes.

Fig. (3.2.4)



Algebraically these 6 equilibria types are categorized as in table (3.2.1).

Table (3.2.1)

L \ X	$\frac{1}{2}\left[\frac{M_o+w\bar{L}}{p} + \left(\frac{c}{1+c}\right)X^*\right]$	X^*	$\frac{1}{2}\left[\frac{M_o+w\bar{L}}{p} - \left(\frac{c}{1-c}\right)X^*\right]$
$\frac{\bar{X}}{A}$	TYPE k_1	TYPE k_2	TYPE k_3
$L^* - \frac{w}{2\phi}$	TYPE k_4	TYPE k_5	TYPE k_6

Figure (3.2.4) (1)-(6) demonstrates the effects uncertainty and adjustment costs have upon the equilibria.[17] In each diagram the broken lines represent the demands the agents would express in the absence of uncertainty and adjustment costs and the points denoted K_o are the equilibria that would thus obtain.[18] The equilibria denoted k_1 - k_6 are defined by the functions incorporating uncertainty and adjustment costs, developed earlier. Inspection of figure (3.2.7) (1)-(6) reveals how the position of the equilibria in goods labour space are modified by our treatment, in each case we see that adjustment, either from L^*, X^* or both has been restricted by adjustment costs. This is the second way in which our treatment is significantly different from the standard Malinvaud type approach. Notice however that in type 6 equilibria the producers adjustment costs of moving away from L^* and the consumers cost of moving away from X^* conflict, and adjustment in one variable may be greater than in the non-adjustment cost case, (k_o), depending upon the form and magnitude of the adjustment costs.

Finally let us examine the comparative static effects of a change in agents expectations, this may be due to a relocation of the mean of their subjective probability distributions, an agent adjusting or introducing skewdness into the distribution, or the adoption of a different form of probability distribution entirely.[19]

Our earlier analysis ensures

$$\frac{\partial L^*}{\partial \psi} \text{ and } \frac{\partial X^*}{\partial \varepsilon} > 0$$

Thus:

$$\frac{\partial X}{\partial \psi}, \frac{\partial L}{\partial \psi}, \frac{\partial X}{\partial \varepsilon}, \frac{\partial L}{\partial \varepsilon} \quad \text{take the sign of}$$

$$\frac{\partial X}{\partial X^*}, \frac{\partial L}{\partial X^*}, \frac{\partial X}{\partial L^*}, \frac{\partial L}{\partial L^*} \quad \text{respectively}$$

We now examine the comparative static effects of change in the subjective probability agents attach to states. From the definition of equilibria presented in table 1 we obtain table (3.2.2).[20]

Table (3.2.2)

		$\frac{\partial X}{\partial X^*} \cdot \frac{\partial X^*}{\partial \varepsilon}$	$\frac{\partial L}{\partial X^*} \cdot \frac{\partial X^*}{\partial \varepsilon}$	$\frac{\partial X}{\partial L} \cdot \frac{\partial L^*}{\partial \psi}$	$\frac{\partial L}{\partial L^*} \cdot \frac{\partial L^*}{\partial \psi}$
EQUILIBRIA	K1	+	+	0	0
	K2	+	+	0	0
	K3	-	-	0	0
	K4	+	0	+	+
	K5	+	0	0	+
	K6	-	0	+	+

Table (3.2.2) tells us how a change in the subjective probability the consumer ($\partial\varepsilon$) and producer ($\partial\psi$) attaches to the state of the world occurring in which he is unrationed will effect final goods and labour sales.

To discuss the results of table 2 in detail would be somewhat repetitive, and inspection of figures (3.2.3), (3.2.4) and (3.2.5) should provide the reader sufficient intuition. We wish to demonstrate how this approach gives different comparative static results from the standard treatments and illustrate this by considering an example using a type K equilibria.

Example

[illegible] an increase in consumer optimism may be represented in a standard type treatment. We shall assume that increased optimism means that the consumer expects to be able to sell more labour, and places a greater probability on selling his choice labour supply, in the next period. We assume that his intertemporal preferences are such that this will imply that he gains less utility from additions to money balance at the current period. Thus if his maximand is as (3.2.42)

$$\max_{X} u = \log X + \log (T-\bar{L}) + \alpha_i \log M_1 \qquad i = 1,2 \tag{3.2.42}$$

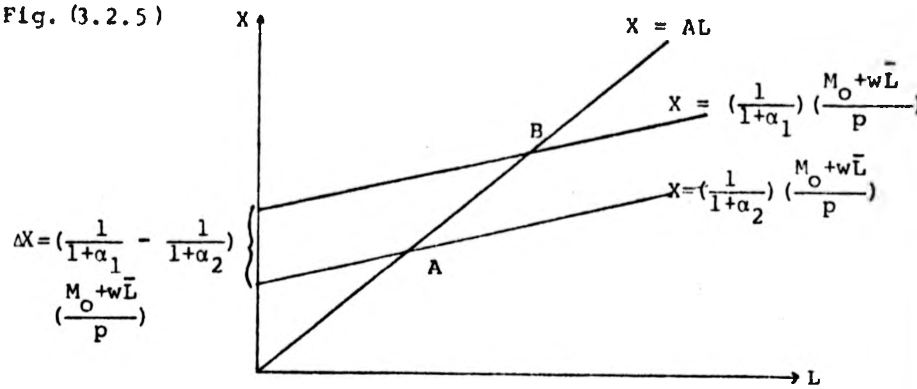
where α_1 is the optimistic and α_2 the pessimistic weight of current period money balances in the utility function, $\alpha_1 < \alpha_2$.

It may easily be shown that an increase in consumer optimism and subsequent reweighting $\alpha_2 \to \alpha_1$ will yield the following change in current period goods demand.

$$\Delta X = \left(\frac{1}{1+\alpha_1} - \frac{1}{1+\alpha_2}\right)\left(\frac{M_0 + wL}{p}\right) \tag{3.2.43}$$

Expression (3.2.43) simply states that an increase in consumer optimism yields an upward shift in his current period goods demand, as indicated in Fig. (3.2.5):

Fig. (3.2.5)



We have imposed the production function upon figure (3.2.5), illustrating that an increase in consumer optimism moves the equilibrium from A to B, thus having the comparative static effect of raising both output and employment. In contrast now consider how an increase in consumer optimism will effect output and employment in our type k_3 equilibrium. From our preceding analysis and table (3.2.2) it follows that figure (3.2.6) describes the comparative statics in this case. We see that the equilibria moves from A to B as the consumer attaches a high probability to the good state of the world occurring, but that this has the comparative static effect of reducing both output and employment, the opposite effect to the standard case. The effect will persist until the equilibria changes to

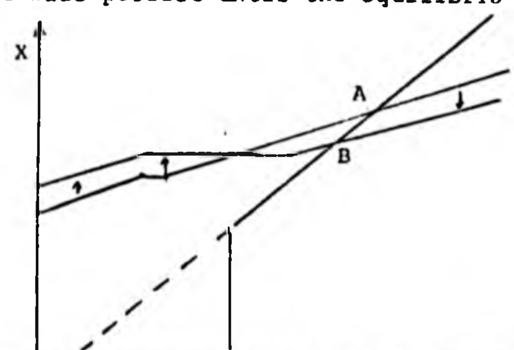


Fig. (3.2.6)

type k_2, then the comparative static effects will revert to those found in the standard type treatments.

Conclusion

In this model we examined the behaviour of agents who are aware of uncertainty and adjustment costs. We argued that they will base their initial transaction demands upon the maximization of Von Neumann-Morgenstern objective functions, and that these initial demands would be revised once the true state of the world has been revealed. It was found that to allow agents to behave in this sophisticated manner, brings traders expectations in the form of subjective probability distribution over states of the world to the fore, giving the model several new features. We noticed that initial transactions demands took different functional forms depending upon parameter magnitudes. Also we saw how adjustment costs and uncertainty altered the shape of the constrained demand curves for both goods and labour. The new demand curves were used to define six different Keynesian short-run equilibria, which were characterized by transaction levels and/or comparative static properties at variance from those found in a more standard treatment. Introducing uncertainty and adjustment costs, has made expectations crucial and has introduced rigidities in quantity adjustment which modify the equilibria, not in any ad hoc fashion but as a result of standard maximization procedures. Thus it may be seen in this section that the introduction of transactions costs has an important impact on this class of Neo-Keynesian models. Indeed it may be argued that the approach becomes more Keynesian in spirit.

FOOTNOTES

1. It may however be the case in Varians model that a Nash type equilibrium obtains where no one individual firm will view it as in its own interest to manipulate its labour demand.

2. The maximizing behaviour of this agent is never considered.

3. This of course ties in with Leijonhufvud comments about futures markets being incomplete.

4. For example, expected unemployment tomorrow leads to a greater supply of labour today and some accumulation of money balances. These money balances will be carried forwards and will be associated with a high marginal disutility of labour tomorrow lowering the notional labour supply, making a Keynesian unemployment equilibrium less likely.

5. In this model it is unclear what happens when new relative prices are announced, agents must express demands before they can receive aggregate signals. These demands can only be based upon prices and expectations. It is difficult to envisage how a signal reproducing equilibrium different from a Muellbauer and Portes standard fix-price equilibrium will arise.

6. Salop (1973) for example has examined involuntary unemployment arising from the presence of training costs.

7. Agents organise their own transactions in this model, and as there are no middle men the adjustment costs will depend upon individual transactions technologies. These are assumed identical for expositional purposes.

8. Since it is assumed that the model is on the 'Keynesian regime' L^c will be assumed to not depend upon goods availability.

9. The functional form is chosen for ease of exposition.

10. Adjustment costs here are symmetric, whereas in reality it may be that strong assymetries exist, these would only modify the results of the model quantatively.

11. This condition states that the initial transactions demand must fall between the planned good and bad transactions levels. A simple proof that this is so may be found in the appendix to this chapter.

12. The case where $x_1 = x^* = x_j$ is the other possibility here but is clearly uninteresting.

13. The demand curve is obtained by rearranging the first order conditions of the following programme.

$$\max_{x} u = \log x + \log(T-\bar{L}) + \log(M_O + w\bar{L}-px)$$

14. Again the mathematical expectation of the truncated subjective probability distribution, where the distribution is truncated at the notional level of output.

15. A linear production function is assumed for ease of exposition.

16. $q_k = 0$ would imply that the producer plans to hoard labour, not apply it to capital, when he can sell all he can produce. Clearly this will never be chosen.

17. It is clear that multiple equilibria will exist when the models parameters take certain values.

18. The Equilibria K_c would occur in the Muellbauer and Portes (1978) or Malinvaud (1977) type structures.

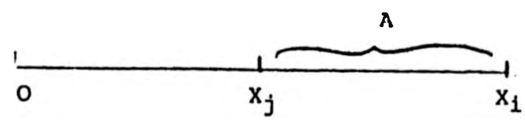
19. We cannot explain exactly how the subjective probability may change without some examination of the learning processes of agents, which is beyond the scope of this analysis.

20. The comparative static effects may be signed but cannot be quantified since x^* and L^* may jump due to the discontinuity in the first stage of agents maximization problems.

APPENDIX

We wish to establish $X_i \geq X^* \geq X_j$.

We know $X_i \geq X_j \ \forall \ p,w$ since X_i represents unconstrained trade and we do not allow forced consumption. Hence to establish our proposition we need only demonstrate X^* falls within the range $OX_i - OX_j$.



Define $OX_i - OX_j$ as A.

Let us choose a point outside the range labelled a and a point inside the range labelled b, such that

$$\varepsilon u[-c(a)] = \varepsilon u[-c(b)]$$

The expected utility loss of adjusting back to X_i from each of the two points is equal.

If we now consider the expected utility loss of adjusting back to X_j from a and b we may write:

$$(1-\varepsilon)u[-c(A+a)] + \varepsilon u[-c(a)] < (1-\varepsilon)u[-c(A-b)] + \varepsilon u[-c(b)]$$

Hence any point above X_i is clearly dominated by a point inside the range $OX_i - OX_j$.

Hence $X_i \geq X^* \geq X_j$ clearly follows if we use the same argument applied to points below X_j .

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4. EXPECTATIONS IN EQUILIBRIUM AND DISEQUILIBRIUM

4.1 Tatonnement and Non-Tatonnement Models

Chapters 2 and 3 examined the roles played by expectations in Neo-Keynesian Models, and the properties of these classes of models which make expectations so important. This chapter looks at expectations in tatonnement and sequential trading models and considers their importance in the dynamic adjustment processes which take place in the different transactions structures. Grossman (1969) poses the basic questions succinctly:

> 'The assumption of recontracting permits an ingenious simplification of the theory of markets. Moreover it may provide a rough approximation of the process by which market equilibrium is attained. However, ever since Walras invented his tatonnement process, economists have conjectured that the assumption of recontracting may seriously obscure understanding of markets while in disequilibrium. The essential theoretical question involved here is the following: what is the causal relationship between trading at non market clearing prices and the true paths of prices and quantities? In particular, how does the absence of recontracting effect the characteristics of the market equilibrium and its stability properties?'

A full answer to this question should explain how individual agents respond to different market situations by adjusting their transactions demands and the prices they wish to trade at, given their information and expectations. Generally the Neo-Keynesian literature has not attempted to answer the question in this manner.[1] Most approaches follow the initial hypothesis of Leijonhufvud (1968) and assume that quantity adjustment precedes price adjustment. An equilibrium in quantities is first established and then price adjustment takes place upon the basis of effective demands. The economy goes through three adjustment processes.

(i) Adjustment to the short-run fix-price equilibrium: agents adjust their trades to be consistent with the constraints they face.

(ii) Adjustment of non-perishable goods or asset stocks, over successive periods of fixed relative prices.

(iii) Adjustment of prices.

The importance of expectations in each of these adjustment processes will now be considered in turn.

Adjustment to the Short-run Fix-Price Equilibrium

Many Neo-Keynesian models do not examine the adjustment process to the fix-price equilibrium. They assume that quantity adjustment occurs sufficiently rapidly that the process may be approximated by a tatonnement on quantities. This obscures both the importance of expectations in the adjustment process and avoids the problem of describing 'false quantity trades'. Benassy (1975 appendix) bring these points out. Assuming that markets are visited in the sequence in which they are numbered, individual i's maximization problem during the adjustment process may be written as (4.1.1)

$$\text{Max } U_i = U_i(e_i + Z_i, M_i) \qquad (4.1.1)$$

$$\text{S.T.} \quad pZ_i + M_i \leq \bar{M}_i(t) \qquad (i)$$

$$e_i + Z_i \geq 0, \quad M_i \geq 0 \qquad (ii)$$

$$Z_{ih'} = \bar{Z}_{ih'} \quad h'<h \qquad (iii)$$

$$Z_{ih'} \leq \bar{\bar{Z}}^e_{ih'}(t) \quad h'>h \quad h \in D_i \qquad (iv)$$

$$Z_{ih'} \geq \bar{\bar{Z}}^e_{ih'}(t) \quad h'>h \quad h' \in S_i \qquad (v)$$

$$M_{ih'} \geq 0 \quad h' \geq h \qquad (vi)$$

Agent i chooses a vector of trades Z_i which given his endowments e_i imply money stock M_i, which maximizes his utility. This optimal trade vector must satisfy the constraints (i)-(vi). (i) is the budget constraint. (ii) the conditions that net trades and final money stocks cannot be negative. (iii) states that on all markets h' which have already been visited the elements of the transactions vector should be equal to those trades that have already been carried out, $\bar{Z}_{ih'}$ describes trades that have already been made. Constraints (iv) and (v) state that on all markets h' to be visited after h, whether the agent is a supplier $h' \varepsilon S_i$ or a demander $h' \varepsilon D_i$ on that market, he should not plan to exceed his expected constraints, $\bar{\bar{Z}}^e_{ih'}(t)$. Finally (vi) states that the individual never plans to hold a negative quantity of money at any point in the transaction sequence. In Benassy's tatonnement model the ex-ante constraints (iii)-(iv) collapse to a set of perceived constraints given for all markets and an overall liquidity constraint as (4.1.1) (ii). This obscures the importance of expectations which are fundamental in the sequential approach. Although Benassy does not examine the stability properties of his model under the sequential trading formulation, some aspects of the problem have been studied in an aggregated context by Varian (1977) and Honkapohja and Ito (1980). The Varian paper has already been extensively discussed in previous sections, its relevance here is because its results depend upon its sequential trading structure. The labour market meets before the goods market and thus employment depends upon sales expectations.

Honkapohja and Ito (1980) also assume that transactions first take place upon the labour market, firms attempt to purchase sufficient labour in order to produce an optimal inventory stock which will satisfy expected goods demand with sufficient stock that any random fluctuation in that demand may be satisfied. Firms are assumed to have rational expectations of goods demand, so when the good market opens a stock-out will only occur if firms have been constrained for labour. Also rational expectations of goods demand imply that no classical unemployment can arise. Inventories are the only dynamic link in the model since households are assumed to supply d units of labour inelastically and to consume a constant proportion of labour income. Thus different initial inventory stocks will cause firms to attempt to purchase different amounts of labour to achieve optimal inventories. Labour purchases, constrained or notional, will determine goods purchases via workers incomes and therefore, end of period inventories.

Honkapohja and Ito examine equilibria where beginning and end of period inventories are in equality, and examine their stability properties under different quantity constraint regimes. Their results are summarised in figure (4.1.1) (a) and (b):

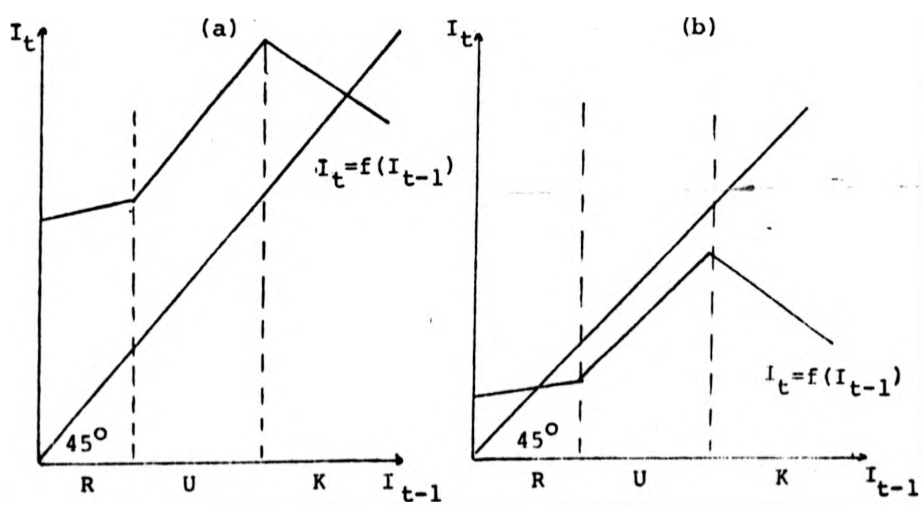

Figure (4.1.1)

In figure (4.1.1)(a) a steady state occurs in the Keynesian region; in (b) a steady state occurs in the repressed inflation region. No steady state occurs in the under consumption region.

Honkapohja and Ito demonstrate that the motion around a Keynesian steady state is always oscillatory, and may be either stable or of limit cycles. If no steady state exists on the Keynesian regime then the steady state in the Repressed Inflation regime is stable with monotonic convergence.

It is interesting to note the part played by producers 'rational' expectations in these dynamics. Expectations are termed rational because their expected value is assumed equal to the actual value of goods sales, as predicted by the model. The implication here being that producers can solve the model to obtain its sales prediction for each regime. The dynamics are given by three first order piecewise linear stocastic difference equations in inventory stocks which describe the following process. Given the models parameters which include initial inventory stocks,firms calculate their rational expectations of goods demand and an associated optional inventory stock. Required labour purchases are calculated and actually achieved if less than labour supply. Goods sales are carried out and provided no constraint on the labour market has been met mean plans of goods sales are achieved without a stock-out. Mean end of period inventories are equal to the planned inventory stock. Next periods calculations are then made using last periods end of period inventory stock as a data, a new sales prediction and associated optional inventory stock obtained. A difference equation in inventory stocks is derived which yields stationary states where the expectation of goods demand and inventory stocks replicate themselves.

Interesting though this treatment is it examines only some of the problems of adjustment to the short run fix-price equilibrium,the dynamics of consumer behaviour and expectations are surpressed and it is assumed that firms have sufficient information that they make only random errors in expectations formation.

Benassy (1976) examines a much more general approach to the determination of dynamics in a short-run disequilibrium model. Again the basic assumption is that the labour market operates before the goods market. The sequence of activities within the market period is as follows. Capitalists advance money to firms on the basis of expected prices, wages, constraints and the rate of profit. Firms then set wages make production plans and demand specific quantities of labour. The labour market then operates. Consumers visit firms and labour transactions take place at the quoted wage rates. These transactions reflect producer and consumer expectations of constraints and prices that will obtain on the forthcoming goods market. Firms then produce and fix goods market prices. The goods market then operates. Consumers again visit firms and transactions take place upon the goods market. Finally, from their receipts firms reimburse capitalists. Benassy does not model the dynamics of this process but simplifies the model and reports a series of simulations. He does however stress the importance of expectations in this process and suggests that expectations should be rational at the steady state of the economy. The idea of expectations as rational at the steady state is a point to which we shall return in section 4.2.

<u>Adjustment of Non-Perishable Goods and Asset Stocks over Successive Periods of Fixed Relative Prices</u>

The preceding section examines sequential adjustments to the short run fix-price equilibrium.[2] In this section the dynamics of asset or inventory accumulation (decumulation) over a succession of short-run fix-price equilibria are examined. Analysis which examine this sort of dynamics implicitly assume a tatonnement upon quantities keeping agents behaviour upon the various markets mutually consistant. Models of this type include the multiplier analysis models of Barro and Grossman (1974) (1976) Honkapohja (1980) and the stability analysis of Böhm (1978).[3]

Barro and Grossman's (1976) analysis of the supply multiplier examines the effect that a decrease in goods availability will have upon output and employment in the repressed inflation regime of their fix-price model. The 'story' behind the supply multiplier is as follows. Faced by a tightening of the goods supply ration households have two types of response, to adjust their savings behaviour or their labour supply or a mixture of the two. If households reduce their labour supply firms who are constrained for labour reduce their output and goods availability to consumers is reduced further. The firms maximand is atemporal in Barro and Grossman and the inter-temporal link in the process is provided by households money balances. The representative households maximization problem is as follows (4.1.2):

$$\text{Max } u = \int_0^{\hat{T}} u\left[\bar{x}(t), L^s(t)\right]dt + \int_{\hat{T}}^{T'} u\left[x^d(t), L^s(t)\right]dt + \int_{T'}^{T} u\left[x^d(t), 0\right]dt \tag{4.1.2}$$

(4.1.2) says that the representative household maximizes utility by planning its goods purchases and labour supply over three distinct periods. During the first period O-$\hat{T}$ the household expects to be constrained to purchasing $\bar{\bar{x}}$ and maximizes utility by planning to work $L'(t)$. In the second period $\hat{T}$-T' the household anticipates facing no constraints and freely chooses $x^d(t)$ and $L'(t)$. In the final period the household has retired $L'(t)=0$ and consumes savings to achieve $x^d(t)$.

Maximization is dependent upon the expected time path of the goods constraint. In the Barro Grossman treatment it is assumed that households expect the current goods constraint $\bar{\bar{x}}$ to remain constant at that given level until time $\hat{T}$. Households have the option of substituting either current period leisure or future consumption, in the period $\hat{T}$-T, for current constrained consumption. The magnitude of the supply multiplier depends upon expectations in two ways, the expected size and the duration of the goods constraint. It is demonstrated that if the constraint is regarded as purely transitory then it will have negligible effect upon the households life plan and will not significantly reduce labour supply, also if the goods constraint expectation is close to the notional goods demand there will be little effect upon labour supply.

In the slightly longer run the effect of the supply multiplier will be to reduce profits, reducing household non-wage income and so increase labour supply, lower goods demand and moderate the impact of the multiplier. In extreme circumstances this effect may reverse the sign of the supply multiplier. Honkapohja (1980) modifies the Barro Grossman short run analysis of fix-price multipliers to the medium-run by utilizing the steady state multiplier of government expenditure taking into account changes in asset stocks. Honkapohja demonstrates that the steady state demand multiplier is larger than the short-run analysis suggests whilst the supply multiplier changes sign.

Analysis of the Barro Grossman (1976) or Honkapohja (1980) type examine the comparative static shifts of the short-run fix-price equilibrium in response to some parameter change. Implicit in such treatments is the assumption that the adjustment processes underlying such a change are stable. Böhm (1978) considers the underlying dynamics. A sequence of fix-price equilibria are considered, a steady state is an equilibrium money stock,real wage pair,at which households desired money stock changes are zero. No saving or disaving occurs. It is demonstrated that such steady states are globally unstable on the repressed inflation regime, but are monotonically convergent upon the Keynesian regime for all initial money stocks which are not too large. Böhms results have been taken as demonstrating that the repressed inflation regime and supply multiplier are unimportant, see Honkapohja (1980), however Böhm's analysis is based upon some special assumptions. The model used assumes that money balances held by households provide the only link between periods. There are no inventory stocks, expectations are static. The government levies a 100% profits tax. Discussion of both demand and supply multipliers given by Barro and Grossman and Honkapohja, suggests that Böhms results are based upon strong assumptions which are at variance with some aspects of multiplier theory. The dynamics of adjustment through a succession of fix-price equilibria requires an explanation of inventory stock, asset stock and expectations adjustment. Comparative static multiplier analysis will be valid upon those regimes and over those ranges of parameter values for which a dynamical system, which places both firms and households in an inter-temporal decision making setting, is stable.

Adjustment of Prices

Most Neo-Keynesian models obtain equilibria in which resources are not fully utilized because relative prices are wrong, and persist in being wrong for some meaningful period.[4] The obvious question is why do not relative prices adjust to their market clearing levels? The literature has approached this question in two distinct ways. One approach is to examine endogeneous price setting by individual economic actors or institutions, and to consider under what circumstances agents will choose to keep prices constant. The second approach has been to consider the dynamics of price adjustment and the possibility of effective demand failures.

In this section the question of price dynamics and effective demand failures will be examined, the question of endogeneous price setting and its implications will be the main theme of chapters 5-7 and will be examined there. Conventional price adjustment equations express the time derivative of prices as a function of the difference between the notional supplies and demands for the good in question, as (4.1.3) and (4.1.4):

$$\dot{p} = f(x^d - x^s) \quad f'>0 \quad f(0)=0 \tag{4.1.3}$$

$$\dot{w} = g(L^d - L^s) \quad g'>0 \quad g(0)=0 \tag{4.1.4}$$

where $\dot{p} = \frac{dp}{dt}$ and $\dot{w} = \frac{dw}{dt}$

and $x^d = x^d(w/p)$, $x^s = x^s(w/p)$, $L^d = L^d(w/p)$, $L^s = L^s(w/p)$

The equations (4.1.3) and (4.1.4) represent the traditional excess demand hypothesis, this price adjustment mechanism can easily be shown to be locally asymptotically stable.[5] The problem with such an adjustment mechanism is that it is based upon the difference between notional supplies and demands which can only be expressed at market clearing prices. As was first pointed out by Leijonhufvud (1973) away from the market clearing price vector the correct functional forms to work with when formulating excess demands are the effective supplies and demands. The form taken by effective excess demands will depend upon which constraint regime the system is upon. In principle there are four basic formulations:[6]

Keynesian:

$$\dot{p} = f^k(x^{d'} - x^s) \qquad f^{k'} > 0 \qquad (4.1.5)$$

$$\dot{w} = g^k(L^{d'} - L^s) \qquad g^{k'} > 0 \qquad (4.1.6)$$

Repressed inflation:

$$\dot{p} = f^R(x^d - x^{s'}) \qquad f^{R'} > 0 \qquad (4.1.7)$$

$$\dot{w} = g^R(L^d - L^{s'}) \qquad g^{R'} > 0 \qquad (4.1.8)$$

Classical:

$$\dot{p} = f^c(x^{d'} - x^{s'}) \qquad f^{c'} > 0 \qquad (4.1.9)$$

$$\dot{w} = g^c(L^d - L^{s'}) \qquad g^{c'} > 0 \qquad (4.1.10)$$

Under Consumption:

$$\dot{p} = f^u(x^d - x^{s'}) \qquad f^{u'} > 0 \qquad (4.1.11)$$

$$\dot{w} = g^u(L^{d'} - L^{s'}) \qquad g^{u'} > 0 \qquad (4.1.12)$$

where: $x^{d'}/w/p,\bar{L})$, $x^{s'}(w/p,\bar{L})$, $L^{d'}(w/p,\bar{x})$, $L^{s'}(w/p,\bar{x})$

Equations (4.1.5)-(4.1.12) describe the effective excess demand hypothesis. Two essentially inter-related controversies are associated with this hypothesis, both of which stem from Leijonhufvud (1973) who argues that if the economy is close to the equilibrium generating price vector, within the corridor, then this system is characterised by good stability properties. If however the system is outside the corridor then effective demand failures will occur and prices will not adjust. Leijonhufvud argues that large unanticipated deviations drain the system of liquid buffer stocks, causing households to revise downwards subjective estimates of permanent income, these are self fulfilling because of large downward multiplier effects. Self fulfilling expectations of permanent income cause effective demand failure. Thus it is argued that prices, when well away from their Walrasian values, may not adjust and the economy may be in a state of lasting unemployment. This is the first of the two inter-related controversies, the idea that for some prices and corresponding effective demands the expectations of permanent income held by households will equate (4.1.5)-(4.1.12) to zero. Grossman (1974) and Veendorp (1975) have disputed Leijonhufvud's idea of effective demand failures. Assuming an underlying 'fast' or rather tatonnement quantity adjustment Grossman argues that prices will adjust towards their full equilibrium values even if the system is initially far away from the Walrasian price vector. Veendorp's paper initiated the second controversy. He provides stability analysis of price adjustments based upon effective excess demands in a two consumer three commodity pure exchange economy. The results obtained are that if the notional excess demand functions have the property of gross substitutability, then both price mechanisms based on notional and effective excess demand functions are locally asymptotically stable. Veendorp also demonstrates numerically that in economy where the excess demand functions lack the property of gross substitutability the system can be stabilised by replacing the notional excess demands with the effective excess demands. This leads Veendorp to conjecture that a sufficient condition for local stability of a price mechanism based upon effective excess demands is that a price mechanism based upon the notional excess demands is stable. This conjecture and indeed Veendorp's other results have been partially refuted by Löfgren (1979). Löfgren examines an economy with production introduced in the form of a simple atemporal production function.[7] Using a version of the Barro-Grossman model it is demonstrated that an economy which is stable under price adjustments based upon notional excess demands can be unstable under the effective excess demand hypothesis.

The two inter-related controversies are whether the effective excess demands may be zero away from the Walrasian price vector as Leijonhufvud argues, and whether or not the price adjustment mechanism based on the effective excess demands is stable.

Even if it seems reasonable to treat the price and quantity adjustment processes separately, and this is perhaps in doubt, there are other problems associated with the effective excess demand hypothesis as described by equations (4.1.5)-(4.1.12). As Veendorp notes it may not be correct to include the notional supplies or demands in the price adjustment equation even when these are on the long side of their respective markets. It may not be optional for agents to express their notional supplies or demands on a market upon which they are rationed if there are costs associated with expressing a demand, as were discussed in chapter 3.[8] It may also be the case that agents will express demands in excess of their notional demands upon markets where they face rationing. Agents may attempt to manipulate the rationing scheme through their demands, this could cause serious problems for the stability of the price adjustment mechanism. Honkapohja and Ito (1980) also suggest that agents may become discouraged by rationing and understate their notional demands, which could cause sluggishness in the price adjustment mechanism. It seems likely that the correct specification of the effective demand hypothesis will depend crucially upon the properties of the rationing scheme in operation, and whether there are costs associated in expressing demands.

The price adjustment process as defined by the effective excess demand hypothesis assumes that the underlying quantity adjustments are stable, establishing a series of fix-price equilibria in which the trades agents conduct upon markets are mutually consistant. Contributions to the literature which make this point usually cite Böhm (1978) as calling this into question. Böhm's analysis is based upon a series of periods of constant fixed prices and medium run asset stock adjustments by consumers.

The analysis itself assumes the short run adjustment to the fix-price equilibria is stable, that agents trade upon markets in any period are consistent with the solutions to their constrained maximization problems. It could be argued that the dynamical process which is relevant to the demands and supplies expressed is the quantity adjustment equation in the short-run adjustment process to the fix-price equilibrium not the evolution of these equilibria over successive periods of were discussed in chapter 3.[8] It may also be the case that agents will express demands in excess of their notional demands upon markets where they face rationing. Agents may attempt to manipulate the rationing scheme through their demands, this could cause serious problems for the stability of the price adjustment mechanism. Honkapohja and Ito (1980) also suggest that agents may become discouraged by rationing and understate their notional demands, which could cause sluggishness in the price adjustment mechanism. It seems likely that the correct specification of the effective demand hypothesis will depend crucially upon the properties of the rationing scheme in operation, and whether there are costs associated in expressing demands.

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4.2 A Neo-Keynesian Disequilibrium Trading Model: Sequential Adjustment to the Short-Run Fix-Price Equilibrium

In the preceding section (4.1) some of the adjustment processes that have been analysed in the Neo-Keynesian literature were discussed. Starting from the conceptual foundation of the Clower/Leijonhufvud fix-price equilibrium three adjustment processes were identified. Adjustment to the short-run fix-price equilibrium. Adjustment of Non-Perishable goods and asset stocks over successive periods of fixed relative prices, and the adjustment of prices. It was argued in the section on the adjustment of prices that it was necessary to examine the stability of the underlying quantity adjustment mechanism to provide an underpinning for price adjustments. The analysis discussed in the section upon adjustments to the short-run fix-price equilibria, Benassy (1975 appendix) Honkapohja and Ito (1980), do not examine inventory and money stock adjustments together, this is required for a satisfactory examination of the quantity adjustment process. In this section a sequential trading fix-price model will be developed in which consumers hold money and firms inventory stocks. Dynamic analysis of the quantity adjustment process to the fix-price equilibrium will be carried out for the Keynesian and Repressed Inflation cases. This analysis, it is argued, provides the correct underpinning for the effective excess demand hypothesis on these regimes. It also places emphasis upon the role of expectations adjustment in the dynamics of quantity adjustment.

It will be shown that the equilibria of this model has for the most part nice stability properties contrary to the results of Böhm (1978) and similar in some cases to Honkapohja and Ito (1980) which were discussed in section (4.1). Here it will be demonstrated that in certain circumstances the Keynesian equilibria have saddle point or cyclically divergent dynamic properties, and that the application of government monetary and fiscal policy may be required to stabilize the system, whilst the Repressed Inflation regime has either stable, spiral or saddle point characteristics.

The Model

There are three types of economic agents in the economy studied, a representative consumer/worker, a representative producer and the government. There are three goods, labour, consumption good and money, the prices of which are w, p and 1 respectively. Money and the consumption good are storeable.

It is assumed that the consumption good and labour markets open sequentially.[9] Producers and consumers current transactions will be dependent upon the trading possibilities they anticipate when the other market opens.[10] In using a representative consumer, representative producer model there are clearly questions associated with sequential trading in disaggregated models which cannot be considered.[11] However it is argued that this model is adequate to describe the fundamental dicotomy between the good and labour markets and the decision making processor therein. This is not of course a description of any real economy but rather an abstraction to examine some of the implications of disequilibrium trading that occurs in the quantity adjustment process to the fix-price equilibrium. Consider first the Keynesian regime.

<u>The Keynesian Regime</u>

There are two possible decision making processes here. First the consumer may decide to purchase consumption goods by spending money balances which he plans to rebuild when the labour market opens. In this case it will be assumed that the producer holds a sufficiently large inventory stock to accommodate the consumers purchases, and when the labour market opens the producer then hires sufficient labour to rebuild his stocks. Second, the producer decides to purchase labour and produce consumption goods in anticipation of the demand he will face when the consumption good market opens. It is assumed that the consumer is willing to supply the producers labour demand, and when the goods market is open to purchase consumption goods consistent with realised labour sales.[12]

Case 1: The first component of any pair of trades occurs on the goods market.

Assume the consumer behaves according to a short-run Keynesian consumption function as (4.2.1):

$$x(t) = x(\bar{\bar{L}}(t), m(t) \mid p, w, \overset{*}{M}) \qquad (4.2.1)$$

where

X(t) is goods purchased at time t, $\bar{\bar{L}}(t)$ anticipated (planned) labour sales, m(t) money stock. $\overset{*}{M}$ is a long run target money stock which reflects the consumers long-run expectations.

The consumer money stock m(t) an any t is given by (4.2.2):

$$m(t) = m(o) + \int_{o}^{t} \dot{M}(t)\,dt + \int_{o}^{t} \dot{Q}(t)\,dt \qquad (4.2.2)$$

where

$\dot{M}(t)$ = dM/dt planned money stock changes,

$\dot{Q}(t)$ = dQ/dt unanticipated money stock changes.

The planned money stock adjustment of the consumer is as (4.2.3)

$$\dot{M}(t) = \Lambda(\overset{*}{M} - m(t)) \qquad (4.2.3)$$

However plans may not be realised and actual money stock adjustment is described by (4.2.4):

$$\dot{m}(t) = \dot{M}(t) + \dot{Q}(t) \qquad (4.2.4)$$

To explain why unanticipated changes in the consumers money stock occur consider the producers behaviour. Let the producer behave according to the inverse production function (4.2.5)

$$\bar{L}(t) = \ell(Y(t)) = \ell(x(t) + g(t)) \qquad (4.2.5)$$

$\bar{L}(t)$ actual employment at t, g(t) is government goods demand.

Transactions are allowed to take place out of equilibrium, hence realised labour sales $\bar{L}(t)$ may not equal planned labour sales $\bar{\bar{L}}(t)$ - there is rationing. This implies that the consumer will accumulate (decumulate) unanticipated money balances as in (4.2.6) and will also have to revise planned labour sales towards the ration $\bar{L}(t)$ as in (4.2.7)

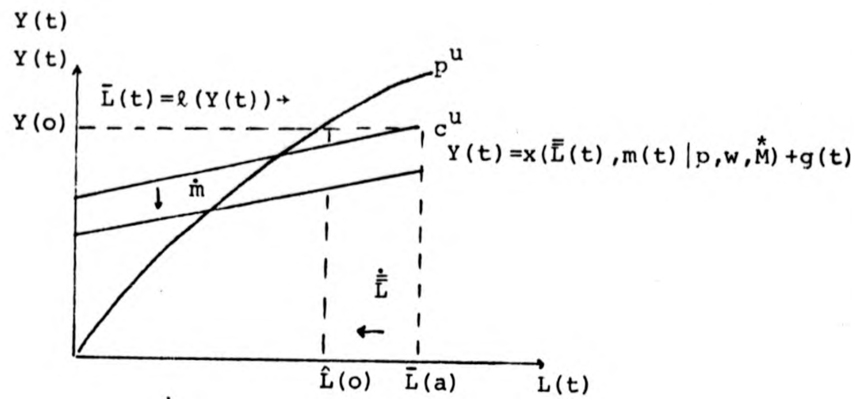
$$\dot{Q}(t) = w(\bar{L}(t) - \bar{\bar{L}}(t)) \qquad (4.2.6)$$

$$\dot{\bar{\bar{L}}}(t) = \phi(\bar{L}(t) - \bar{\bar{L}}(t)) \qquad (4.2.7)$$

ϕ is a constant adjustment parameter.

A clearer understanding of the processes involved in the model may be achieved by considering figure (4.2.1)

Fig. (4.2.1)



where

$p^u = (Y^F, L^F)$ is the producer unconstrained choice point

$c^u = (X^C, L^C)$ is the consumers unconstrained choice point

To explain figure 1 hypothesize the following, the relative price vector has just been announced and both agents trade first upon the goods market. The initial goods market trade will be $Y(O) = \min(Y^F, X^C + g(o))$ and we see from the diagram that the producer faces a shortfall in effective demand ($Y(o)$ represents a ration). The consumers goods purchase was based upon the anticipation that $\bar{L}(o)$ labour could be sold in the next instant such that he may replace

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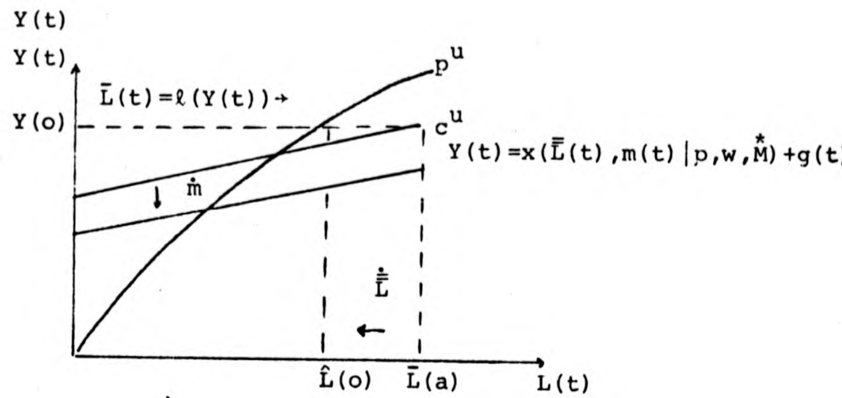

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By manipulation of (4.2.1), (4.2.3)-(4.2.7) we may describe the dynamics of our model by the differential equations (4.2.8) and (4.2.9).[14]

$$\dot{m}(t) = \Lambda(\overset{*}{M}-m(t)) + w(\ell[x(\bar{\bar{L}}(t),m(t)\,|\,p,w,\overset{*}{M}) + g(t)] - \bar{\bar{L}}(t)) \tag{4.2.8}$$

$$\dot{\bar{\bar{L}}}(t) = \phi(\ell[x(\bar{\bar{L}}(t),\ m(t)\,|\,p,w,\overset{*}{M}) + g(t)] - \bar{\bar{L}}(t)) \tag{4.2.9}$$

An equilibrium is thus defined when (4.2.8) and (4.2.9) are both zero, when $m(t) = \overset{*}{M}$ and $\bar{L}(t) = \bar{\bar{L}}(t)$.[15]

Before examining the stability properties of this economy first note that (4.2.4) implies (4.2.1) may be rewritten:[16]

$$X(t) = x(\bar{\bar{L}}(t), m(t) \mid p, w, \overset{*}{M}) = \frac{w\bar{\bar{L}}(t) - \Lambda(\overset{*}{M} - m(t))}{p} \qquad (4.2.10)$$

the linear consumption function (4.2.10) yields:

$$\frac{\partial x(t)}{\partial \bar{\bar{L}}(t)} = \frac{w}{p} \quad \text{and} \quad \frac{\partial x(t)}{\partial m(t)} = \frac{\Lambda}{p} \qquad (4.2.11)$$

These properties will be useful in what follows.

It may perhaps be anticipated that as we allow income effects the stability properties of our model will be problematic, but counter to our intuition, this does not prove to be the case. Taking first order Taylors approximation to (4.2.1) and (4.2.9) and using (4.2.11) write[2]

$$\dot{Z}_m(t) = f_1 Z_m(t) + f_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.12)$$

$$\dot{Z}_{\bar{\bar{L}}}(t) = g_1 Z_m(t) + g_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.13)$$

where

$$\dot{Z}_m(t) = \dot{m}(t), \quad \dot{Z}_{\bar{\bar{L}}}(t) = \dot{\bar{\bar{L}}}(t), \quad Z_m(t) = m(t) - \overset{*}{M}, \quad Z_{\bar{\bar{L}}}(t) = \bar{\bar{L}}(t) - \overset{*}{\bar{\bar{L}}}.$$

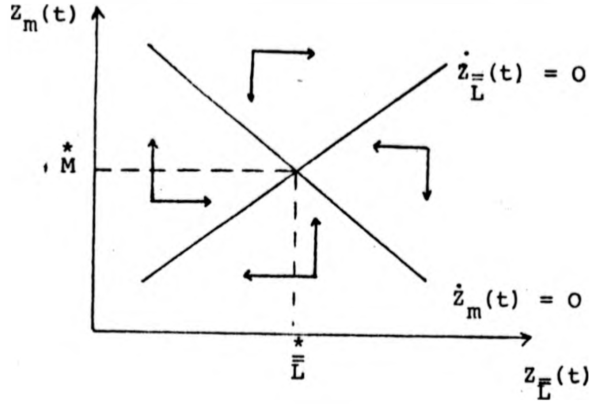
$$\left.\begin{array}{ll} f_1 = \Lambda \left[\dfrac{\partial \ell}{\partial x}(t) \quad \dfrac{w}{p}^{-1}\right] < 0 & f_2 = w\left[\dfrac{\partial \ell}{\partial x}(t) \quad \dfrac{w}{p}^{-1}\right] < 0 \\[2ex] g_1 = \phi \quad \dfrac{\partial \ell}{\partial x}(t) \quad \dfrac{\Lambda}{p} > 0 & g_2 = \phi\left[\dfrac{\partial \ell}{\partial x}(t) \quad \dfrac{w}{p}^{-1}\right] < 0 \end{array}\right| \qquad (4.2.14)$$

Each partial in (4.2.14) is evaluated at the equilibrium.

Assuming no forced trading, the standard condition $w/p \leqslant MPL$ allows us to sign the partials in (4.2.14).

Using (4.2.14) we may now describe the dynamics of our economy by the phase diagram figure (4.2.2):

Fig. (4.2.2)



This equilibrium is clearly stable, since $f_1, g, < 0$, the real parts of both complex conjugate roots of the system are negative.

Since the equilibria are stable the comparative statics of a change in government expenditure will be the standard multipliers (4.2.15) and (4.2.16):

$$\frac{dY(t)}{dg(t)} = \frac{w}{p} \; \frac{\partial \ell}{\partial g(t)} + 1 \tag{4.2.15}$$

$$\frac{d\bar{\bar{L}}(t)}{dg(t)} = \ell \left[\frac{w}{p} \cdot \frac{\partial \ell}{\partial g(t)} + 1 \right] \tag{4.2.16}$$

at $m(t) = \overset{*}{M}$ all changes in consumers wage income are translated into consumption demand (MPC=1).

<u>Note</u>: The slopes of the stationaries are:

$$\left.\frac{\partial Z_m(t)}{\partial Z_{\bar{\bar{L}}}(t)}\right|_{\dot{Z}_m(t)=0} = -\frac{f^2}{f_1} = -\frac{w}{\pi} < 0$$

$$\left.\frac{\partial Z_m(t)}{\partial Z_{\bar{\bar{L}}}(t)}\right|_{\dot{Z}_{\bar{\bar{L}}}(t)=0} = \frac{-g_2}{g_1} \quad \frac{\partial \ell \,|\, \partial x(t).w/p-1}{\partial \ell \,|\, \partial x(t). \;/p} > 0$$

Arrows on phase diagram

$$\frac{\partial \dot{Z}_m(t)}{\partial Z_m(t)} = f_1 < 0 \; , \; \frac{\partial \dot{Z}_{\bar{\bar{L}}}(t)}{\partial Z_{\bar{\bar{L}}}(t)} = g_2 < 0$$

So far it has been demonstrated that an economy characterised by a decision making process where transactions upon the goods market essentially precede those on the labour market will, given the qualifications previously stated, display a stable Keynesian equilibria.

The foregoing analysis assumes consumers expectations and hence their target money stock to be unaffected by current market experience. Now relax this heroic assumption by rewriting (4.2.4) as (4.2.4)'

$$\dot{M}(t) = \Lambda(\overset{*}{M}(\bar{\bar{L}}(t)\,|\,p,w) - m(t)) \qquad (4.2.4)'$$

Thus the money stock the consumer wishes to carry forward to the next period depends upon the expected current labour ration $\bar{\bar{L}}(t)$[17]. To know how $\bar{\bar{L}}(t)$ effects $\overset{*}{M}$ it is necessary to specify an expectations formation mechanism and then the consumers preferences over periods, this is omitted since here interest is focused on the question of whether the expectations effect through $\overset{*}{M}$ be destabilizing?[18] It will be found that the answer to this question is somewhat counter intuitive, the effect of expectations as expressed through (4.2.4)' can at worst cause cyclically unstable equilibria but in most cases will generate equilibria which are stable, or may be stabilized by the operation of standard government policy instruments.

Carrying through the previous analysis with (4.2.4)' replacing (4.2.4) gives a new set of partial derivatives for (4.2.14)

$$\left.\begin{array}{ll} f_1' = \Lambda\left[\dfrac{\partial \ell}{\partial x(t)}\ \dfrac{w}{p} - 1\right] < 0 & f_2' = \Lambda\ \dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)} + \dfrac{w}{p}\ \dfrac{\partial \ell}{\partial x(t)}\left[w - \dfrac{\Lambda\,\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)}\right] - w \lesseqgtr 0 \\ g_1' = \phi\ \dfrac{\partial \ell}{\partial x(t)}\ \dfrac{\Lambda}{p} > 0 & g_2' = \phi\left[\dfrac{\partial \ell}{\partial x(t)}\left(\dfrac{w}{p} - \dfrac{\Lambda}{p}\ \dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)}\right) - 1\right] \lesseqgtr 0 \end{array}\right| \quad (4.2.\ 4)'$$

From (4.2.14)' it is clear that to describe the stability properties of the model with endogenous expectations, information about the signs of f_2' and g_2' is required. The new component in the partials (4.2.14)' over (4.2.14) is the expectations effect $\partial \overset{*}{M}/\partial \bar{\bar{L}}(t)$, this term cannot be signed without specifying a multi-period model with functional form. Now examine the possibilities.

(i) Let $\dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)} < 0$: A tightening of the current period labour ration raises the consumers desired savings.

This might be called the 'Pessimistic' case. The consumer anticipates that things will become worse in the next period. The signs of the partials in (4.2.14)' are now as in table (4.2.1)

Table (4.2.1)

	f_1'	f_2'	g_1'	g_2'
a < o	-	-	+	-
o<a<b	-	-	+	+
o<b<a	-	+	+	+

where

$$a = \frac{w}{p} \frac{\partial \ell}{\partial x(t)} \left[w - \frac{\Lambda \partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)} \right]$$

$$b = \left| \Lambda \frac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)} - w \right|$$

Note that a<o is only possible with negative production or prices and will not be examined in what follows. Inspection of table 1 reveals three possible cases represented in figure (4.2.3) (a) (b) and (c).

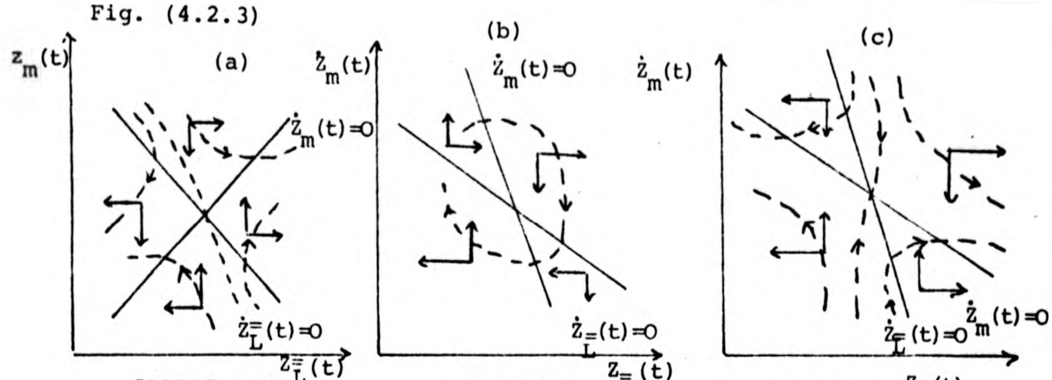
In figure (4.2.3)

(a) $f_1' < 0,\ f_2',\ g_1',\ g_2' > 0$

(b) $f_1',\ f_2' < 0,\ g_1',\ g_2' > 0$ and $-f_2'/f_1' < -g_2'/g_1'$

(c) $f_1',\ f_2' < 0,\ g_1',\ g_2' > 0$ and $-f_2'/f_1' > -g_2'/g_1'$

Fig. (4.2.3)



SADDLE POINT — CYCLICAL — SADDLE POINT

As the phase diagrams in figure (4.2.3) indicate, endogenizing expectations changes the stability properties of the model.

<u>Note</u>: The slopes of the stationaries are:

$$\left.\frac{\partial Z_m(t)}{\partial Z_{\bar{\bar{L}}}(t)}\right|_{\dot{Z}_m(t)=0} = -\frac{f'_2}{f_1} = -\Lambda\,\frac{\dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}} + \dfrac{w}{p}\dfrac{\partial \ell}{\partial x(t)}\left[w - \Lambda\dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)}\right] - w}{\Lambda\left[\dfrac{\partial \ell}{\partial x(t)}\ \dfrac{w}{p} - 1\right]} \gtrless 0$$

$$\left.\frac{\partial Z_m(t)}{\partial Z_{\bar{\bar{L}}}(t)}\right|_{\dot{Z}_{\bar{\bar{L}}}(t)=0} = \frac{-g_2'}{g_1'} = \frac{\phi\left[\dfrac{\partial \ell}{\partial x(t)}\left(\dfrac{w}{p} - \dfrac{\Lambda}{p}\ \dfrac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)}\right) - 1\right]}{\phi\ \dfrac{\partial \ell}{\partial x(t)}\ \dfrac{\Lambda}{p}} \lesseqgtr 0$$

Arrows in phase diagram

$$\frac{\partial \dot{Z}_m(t)}{\partial Z_m(t)} = f_1' < 0$$

$$\frac{\partial \dot{Z}_{\bar{\bar{L}}}(t)}{\partial Z_{\bar{\bar{L}}}(t)} = g_2' = \phi\left[\frac{\partial \ell}{\partial x(t)}\left(\frac{w}{p} - \frac{\Lambda}{p}\ \frac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)}\right) - 1\right] \lesseqgtr 0$$

There are two questions which immediately spring to mind on examining figure (4.2.3). Firstly, is the cyclical equilibrium stable? Secondly, is there any action that the government may undertake which will ensure that the system is on the stable manifold in the saddle point cases? To examine these questions rewrite the differential equation system in matrix form.

where $\dot{Z}(t) = \begin{bmatrix} \dot{Z}_m(t) \\ \dot{Z}_{\bar{\bar{L}}}(t) \end{bmatrix}$, $A = \begin{bmatrix} f_1{}' & f_2{}' \\ g_1{}' & g_2{}' \end{bmatrix}$, $Z(t) = \begin{bmatrix} Z_m(t) \\ Z_{\bar{\bar{L}}}(t) \end{bmatrix}$ (4.2.15)

The characteristic equation of the system (4.2.15) is (4.2.16)

$$\beta^2 - (f_1{}' + g_2{}')\,\beta + f_1{}'\,g_2{}' - f_2{}'\,g_1{}' = 0 \qquad (4.2.16)$$

To examine the stability properties the eigenvalues are obtained by applying the quadratic formula to (4.2.16)

$$\beta_1, \beta_2 = \tfrac{1}{2}\left[(f_1{}' + g_2{}') \pm \sqrt{(f_1{}' + g_2{}')^2 - 4(f_1{}'g_2{}' - f_2{}'g_1{}')}\right] \qquad (4.2.17)$$

If the solution path is cyclical as in figure (4.2.3)(b) then stability requires that the real part of the complex conjugate roots β_1, β_2 of (4.2.17) be negative. Hence stability requires

$$(f_1{}' + g_2{}') < 0$$

which is equivalent to

$$\left|(\Lambda+\phi)\left[\frac{\partial \ell}{\partial x(t)}\ \frac{W}{P}\ -1\right]\right| > \phi\ \frac{\partial \ell}{\partial x(t)}\ \frac{\Lambda}{P}\ \frac{\partial \overset{*}{M}}{\partial \bar{\bar{L}}(t)} \qquad (4.2.18)$$

Can the government ensure that (4.2.18) holds, and ensure that the solution path of the system is a stable spiral? This is likely, inspection of (4.2.18) reveals that the left hand side is decreasing in x(t) whilst the right hand side is increasing in x(t), from (4.2.5) it is clear that $\partial\ell/\partial x(t) = \partial\ell/\partial g(t)\ \forall g(t),\ x(t), g(t) \neq 0,\ x(t) \neq 0$. Hence by lowering government expenditure the inequality in (4.2.18) may be ensured and the solution path will be a stable spiral.[19] Heuristically regard this as the case where the system becomes over responsive, small changes in goods purchases lead to large employment level responses, due to a very low marginal product of labour, and large expectations and desired money balance adjustments in response to employment fluctuations. By cutting back government expenditure, the system is forced back down the production function, employment fluctuations are diminished and the system becomes more stable.

If the equilibrium is a saddle point as in figure (4.2.3)(a) and (c), then both roots of (4.2.16) are real. Denote β_2 as the stable (negative) root. The Government in such an equilibrium can stabilise the system if it can place the system upon the stable manifold. The equation of the stable path of a saddle point of our system (4.2.15) is given by (4.2.19).

$$m(t) = \text{Tan}\left(\frac{f_1-\beta_2}{f_2}\right)\overset{*}{\bar{\bar{L}}}(t) + \overset{*}{M}(t) - \left(\frac{f_1-\beta_2}{f_2}\right)\bar{\bar{L}}(t) \qquad (4.2.19)$$

(4.2.19) is only an approximation to the stable path since we are working with linear approximations. From (4.2.19) an (approximate) lump sum transfer rule may be defined, which the government may follow to stabilize the system. If $\bar{\bar{L}}(o)$, m(o) are the initial conditions the lump sum transfer rule is (4.2.20)

$$LST(o) = \text{Tan}\left(\frac{f_1-\beta_2}{f_2}\right)\overset{*}{\bar{\bar{L}}}(t) + \overset{*}{M}(t) - \left(\frac{f_1-\beta_2}{f_2}\right)\bar{\bar{L}}(o)-m(o) \qquad (4.2.20)$$

<u>Note</u>: To obtain the equation of the stable branch use the following method (due to Dixit, 1980):

$$\begin{bmatrix} f_1-\beta_2 & f_2 \\ g_1 & g_2-\beta_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ where } \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ is the eigenvector}$$

Choose $v_1 = 1$ and solve for v_2 to obtain $v_2 = \frac{f_1 - \beta}{f_2}$ and using the properties of the unit circle write the slope of the stable branch as $\frac{v_2}{v_1} = -\left|\frac{f_1 - \beta_2}{f_2}\right|$

Given a point on the line $\overset{*}{M}(t)$, $\overline{\overline{L}}(t)$ and its slope v_2/v_1 the equation of the stable branch now follows from basic trigonometry.

Note that as (4.2.20) only places the system upon an approximation to the stable manifold it will be necessary for the government to repeatedly adjust money balances according to (4.2.20) until the system is within a neighbourhood of the equilibrium $\overset{*}{M}(t)$, $\overset{*}{\overline{\overline{L}}}(t)$. Effectively (4.2.20) defines a money supply rule to stabilise the system.

Having thus examined the possible equilibria that will occur upon the Keynesian regime when consumers are pessimistic, in the sense that a tightening of the current period labour ration induces them to believe that things will become worse in the next period and hence raises their desired level of money balances. Next consider the 'optimistic' case.

(ii) Let $\frac{\partial \overset{*}{m}}{\partial \overline{\overline{L}}(t)} > 0$: A tightening of the current period labour ration lowers desired savings.

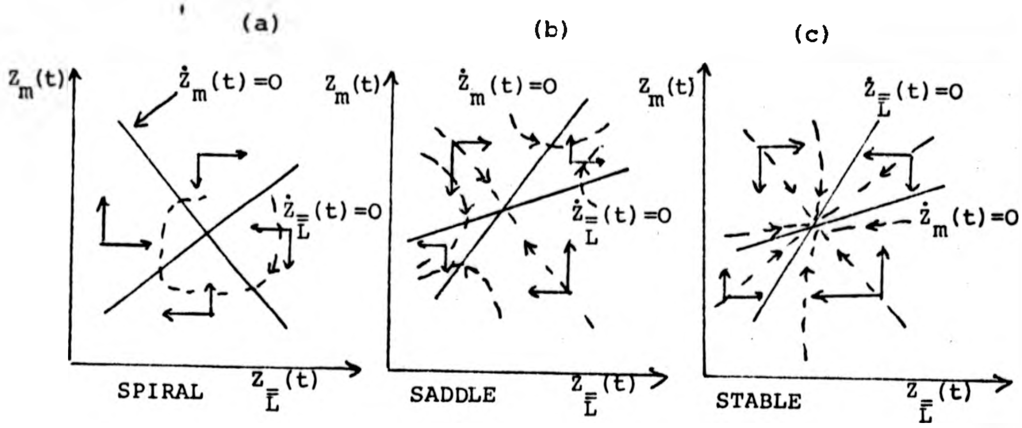
It may be shown that the sign of the partials in (4.2.14) are now as in table (4.2.2)[20].

Table (4.2.2)

	f_1'	f_2'	g_1'	g_2'
$w > \Lambda \frac{\partial \overset{*}{M}}{\partial \overline{\overline{L}}(t)}$	-	-	+	-
$w < \Lambda \frac{\partial \overset{*}{M}}{\partial \overline{\overline{L}}(t)}$	-	+	+	-

Again there are three possible cases, as described by figure (4.2.4).

<u>Fig. (4.2.4)</u>



In figure (4.2.4)

(a) $f_1', f_2', g_2' < 0 \quad g_1' > 0$

(b) $f_1', g_2' < 0 \quad f_2', g_1' > 0 \quad -f_2'/f_1' > -g_2'/g_1'$

(c) $f_1', g_2' < 0 \quad f_2', g_1' > 0 \quad -f_2'/f_1' < -g_2'/G_1'$

Again is it possible for the government to utilize standard policy instruments to stabilise the system. It would appear that stabilization policy is required in the cases depicted by figure (4.2.4)(a) and (b) but it may be easily shown that (a) is a stable spiral. The instability of (b) is due to the system being off the (unique) stable branch the equation of which will be (4.2.19) as in the previous case. Thus again the government may stabilize the system by following the approximate lump sum transfer rule defined by expression (4.2.20). Thus if consumers are optimistic, in the sense that a tightening of the current period labour ration lowers their desired savings, then on the Keynesian regime the government can stabilize the system by a lump sum transfer/money supply rule.

Having examined the stability properties of a fix-price representative consumer, producer model, where transactions are permitted away from the quantity tatonnement equilibrium, and where the first leg of any pair of transactions takes place upon the goods market. We now turn attention to the case where trading takes place first on the labour market.

Case 2: The first component of any pair of trades occurs on the labour market.

Again assume that the consumer behaves according to some short-run Keynesian consumption function.

$$X(t) = x(\bar{L}(t), m(t) \mid p, w, \overset{*}{M}) \qquad (4.2.21)$$

(4.2.21) differs from (4.2.1) the consumption function in case 1, since $\bar{L}(t)$ actual labour sales replaces, $\bar{\bar{L}}(t)$ expected labour sales, because in the second case $\bar{L}(t)$ is experienced prior to trading on the goods market.

Since trading takes place first upon the labour market, the onus is upon the producer to predict the level of goods sales that will be achieved when that market opens, and purchase sufficient labour $\bar{L}(t)$, to produce sufficient goods to meet predicted goods demand net of undesired inventories. Labour demand is thus given by the inverse production function (4.2.22):

$$\bar{L}(t) = \ell(\bar{\bar{y}}(t) - I(t)) \qquad (4.2.22)$$

where $\bar{\bar{y}}(t)$ is predicted goods demand, I(t) undesired inventories.

Clearly then the dynamics of this case are generated by the discrepancy between predicted and actual goods demand. Producers will respond to this discrepancy by adjusting their prediction of demand and hence their labour purchases. Consumers will respond to changes in labour sales by adjusting money balances and goods purchases.

The difference between goods demand and predicted demand will be observed in the level of undesired inventories, as (4.2.23)[21]:

$$\dot{I}(t) = \bar{\bar{y}}(t) - I(t) - X(t) - g(t) \qquad (4.2.23)$$

The producers response will be to adjust anticipated demand as (4.2.24) and thus employment as (4.2.25)

$$\dot{\bar{\bar{y}}}(t) = \rho\left[X(t) + g(t) - \bar{\bar{y}}(t)\right] \qquad (4.2.24)$$

$$\dot{\bar{L}}(t) = \ell\left[\bar{\bar{y}}(t) - I(t) + \dot{\bar{\bar{y}}}(t) - \dot{I}(t)\right] - \bar{L}(t) \qquad (4.2.25)$$

In response to change in the labour ration the consumer will adjust his goods purchases and money stock.[22] Thus write

$$\dot{m}(t) = w\bar{L}(t) - px(\bar{L}(t), m(t)\,|\,p,w,\overset{*}{M}) \qquad (4.2.26)$$

Expressions (4.2.25) and (4.2.26) represent the dynamics of the model. With some substitution (4.2.25) may be rewritten as (4.2.27).

$$\dot{\bar{L}}(t) = \ell\left[(1+\rho)\ \left[x(\bar{L}(t), m(t)\,|\,p,w,\overset{*}{M}) + g(t)\right] - \rho\bar{\bar{y}}(t)\right] - \bar{L}(t) \qquad (4.2.27)$$

The stability properties of the model may be analysed in the same manner as case 1. Using a Taylors approximation rewrite the differential equation system (4.2.26), (4.2.27) in matrix form as (4.2.28).

$$\dot{Z}(t) = AZ(t)$$

where

$$\dot{Z}(t) = \begin{bmatrix} \dot{Z}_m(t) \\ \dot{Z}_{\bar{L}}(t) \end{bmatrix} \quad A = \begin{bmatrix} f_1 & f_2 \\ g_1 & g_2 \end{bmatrix} \quad Z(t) = \begin{bmatrix} Z_m(t) \\ Z_{\bar{L}}(t) \end{bmatrix}$$

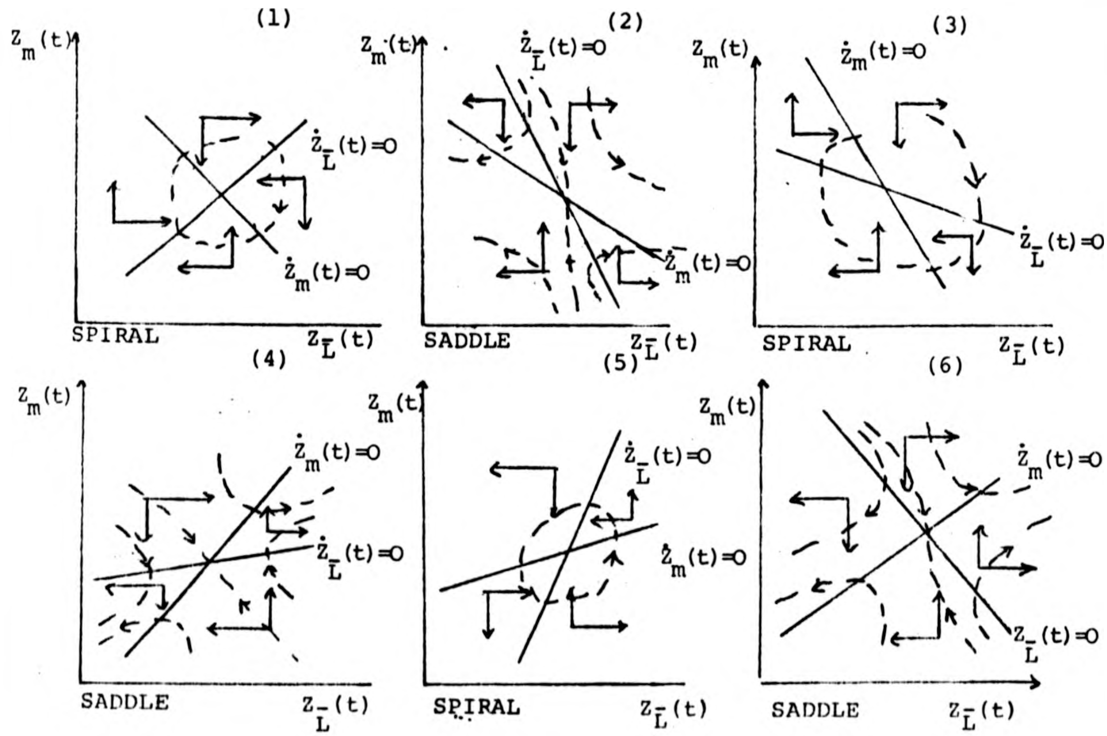
and

$$f_1 = -p \frac{\partial x}{\partial m(t)} < 0, \quad f_2 = w-p \frac{\partial x}{\partial \bar{L}(t)} \gtreqless 0$$

$$g_1 = (1+p)\frac{\partial \ell}{\partial x}\frac{\partial x}{\partial m(t)} > 0, \quad g_2 = (1+p)\frac{\partial \ell}{\partial x}\frac{\partial}{\partial \bar{L}(t)} - \gtreqless 0$$

Following the same procedure as in case 1 it may be shown that the phase diagrams of figure (4.2.5) describe the systems possible dynamic characteristics.

Fig. (4.2.5)



The possible outcomes demonstrated in figure (4.2.5) (1)-(6) are characterised as follows:

(1) $f_1, f_2, g_2 < 0, \quad g_1 > 0$

(2) $f_1, f_2 < 0, \quad g_1, g_2 > 0 - f_2/f_1 > -g_2/g_1$

(3) $f_1, f_2 < 0,\ g_1, g_2 > 0 \qquad -f_2/f_1 < -g_2/g_1$

(4) $f_1, g_2 < 0,\ f_2, g_1 > 0 \qquad -f_2/f_1 > -g_2/g_1$

(5) $f_1, g_2 < 0,\ f_2, g_1 > 0 \qquad -f_2/f_1 < -g_2/g_1$

(6) $f_1 < 0,\ f_2, g_1, g_2 > 0$

The solution path of each of the systems dynamics depicted in figure (4.2.5) has either a spiral or saddle point pattern. Here the government attempting to stabilize the system has one of two problems to solve, if there is a saddle point case, can it ensure that the economy stays on or approximately upon the stable manifold? If there is a spiral solution path, can it be ensured that it is convergent? The answers to the first question is a qualified affirmative, the lump sum transfer rule defined by expression (4.2.20) can be rewritten as (4.2.29)[23]

$$LST(t) = \text{Tan}\left[\frac{f_1-\beta_2}{f_2}\right]\overset{*}{\bar{L}}(t) + \overset{*}{M} - \left[\frac{f_1-\beta_2}{f_2}\right]\bar{L}(t) - m(t) \qquad (4.2.29)$$

If the government effects the transfers defined by (4.2.29) the system will be repeatedly returned to the approximation to the stable branch, and will achieve a stable equilibrium at $(\overset{*}{\bar{L}},\overset{*}{M})$. In the case depicted in figure (4.2.5) (1), (3) and (5) the problem faced by the government is to ensure that the spiral converges. This requires that the real parts of the systems characteristic roots be negative. That $f_1+g_2 < 0$. This clearly follows in (1) and (5) since $f_1, g_2 < 0$ in these cases. In (3) $f_1 < 0$, $g_2 > 0$ thus stability requires $|f_1| > g_2$. To achieve this stability condition the government needs to cut its expenditure, thus moving the equilibrium down the production function, raising the marginal product of labour and so reducing g_2, since g_2 contains the term $\partial\ell/\partial x = \partial\ell/\partial g(t)$.

Hence on the Keynesian Regime of a representative consumer, producer model, with fixed prices and no quantity tatonnement, and where the decision making process involves transactions effectively taking place first upon the labour market, the Government may stabilize the system by utilizing lump sum transfers or expenditure policies. Consider next the possibilities that may occur upon the Repressed Inflation Regime.

<u>The Repressed Inflation Regime</u>

As on the Keynesian regime there are two possible decision making processes that may occur here. First the producer may decide to sell from stock a certain quantity of consumption good given his expectation of how much labour will be available, and their replenishment of stock, when the labour market opens. Here it will be assumed consumers are willing to purchase however much stock the producer releases for sale. Second the consumer may decide to sell its labour services in anticipation of goods availability when the labour market opens. In this case it will be assumed that producers are willing to employ all consumers who wish to work.[24]

Case 1: The first component of any pair of trades occurs on the goods market.

Assume the consumer behaves according to the short-run labour supply function (4.3.30)

$$L(t) = l(x(t), m(t) \mid p, w, \overset{*}{M}) \qquad (4.2.30)$$

where

L(t) is the planned labour supply, x(t) is goods available for consumption by the consumer at time t.

m(t) is money stock at t.

Since x(t) is known to the consumer and his planned labour sales (L(t)) will be realised the change in the consumers money stock may be written as (4.2.31).

$$\dot{m}(t) = w\,l(x(t), m(t)\,|\,p, w, \overset{*}{M}) - px(t) \qquad (4.2.31)$$

The amount of consumption good made available to the consumer for consumption at t, will be producers planned output net of any desired inventory accumulation and government purchases as (4.2.32)

$$x(t) = y(t) - \dot{I}(t) - g(t) \qquad (4.2.32)$$

planned output will depend upon the producers expectation of labour availability when the labour market opens.

$$y(t) = \ell[\bar{\bar{L}}(t)] \qquad (4.2.33)$$

Thus using (4.2.33), (4.2.32) may be written as (4.2.34)

$$x(t) = \ell[\bar{\bar{L}}(t)] - \dot{I}(t) - g(t) \qquad (4.2.34)$$

Adjustment of producers expectations may be described as (4.2.35)

$$\dot{\bar{\bar{L}}}(t) = \psi[\bar{L}(t) - \bar{\bar{L}}(t)] \qquad (4.2.35)$$

where ψ is a constant adjustment parameter. $\bar{L}(t) = L(t)$ from (4.2.30). With some substitutions the dynamics of the quantity adjustment process may be described as (4.2.36) and (4.2.37).

$$\dot{m}(t) = wl(\ell[\bar{\bar{L}}(t)] - \dot{I}(t) - g(t), m(t)\,|\,p, w, \overset{*}{M}) - p[\ell[\bar{\bar{L}}(t)] - \dot{I}(t) - g(t)] \qquad (4.2.36)$$

$$\dot{\bar{\bar{L}}}(t) = \psi[l(\ell[\bar{\bar{L}}(t)] - \dot{I}(t) - g(t), m(t)\,|\,p, w, \overset{*}{M}) - \bar{\bar{L}}(t)] \qquad (4.2.37)$$

A repressed inflation equilibrium is defined by (4.2.36) and (4.2.37) both being zero, when $m(t) = \overset{*}{M}$ and $\bar{L}(t) = \bar{\bar{L}}(t)$. Initially $\dot{I}(t)$, the producers desired change in inventory stocks, will be assumed to represent the producers response to the previous instants experience and may be treated as exogenously determined at time t. This is, of course, the effect that labour supply rationing in the previous instance has upon the firms behaviour if the firm does not adjust its desired level of inventories.

To examine the stability properties of the system take first order Taylors approximations to linearise (4.2.36) and (4.2.37) which may now be written as (4.2.38) and (4.2.39)

$$Z_m(t) = f_1 Z_m(t) + f_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.38)$$

$$\dot{Z}_{\bar{\bar{L}}}(t) = g_1 Z_m(t) + g_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.39)$$

where

$$\dot{Z}_m(t) = \dot{m}(t) \;.\; \dot{Z}_{\bar{\bar{L}}}(t) = \dot{\bar{\bar{L}}}(t),\; Z_m(t) = m(t) - \overset{*}{M},\; Z_{\bar{\bar{L}}}(t) = \bar{\bar{L}}(t) - \overset{*}{\bar{\bar{L}}}(t)$$

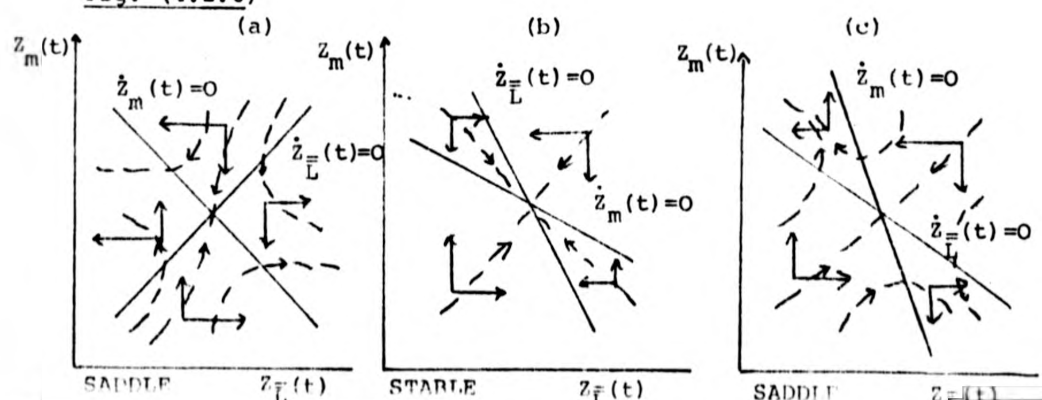
and

$$\left.\begin{aligned} f_1 &= w\frac{\partial l}{\partial m(t)} < 0, \quad f_2 = w\,\frac{\partial l}{\partial \ell}\,\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - p\,\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} < 0 \\ g_1 &= \psi\frac{\partial l}{\partial m(t)} < 0, \quad g_2 = \psi\left[\frac{\partial l}{\partial \ell}\,\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - 1\right] \lesseqgtr 0 \end{aligned}\right) \qquad (4.2.40)$$

Again each partial is evaluated at the equilibrium. f_1 and g_1 are negative since it is assumed an increase in the consumers money stock reduces his labour supply. $f_2 < 0$ since it is assumed $|-p/w| > \partial l/\partial \ell$.

Using (4.2.40) the dynamics of the economy upon this regime may be described by the phase diagrams in figure (4.2.6)

Fig. (4.2.6)



determined at time t. This is, of course, the effect that labour supply rationing in the previous instance has upon the firms behaviour if the firm does not adjust its desired level of inventories.

To examine the stability properties of the system take first order Taylors approximations to linearise (4.2.36) and (4.2.37) which may now be written as (4.2.38) and (4.2.39)

$$Z_m(t) = f_1 Z_m(t) + f_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.38)$$

$$\dot{Z}_{\bar{\bar{L}}}(t) = g_1 Z_m(t) + g_2 Z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.39)$$

where

$$\dot{Z}_m(t) = \dot{m}(t) \ . \ \dot{Z}_{\bar{\bar{L}}}(t) = \dot{\bar{\bar{L}}}(t), \ Z_m(t) = m(t) - \overset{*}{M}, \ Z_{\bar{\bar{L}}}(t) = \bar{\bar{L}}(t) - \overset{*}{\bar{\bar{L}}}(t)$$

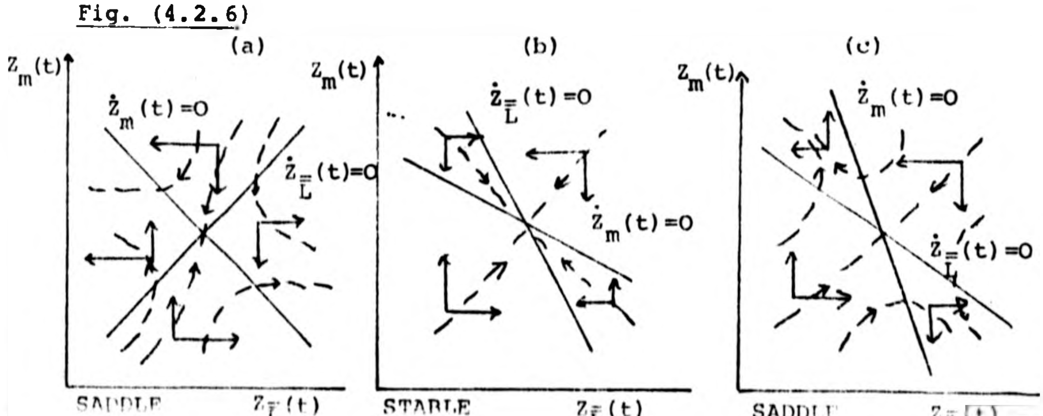
and

$$\left.\begin{array}{l} f_1 = w\dfrac{\partial l}{\partial m(t)} < 0, \quad f_2 = w\dfrac{\partial l}{\partial \ell}\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)} - p\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)} < 0 \\ g_1 = \psi\dfrac{\partial l}{\partial m(t)} < 0, \quad g_2 = \psi\left[\dfrac{\partial l}{\partial \ell}\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)} - 1\right] \lesseqgtr 0 \end{array}\right) \qquad (4.2.40)$$

Again each partial is evaluated at the equilibrium. f_1 and g_1 are negative since it is assumed an increase in the consumers money stock reduces his labour supply. $f_2 < 0$ since it is assumed $|-p/w| > \partial l/\partial \ell$.

Using (4.2.40) the dynamics of the economy upon this regime may be described by the phase diagrams in figure (4.2.6)

Fig. (4.2.6)



Note: The slopes of the stationaries are:

$$\frac{\partial z_m(t)}{\partial z_{\bar{\bar{L}}}(t)}\Bigg|_{\dot{z}_m(t)=0} = -\frac{f_2}{f_1} = -\frac{w\dfrac{\partial 1}{\partial m(t)}}{w\dfrac{\partial 1}{\partial \ell}\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)} - p\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)}} < 0$$

$$\frac{\partial z_m(t)}{\partial z_{\bar{\bar{L}}}(t)}\Bigg|_{\dot{z}_{\bar{\bar{L}}}(t)=0} = -\frac{g_2}{g_1} = \frac{\psi\left[\dfrac{\partial 1}{\partial \ell}\dfrac{\partial \ell}{\partial \bar{\bar{L}}(t)}-1\right]}{\psi\dfrac{\partial 1}{\partial m(t)}} \gtreqless 0$$

Arrows in phase diagrams

$$\frac{\partial \dot{z}_m(t)}{\partial z_m(t)} = f_1 < 0 \qquad \frac{\partial \dot{z}_{\bar{\bar{L}}}(t)}{\partial z_{\bar{\bar{L}}}(t)} = g_2 \gtreqless 0$$

Figure (4.2.6) (a), (b) and (c) demonstrates the stability properties of the repressed inflation regime where the decision making process is such that the first component of any pair of trades effectively takes place upon the labour market. The system has either stable or saddle point characteristics.

In an entirely similar manner to the Keynesian saddle point cases examined earlier an approximate government lump sum transfer/money supply rule may be defined, which maintains the system upon the stable branch in the saddle point equilibrium possibilities described by figure (4.2.6)(a) and (c). The stable branch is defined by (4.2.41)

$$m(t) = \overset{*}{M}(t) - \mathrm{Tan}\left[\frac{f_1-\beta_2}{f_2}\right]\overset{*}{\bar{\bar{L}}}(t) + \left[\frac{f_1-\beta_2}{f_2}\right]\bar{\bar{L}}(t) \qquad (4.2.41)$$

where β_2 is the stable root of the dynamical system (4.2.38) and (4.2.39). Thus the approximate lump sum transfer rule may be defined as (4.2.42)[25]:

$$LST(o) = \overset{*}{M}(t) - \mathrm{Tan}\left[-\frac{f_1-\beta_2}{f_2}\right]\overset{*}{\bar{\bar{L}}}(t) + \left[\frac{f_1-\beta_2}{f_2}\right]\bar{\bar{L}}(o) - m(o) \quad (4.2.42)$$

The government can thus stabilize the system by levying successive lump sum transfers according to (4.2.42).

There are two immediate implications of the result that repressed inflation equilibria may be stable, firstly the comparative statics associated with the supply multiplier may be a legitimate exercise, and secondly the excess effective demand hypothesis may be reasonably analysed in the context of the Repressed Inflation regime. This contradicts the results obtained by Böhm (1978) who found the repressed inflation regime unstable and Honkapohja and Ito who argue that the dynamics are monotonically convergent. This is because the analysis considered here is considerably different from their treatments as discussed previously.

The treatment of inventories in the preceding analysis argues that producers will adjust stocks to their desired level by withholding goods from the market. Stock discrepancies were generated by an inability on the part of the producer to achieve expected labour supplies, causing output to fall short of its planned level. Thus the quantity of consumption goods withheld from the market could be taken as predetermined. Such an approach assumes that the desired level of inventory stocks is not influenced by expectations at t. It may be argued that the target inventory stock is positively correlated with expected labour supply and output. Thus write the desired inventory stock adjustment as (4.2.43)

$$\dot{I}(t) = \xi\left[\overset{*}{I}\left[\ell\left[\bar{L}(t)\right]\right] - \hat{I}(t)\right] \quad (4.2.43)$$

where

ξ is a constant adjustment parameter, $\hat{I}(t)$ is the previous instants desired inventory stock, which was achieved. To examine the dynamics of the regime substitute (4.2.43) into (4.2.36) and (4.2.37) to obtain (4.2.44) and (4.2.45)

$$\dot{m}(t) = wl(\ell[\bar{\bar{L}}(t)] - \xi[\overset{*}{I}[\ell[\bar{\bar{L}}(t)]] - \hat{I}(t) - g(t), m(t) \mid p, w, \overset{*}{M}) - p[\ell[\bar{\bar{L}}(t)] - \xi[\overset{*}{I}[\ell[\bar{\bar{L}}(t)]] - \hat{I}(t) - g(t)] \qquad (4.2.44)$$

$$\dot{\bar{\bar{L}}}(t) = \psi[l(\ell[\bar{\bar{L}}(t)] - \xi[\overset{*}{I}[\ell[\bar{\bar{L}}(t)]] - \hat{I}(t)] - g(t), m(t) \mid p, w, \overset{*}{M}) - \bar{\bar{L}}(t)] \qquad (4.2.45)$$

From (4.2.44) and (4.2.45) differentiation yields the partial derivatives (4.2.46)

$$f_1' = w\frac{\partial l}{\partial m(t)} < 0, \quad f_2' = w\left[\frac{\partial l}{\partial \ell}\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} + \xi\frac{\partial l}{\partial \overset{*}{I}}\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial \bar{\bar{L}}(t)}\right] - p\left[\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - \xi\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial \bar{\bar{L}}(t)}\right] \gtreqless 0$$

$$(4.2.46)$$

$$g_1' = \psi\frac{\partial l}{\partial m(t)} < 0 \qquad g_2' = \psi\left[\frac{\partial l}{\partial \ell}\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - \xi\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - 1\right] \gtreqless 0$$

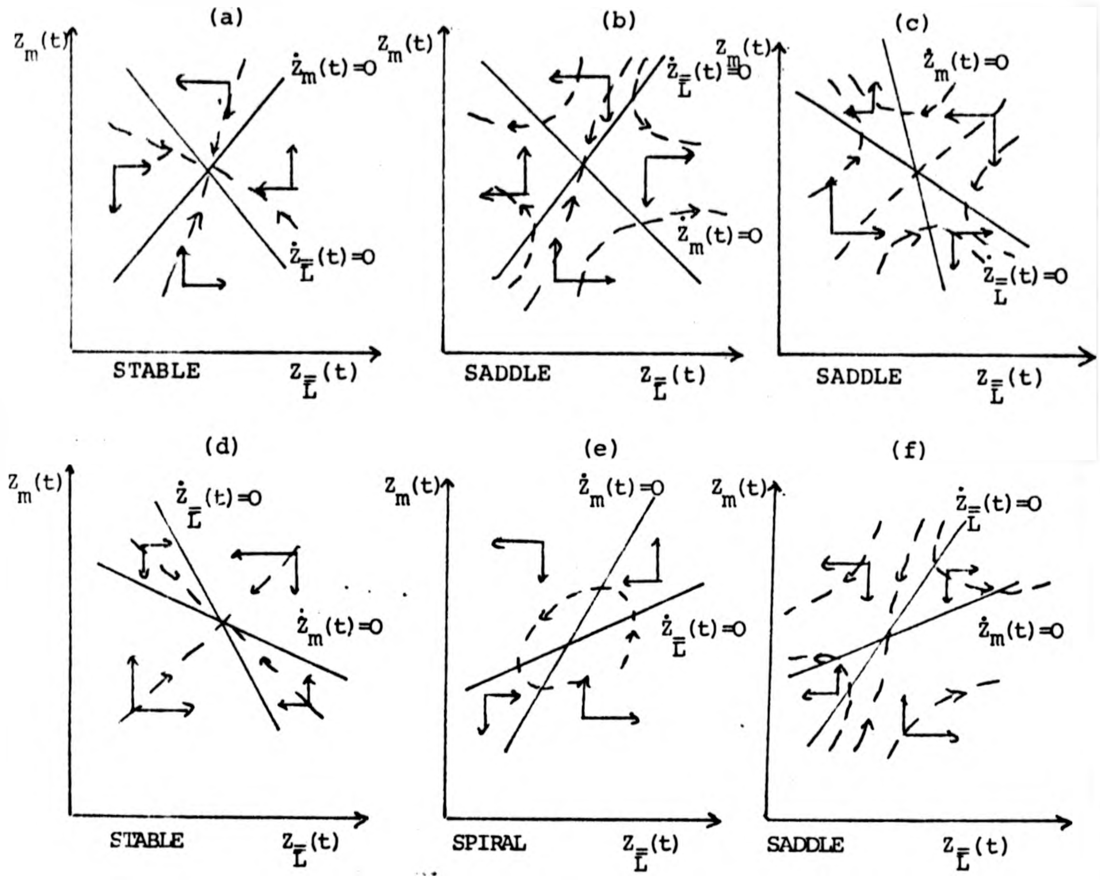
Thus the Taylors approximation may be used to linearise the system (4.2.44) and (4.2.45) to yield

$$\dot{z}_m(t) = f_1' z_m(t) + f_2' z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.47)$$

$$\dot{z}_{\bar{\bar{L}}}(t) = g_1' z_m(t) + g_2' z_{\bar{\bar{L}}}(t) = 0 \qquad (4.2.48)$$

The possible configurations of the systems dynamics are described by the phase diagrams in figure (4.2.7). The non-postivity of the partial, $f_1' < 0$, ensures that the system does not display a case of complete instability. The real parts of the characteristic roots of the system are not unambiguously positive.

Fig. (4.2.7)



In figure (4.2.7)

(a) $f_1', g_1', g_2' < 0, \quad f_2' > 0$

(b) $f_1', f_2', g_1' < 0, \quad g_2' > 0$

(c) $f_1', f_2', g_1', g_2' < 0$ and $-f_2'/f_1' < -g_2'/g_1'$

(d) $f_1', f_2', g_1', g_2' < 0$ and $-f_2'/f_1' > -g_2'/g_1'$

(e) $f_1', g_1' < 0 \quad f_2', g_2' > 0$ and $-f_2'/f_1' > -g_2'/g_1'$

(f) $f_1', g_1' < 0 \quad f_2', g_2' > 0$ and $-f_2'/f_1' < -g_2'/g_1'$

As figure (4.2.7) demonstrates in cases (a) and (d) the dynamics of the repressed inflation regime quantity adjustment may be stable. Cases (b),(c),(d) and (f) show that saddle point outcomes are possible, here the stabilization rule for lump sum transfer will be as (4.2.49).

$$LST(o) = \overset{*}{M}(t) - \text{Tan}\left[\frac{f_1' - \beta_2'}{f_2}\right]\overset{*}{\bar{\bar{L}}}(t) + \left[\frac{f_1' - \beta_2'}{f_2}\right]\bar{\bar{L}}(o) - m(o) \quad (4.2.49)$$

where β_2' is the stable characteristic root of the system (4.2.47), (4.2.48).

Instability may arise in case (e), spiraling divergent behaviour may be observed. Stability would require the real parts of the characteristic roots of the system to be negative, this may easily be shown to require that (4.2.50) holds.

$$-f_1' > g_2' \quad (4.2.50)$$

OR EQUIVALENTLY

$$-w\,\frac{\partial l}{\partial m(t)} > \psi\left[\frac{\partial l}{\partial \ell}\,\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - \xi\,\frac{\partial \overset{*}{I}}{\partial \ell}\,\frac{\partial \ell}{\partial \bar{\bar{L}}(t)} - 1\right]$$

A reversal of the inequalities in (4.2.50) cannot be ruled out, however inspection suggests that this is improbable.

Despite the potential source of instability, the repressed Inflation regime, when the first component of any pair of trades occur on the goods market, has reasonably good stability properties. Consider now the behaviour of the model when the labour market opens first.

Case 2: The first component of any pair of trades occurs upon the labour market.

Again assume the consumer to behave according to the short-run labour supply function (4.2.51)

$$L(t) = l(\bar{\bar{x}}(t),\ m(t)\,|\,p, w, \overset{*}{M}) \quad (4.2.51)$$

Here $\bar{x}(t)$ is the quantity of consumption good the consumer expects to be available when the goods market opens.

The consumer thus accumulates money balances by selling labour in the anticipation of feasible good purchases. The actual change in the money stock will depend upon the goods ration he actually faces. (4.2.52) describes the consumers money stock dynamics

$$\dot{m}(t) = wL(t) - px(t) \qquad (4.2.52)$$

where x(t) is the realised goods ration.

The actual goods availability will depend upon the output producers can achieve from the labour supply, L(t), net of any inventory stock adjustment and government purchases. Goods supply to the consumer is defined by (4.2.53).

$$x(t) = y(t) - g(t) - \dot{I}(t) \qquad (4.2.53)$$

Which may be rewritten, (4.2.54), substituting the production function for the output level to recognise the producers labour supply constraint.

$$x(t) = \ell[L(t)] - g(t) - \dot{I}(t) \qquad (4.2.54)$$

It will initially be assumed that desired inventory stock adjustment $\dot{I}(t)$ represents the accumulation (decumulation) of producers inventory stocks, that will rectify the discrepancy between target inventory stock and the last instants market experience, and may then be treated as exogenous.

Adjustment of consumers goods rationing expectations will depend upon the expectations held when the labour market meets and the actual transaction carried out when the goods market meets as (4.2.55)

$$\dot{\bar{\bar{x}}}(t) = \phi\left[x(t) - \bar{\bar{x}}(t)\right] \qquad (4.2.55)$$

where ϕ is a constant adjustment parameter.

With some substitution into (4.2.52) and (4.2.55) the dynamics of the repressed inflation regime in this case may be described as (4.2.56) and (4.2.57).

$$\dot{m}(t) = wL(\bar{\bar{x}}(t), m(t)\,|\,p,w,\overset{*}{M}) - p\left[\ell\left[1(\bar{\bar{x}}(t), m(t)\,|\,p,w,\overset{*}{M}\right]\right]$$

$$+ p\dot{I}(t) + pg(t) \qquad (4.2.56)$$

$$\dot{\bar{\bar{x}}}(t) = \phi\left[\ell\left[1(\bar{\bar{x}}(t), m(t)\,|\,p,w,\overset{*}{M})\right] - g(t) - \dot{I}(t) - \bar{\bar{x}}(t)\right] \qquad (4.2.57)$$

A repressed inflation equilibrium in this case is defined by (4.2.56) and (4.2.57) being equal to zero, $m(t)=\overset{*}{M}$ and $\bar{\bar{x}}(t) = x(t)$.

Again the system is linearised by taking a first order Taylors approximation as (4.2.58) and (4.2.59)

$$\dot{Z}_m(t) = f_1 Z_m(t) + f_2 Z_{\bar{\bar{x}}}(t) = 0 \qquad (4.2.58)$$

$$\dot{Z}_{\bar{\bar{x}}}(t) = g_1 Z_m(t) + g_2 Z_{\bar{\bar{x}}}(t) = 0 \qquad (4.2.59)$$

where

$$\dot{Z}_m(t) = \dot{m}(t),\ \dot{Z}_{\bar{\bar{x}}}(t) = \dot{\bar{\bar{x}}}(t),\ Z_m(t) = m(t) - \overset{*}{M},\ Z_{\bar{\bar{x}}}(t) = \bar{\bar{x}}(t) - \overset{*}{\bar{\bar{x}}}(t)$$

and

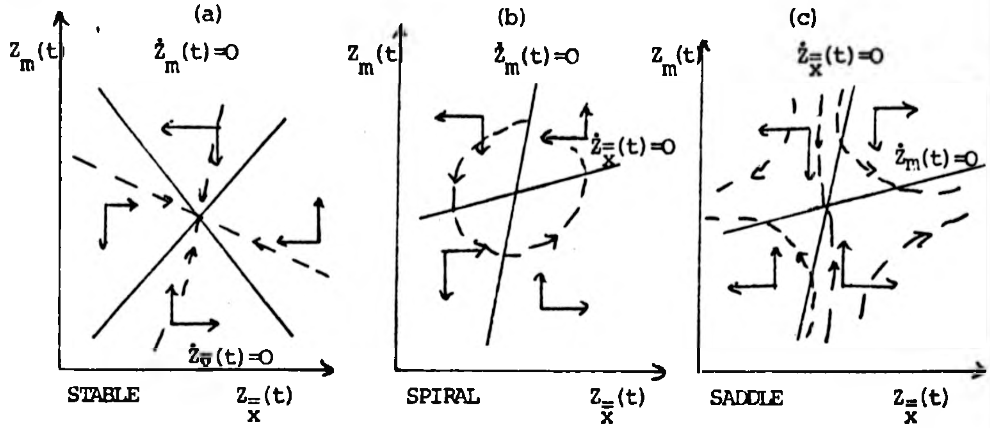
$$\left.\begin{aligned} f_1 &= w\frac{\partial 1}{\partial m(t)} - p\frac{\partial \ell}{\partial 1}\frac{\partial 1}{\partial m(t)} < 0,\ f_2 = w\frac{\partial 1}{\partial \bar{\bar{x}}(t)} - p\frac{\partial \ell}{\partial 1}\frac{\partial 1}{\partial \bar{\bar{x}}(t)} > 0 \\ g_1 &= \phi\frac{\partial \ell}{\partial 1}\frac{\partial 1}{\partial m(t)} < 0,\quad g_2 = \phi\left[\frac{\partial \ell}{\partial}\frac{\partial 1}{\partial \bar{\bar{x}}(t)} - 1\right] \gtrless 0 \end{aligned}\right)$$

$$(4.2.60)$$

Again each of the partials is evaluated at the equilibrium. The signs of the partials follow from the properties $w/p > \partial\ell/\partial 1$ $\partial 1/\partial m(t) < 0$, $\partial\ell/\partial 1 > 0$, $\partial 1/\partial \bar{\bar{x}}(t) > 0$.

Figure (4.2.8) describes the systems dynamics.

Fig. (4.2.8)



In figure (4.2.8)

(a) $f_1, g_1, g_2 < 0 \quad f_2 > 0$

(b) $f_1, g_1 < 0 \quad f_2, g_2 > 0$ and $-f_2/f_1 > -g_2/g_1$

(c) $f_1, g_1 < 0 \quad f_2, g_2 > 0$ and $-f_2/f_1 < -g_2/g_1$

The slopes of the stationaries and the arrows on the phase diagrams are derived as in previous cases.

In case (a) there is stability of the quantity adjustment process. In (c) a saddle point equilibrium arises, and the lump sum transfer/money supply rule which places the system upon the stable branch will be as (4.2.61)

$$LST(t) = \overset{*}{M}(t) - \text{Tan}\left[\frac{f_1-\beta_2}{f_2}\right]\overset{*}{\bar{\bar{x}}}(t) + \left[\frac{f_1-\beta_2}{f_2}\right]\bar{\bar{x}}(o)-m(o) \qquad (4.2.61)$$

where β_2 is the stable root of the system (4.2.58)(4.2.59).

In case (b) the system may be unstable, stability would require that (4.2.62) holds.

$$f_1 + g_2 < 0 \qquad (4.2.62)$$

Equivalently $\quad p \frac{\partial \ell}{\partial l} \frac{\partial l}{\partial m(t)} - w \frac{\partial l}{\partial m(t)} > \phi \left[\frac{\partial \ell}{\partial l} \frac{\partial l}{\partial \bar{\bar{x}}(t)} - 1 \right]$

There appears to be no obvious policy rule which will correct this situation. However inspection of (4.2.62) suggests that if the marginal product of labour is low, the economy is operating at a high level of activity, then case (b) will be a stable spiral.

These results again reinforce the previous findings that the repressed inflation regime may not be as unstable as suggested in some studies.

In this case, where the labour market precedes the goods market, producers may adjust their target inventory stocks in response to their labour constrained output. Thus let desired inventory stock adjustment be written as (4.2.63)

$$\dot{I}(t) = \tau \left[\overset{*}{I} \left[Y(t) \right] - \hat{I}(t) \right] \qquad (4.2.63)$$

where τ is a constant adjustment parameter, $\hat{I}(t)$ is the initial inventory stock at t.

The dynamics of this case may now be rewritten as (4.2.64) and (4.2.65)

$$\dot{m}(t) = wl(\bar{\bar{x}}(t), m(t) \mid p, w, \overset{*}{M}) - p\left[\ell\left[l(\bar{\bar{x}}(t), m(t) \mid p, w, \overset{*}{M})\right]\right]$$
$$+ p\tau\left[\overset{*}{I}(\ell\left[l(\bar{\bar{x}}(t), m(t) \mid p, w, \overset{*}{M})\right]) - \hat{I}(t)\right] + pg(t) \qquad (4.2.64)$$

$$\dot{\bar{\bar{x}}}(t) = \phi\left[\ell\left[l(\bar{\bar{x}}(t), m(t) \mid p, w, \overset{*}{M})\right] - g(t) - \tau\left[\overset{*}{I}(\ell\left[l(\bar{\bar{x}}(t), m(t) \mid p, w, \overset{*}{M})\right] - \hat{I}(t)\right|$$
$$- \bar{\bar{x}}(t) \Big| \qquad (4.2.65)$$

The system is linearised using first order Taylors approximations to yield (4.2.66) and (4.2.67)

$$\dot{z}_m(t) = f_1{}' z_m(t) + f_2{}' z_{\bar{\bar{x}}}(t) = 0 \qquad (4.2.66)$$

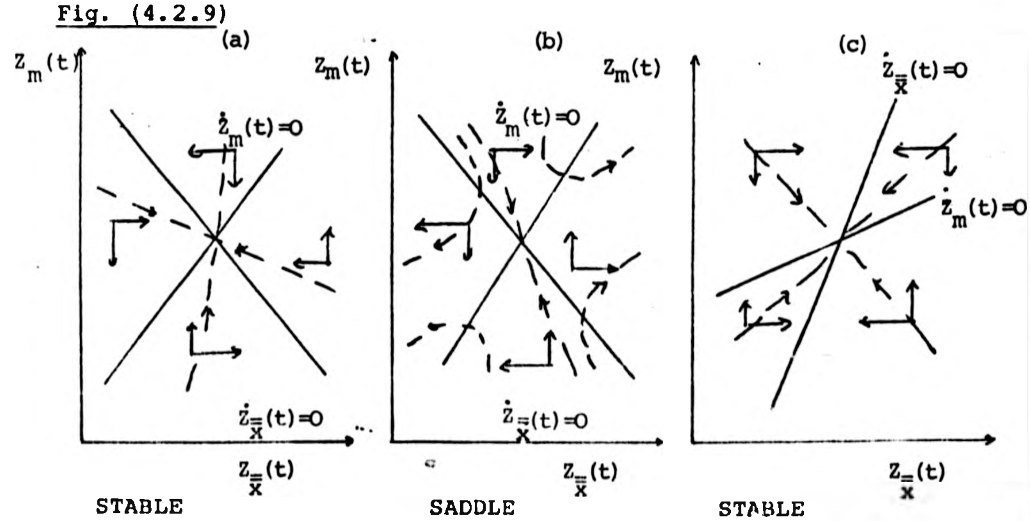
$$\dot{z}_{\bar{\bar{x}}}(t) = g_1{}' z_m(t) + g_2{}' z_{\bar{\bar{x}}}(t) = 0 \qquad (4.2.67)$$

Here the partials are defined as (4.2.68)

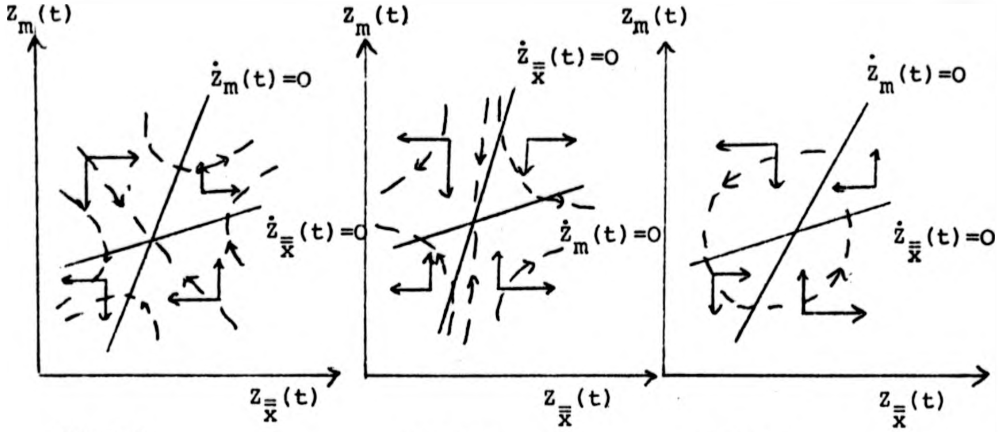
$$\left.\begin{aligned}
f_1{}' &= w\frac{\partial l}{\partial m(t)} - p\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial m(t)} + p\tau\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial m(t)} < 0\\
f_2{}' &= w\frac{\partial l}{\partial \bar{\bar{x}}(t)} - p\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial \bar{\bar{x}}(t)} + p\tau\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial \bar{\bar{x}}(t)} > 0\\
g_1{}' &= \phi\left[\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial m(t)} - \tau\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial m(t)}\right] \gtrless 0\\
g_2{}' &= \phi\left[\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial m(t)} - \tau\frac{\partial \overset{*}{I}}{\partial \ell}\frac{\partial \ell}{\partial l}\frac{\partial l}{\partial \bar{\bar{x}}(t)} - 1\right] \gtrless 0
\end{aligned}\right\} (4.2.68)$$

again the partial derivatives in (4.2.68) are evaluated at equilibrium. The signs of the terms $f_1{}'$ and $f_2{}'$ follow from the previous arguments and the negativity of $\partial l/\partial m(t)$ and positivity of $\frac{\partial l}{\partial \bar{\bar{x}}(t)}$. The phase diagrams in (4.2.9) illustrate the possibilities.

Fig. (4.2.9)



(d) (e) (f)



SADDLE SADDLE SPIRAL

In figure (4.2.9)

(a) $f_1', g_1', g_2' < 0$, $f_2' > 0$

(b) $f_1' < 0$, $f_2', g_1', g_2' > 0$

(c) $f_1', g_2' < 0$, $f_2', g_1' > 0$ and $-f_2'/f_1' < -g_2'/g_1'$

(d) $f_1', g_2' < 0$, $f_2', g_1' > 0$ and $-f_2'/f_1' > -g_2'/g_1'$

(e) $f_1', g_1' < 0$, $f_2', g_2' > 0$ and $-f_2'/f_1' < -g_2'/g_1'$

(f) $f_1', g_1' < 0$ $f_2', g_2' > 0$ and $-f_2'/f_1' > -g_2'/g_1'$

As the diagram figure (4.2.9) illustrates, cases (a) and (c) are stable. Cases (b) and (d) are saddle point equilibria which may be stabilized by the lump sum transfer rule (4.2.69)

$$LST(o) = \overset{*}{M}(t) + \mathrm{Tan}\left[\frac{f_1'-\beta_2}{f_2'}\right]\overset{*}{\bar{\bar{x}}}(t) - \left[\frac{f_1'-\beta_2}{f_2'}\right]\bar{\bar{x}}(o)-m(o) \qquad (4.2.69)$$

The saddle point equilibrium of case (e) can be stabilized by (4.2.70)

$$LST(o) = \overset{*}{M}(t) - \mathrm{Tan}\left[\frac{f_1'-\beta_2}{f_2'}\right]\overset{*}{\bar{\bar{x}}}(t) + \left[\frac{f_1'-\beta_2}{f_2'}\right]\bar{\bar{x}}(o)-m(o) \qquad (4.2.70)$$

where β_2 is again the stable root.

The spiral case may be unstable, stability would require,

$f_1{}' + g_2{}' < 0$ inspection of these terms in (4.2.68) suggests that this cannot be determined.

Thus on the repressed inflation regime in the case where the labour market transactions precede those on the goods market, and target inventory stock depends upon firms labour constrained output, the system displays no completely unstable case. Government lump sum transfer rules may be devised to place the system on the stable manifold in the saddle point cases.

Conclusions

In the preceding analysis a simple aggregate disequilibrium trading model was considered. Trade occurred sequentially upon the good and labour markets. Since an agent cannot be simultaneously on both markets some trades must be made in anticipation of the trading opportunities that will be available upon the other. Keynesian unemployment and Repressed inflation regimes were examined under two scenarios. Case 1, where the first component of any pair of trades occurs upon the goods market, or the decision making process is as if the goods market opens first. Case 2, where the decision making process is as if the first component of any pair of trades occurs on the labour market. Both regimes and cases were studied under different assumptions about how consumers decide upon their desired money stock holdings and how firms decide upon their desired inventory stock. Firstly, inventory and money stock adjustments were considered to be corrective measures in response to the discrepancy between the previous instants actual and desired stock levels. Secondly, stock adjustments were assumed forward looking and to depend upon anticipated trades.

The dynamics of the quantity adjustment process were analysed by examining differential equations in money stock and expectations. The various forms which the differential equation in expectations adjustment took generated the numerous dynamic possibilities. The manner in which expectations are formed and revised is therefore very important. Tatonnement treatments of the quantity adjustment process obscure the importance of expectations adjustment.

The results obtained from the analysis show that when both stocks'of goods and money are examined numerous dynamic possibilities are generated. Stable, saddle point and cyclical possibilities arose. On the Keynesian regime it was argued that in the saddle point cases a lump sum transfer/money supply rule can be derived which keeps the system upon the stable manifold, whilst in spiral cases, a cut in government expenditure may result in generating a stable spiral. This generated the implication that in the spiral cases in this regime there may be a trade off between the stability of the equilibrium which may require a cut in government expenditure, and the level of unemployment displayed at that equilibrium.

Upon the Repressed inflation regime it was also possible to derive lump sum transfer/money supply rules which would yield stability in saddle point cases. However in the spiral possibilities no government stabilization rule is immediately apparent.

The analysis in this section cannot be compared directly with either Böhm's (1978) or Honkapohja and Ito's (1980). Here money and inventory stock adjustments are taken together, and there are consequently considerable complexities.

Firm conclusions cannot be drawn without further information about parameter magnitudes and expectations formation. However, the analysis does provide certain insights, many of the configurations examined suggest that there is a role for government intervention to stabilize the equilibria. Further, it may be concluded that, in the absence of government intervention, comparative statics exercises such as calculating supply and demand multipliers may not be legitimate.

These conclusions are, however, tentative. An unstable dynamical process on a regime may be terminated by a regime switch which halts the movement. Further, the introduction of endogenous price adjustment may change the picture.

FOOTNOTES

1. Hahn (1977) (1978), Varian (1977) and Futia (1977) have made contributions in which prices and quantities are endogenously simultaneously determined.

2. Models in the preceding section, although expressed as operating over a sequence of periods, may also be interpreted as adjustment processes within a period.

3. Böhm's (1978) analysis is about the dynamics of household money stock adjustment over a succession of periods, and may be regarded as different from the actual adjustment to the short-run fix-price equilibrium.

4. The period is sufficiently long for quantity adjustments to establish the fix-price equilibria.

5. This can be established by linearising the system using a Taylors approximation and demonstrating that the characteristic roots of the system have negative real parts.

6. There will of course be intermediate formulations where one market clears. This will occur upon the boundary between regimes.

7. This of course rules out the possibility of an under-consumption regime and this may be important for Löfgrens results.

8. Benassy (1977) provides such a treatment of transaction costs.

9. Benassy (1977) considers the maximization problem faced by an individual trader in a sequential trading context.

10. In some sense this analysis combines the approaches taken by Böhm (1978) and Honkapohja and Ito (1980).

11. The sequence in which agents visit markets would in a disaggregated model be part of an agents choice calculus, and may well effect the nature of the equilibrium of the economy.

12. These assumptions simply state that the Keynesian case is being analysed here.

13. At low levels of economic activity the producer may wish to hold smaller stocks, and if this occurs, the stability properties of the model may worsen. However since this is a short-run approach such adjustments associated with long run expectations will be neglected.

14. It is assumed that the government levies a 100% profits tax, see Böhm (1978) and Futia (1977) who use the same assumption.

15. If there is no exogenous government expenditure the existence of a non-trivial equilibria may be problematic. It may be shown that if $0<g(t)\leq y(t)^F[\mathrm{Tan}(w/p)]$ (F indicates full employment) then a non-trivial Keynesian equilibrium exists for the model.

16. The assumption that consumers adjust to a given target money stock is a first approximation, the target will be made endogenous in subsequent analysis.

17. A period is defined by the length of time the price vector is fixed.

18. For an explicit multi-period model see Muellbauer and Portes (1978) or Neary and Stiglitz (1980).

19. If when government expenditure is zero, $g(t)=0$, the system is unstable then government expenditure policy is ineffective upon this regime.

20. The signs of the partial derivatives again follow from the standard $w/p \leq$ MPL condition, which must hold in the absence of forced trading.

21. It is assumed that the total inventory stock of the firm is $S(t)=\overset{*}{I}+I(t)$ and that $\overset{*}{I}$ is sufficiently large that any excess consumer demand over production may always be met.

22. The two decisions are not independent because of the budget constraint.

23. Here anticipated labour sales have replaced actual labour sales, $\overset{*}{L}(t)$ by $\bar{L}(t)$ and the terms f_1, f_2, g_1, g_2 and β_2 have been redefined according to the parameters of this problem. The technique is identical to case 1.

24. These assumptions simply indicate the economy is upon the repressed inflation regime.

25. The definition of the stable branch on the Repressed inflation regime differs from that on the Keynesian regime since the stable manifold has positive slope in $(m,\bar{L})$ space in the former and negative slope in the latter.

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5. THE DETERMINATION OF PRICES

5.1 Desirable Properties of a Price Adjustment Mechanism

The preceding four chapters examined various characteristics of disequilibrium macroeconomic models and introduced some new extensions. Most of the models presented do not provide an adequate explanation of price determination and adjustment. This chapter considers various explanations of price determination and adjustment that have been suggested in the literature and discusses them in the light of some desirable properties to be outlined below.

It should perhaps be remarked that although disequilibrium models have been heavily criticised for failing to explain why market clearing prices are not realised, the problem of price determination is equally pertinent to models of continuous equilibrium which fail to provide an adequate explanation for continuous perfect price adjustment.

Before examining suggested explanations of price adjustment and determination it is useful to consider what might be desirable properties for an adjustment mechanism. Most of these are self-evident and well known however a clear statement of each will clarify the arguments to follow.

(a) Price adjustments should be made simultaneously with quantity adjustments, and should be determined as part of the solution to individual or groups of agents maximization problems.

This requirement is of course closely related to often quoted observation of Arrow (1959), that when there is excess demand (supply) agents must abandon the perfectly competitive assumption that they can buy (sell) all that they wish at the going market price. Agents are price setters who must choose both prices and consequently quantities simultaneously according to some known or perceived finite elasticity supply and demand curves.

(b) Price and Quantity adjustments should be related explicitly to the non-synchronized nature of trading, and consequent discontinuous reception of signals.

Since agents visit markets sequentially they cannot base price adjustments upon information to be gained upon markets as yet unvisited. Consequently both expected signals, extrapolated from past experience, and information deducible from other agents behaviour should play an important role in an adjustment process. In this context information transferred between agents about their future market behaviour must be incentive compatible to be useful.

(c) Price adjustments or agreements should be made upon the basis of verifiable information.

If prices upon one market are to be agreed or adjusted upon the basis of anticipations or 'second hand' information then agents should be able to verify whether the information or anticipations were correct. This form of requirements for a price agreement or adjustment will be important in the context of implicit contracts and asymmetric information to be discussed in relation to the contributions of Bailey (1974) Azariades (1975) Grossman and Hart (1981) and others.

(d) Adjustment of both prices and quantities should not be costless.

Either to search for another agent who is willing to complete the other side of a desired new transaction, or to disseminate information to potential trading partners is a costly process.

Such costs should be explicit in planned optimal adjustments.

(e) Any mechanism which explains price adjustment should explain as extreme cases both fixed and perfectly flexible prices.

This is a more prosaic requirement but observation of reality quickly indicates that prices are in some periods and on some markets perfectly flexible, giving the appearance of perfect auction markets, yet are, on other occasions, fixed. It would be desirable that a proposed adjustment mechanism should explain why under certain parameter configurations the two extremes occur.

In the next section candidates for an adequate price adjustment mechanism will be considered in the light of (a)-(e). However, before doing so a remark about price determination is perhaps required. Several of the mechanisms to be considered shortly involve interesting arguments relating to when prices and when quantities adjust, however, frequently the initial price vector adopted is arbitrary (or historically given), and the models explain why and how prices adjust but they do not determine the actual level of prices.

5.2 A Consideration of Price Adjustment Mechanisms

It is widely understood that if prices are fixed at non-market clearing values then an equilibrium must be established by some quantity allocation rule. (Rationing Scheme) Despite the attractive macroeconomic models that such an approach generates, the theory cannot be considered a reliable tool for making policy prescriptions unless an acceptable price adjustment mechanism is incorporated.

In this section various adjustment mechanisms will be considered firstly in the light of the discussion about the desirable properties of an adjustment mechanism given in Section 5.1, and secondly in the light of the implications the mechanisms have for the macroeconomics of the theory.

Numerous arguments for either fixed or imperfectly adjusting prices have been advanced, however five basic approaches, or potential approaches to the problem may be identified.[1]

(i) The Effective Excess Demand Approach

Previously discussed in section 4.1, the effective excess demand approach develops Leijonhufvud's initial idea that prices may not adjust due to effective demand failures. He argues that quantities adjust very rapidly and thus the correct formulation of excess demand functions must include effective rather than notional demands and supplies. The reasons why quantities move faster than prices are based upon arguments about informational problems and liquidity constraints. Effective excess demand functions it is then argued may not cause prices to return to their market clearing values. Whatever the characteristics of an effective excess demand function may be it is clear that such a price adjustment mechanism is not adequate given the desirable criteria that have been proposed.[2] Most importantly it can be seen that this adjustment mechanism does not allow simultaneous price and quantity adjustment as part of the solution to individual agents maximization problems.

Benassy (1976, 1980) suggests a price adjustment mechanism conceptually somewhat similar to Leijonhufvud but explicitly introducing monopolistic price setting. He assumes that there are two types of good in the economy studied, H_o being the set of goods the prices of which are exogenously fixed, and H_i the set of goods the prices of which are determined by agents. Further it is assumed that $H_i \cap H_j = \{\phi\}$ $\forall\ i \neq j$ i.e. each good is priced by a monopolist, also it is assumed that only suppliers set prices and each produces only one good. The economy is assumed to behave as follows. Monopolists perceive demand curves for their goods

$$\hat{z}_{ih}(p_i | \sigma_i) \qquad (5.2.1)$$

where $\sigma_i = \{\bar{p}, \bar{z}_i, \underline{z}_i\}$ is the signal of prices and upper and lower bounds that trader i has observed. The perceived demand curves are assumed to have the natural property of going through the currently observed point.

Traders maximize their utility subject to the perceived demand curves, the solution to which is an optimal price as (5.2.2).

$$p_i^*(\sigma_i) = p_i^*(\bar{p}, \bar{z}_i, \underline{z}_i) \qquad (5.2.2)$$

Once each agent has calculated and announced his optimal price, each then maximizes again subject to the set of <u>all</u> quoted prices and the set of fixed prices H_o.

By the usual mechanism a Benassy K-equilibrium in quantities is then established. If an excess of effective demands $\tilde{Z}$ over realised trades Z^* is observed agents re-estimate their perceived demand curves and quote a new set of optimal prices. This process continues until a monopolistic equilibrium is established which is defined as a price vector p^*, net trades Z^*_i, effective demands $\tilde{Z}_i$, perceived constraints $\bar{Z}_i$ and $\underline{Z}_i$ such that

(1) (Z^*_i), $(\tilde{Z}_i)$, $(\bar{Z}_i, \underline{Z}_i)$ are a fix price equilibrium with respect to p^*

(2) $p^*_i = p^*_i(p^*_i, \bar{Z}_i, \underline{Z}_i) \quad \forall i \in H_i$

Heuristically a monopolistic equilibrium is a fix price Benassy K-equilibrium, where upon the basis of perceived demand curves agents have no desire to adjust the prices in their control.

This mechanism may perhaps appear superficially attractive, especially as Benassy presentsit with his customary elegance. Indeed the mechanism is consistant with the desirable property (e) discussed above. If the set of prices H_o are Walrasian and the perceived demand curves $\hat{Z}_{ih}(p_i, \sigma_i)$ are identical to the market demand curves of atomistic traders a full flex-price general equilibrium will result. If however perceived demand curves are finitely elastic, but produce mutually compatible behaviour by agents, then a fix price quantity constrained equilibrium will obtain since agents cannot express the effective demand signals to induce further price adjustment.[3]

However Benassy's approach is not satisfactory in a number of respects. Properties (a) and (b) in section 5.1 argue that agents should select optimal price and quantity adjustments upon an individual market simultaneously, but that adjustments should be made sequentially across markets. Benassy's mechanism does the opposite, price adjustments occur simultaneously across all markets and then are followed sequentially by simultaneous quantity adjustments across markets. Evidently Leijonhufvud's dictum that quantities adjust faster than prices is being adopted without any justification as to why agents should decide to make adjustments in this manner. The rule imposed that only suppliers set prices is arbitrary (the 1980 paper makes the slightly weaker assumption that only one side of the market may set prices) and is only justified by the author expressing the desire not to get involved in any game theoretic aspects of price setting. One brief comment in support for Benassy should be made. Drazen (1980) criticises the mechanism because

> 'excess demands $\tilde{\hat{z}}$ sent to a market out of K-equilibrium are not reasonable indicators of the size of the disequilibrium and, therefore, do not present price setting agents with the information necessary to change prices.'

This criticism is ill directed as given Benassy's formulations only those signals received when the system is in a K-equilibrium are allowed to have any effect upon price setting agents. Further adjustments can only be made upon the basis of signals agents may receive, and the only information upon excess demand they receive when prices 'require' adjustment is based upon effective excess demands. In this sense Benassy is correct.

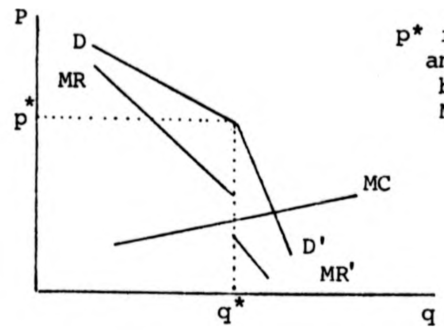
Grandmont and Laroque (1976) present an endogenous price model somewhat similar to Benassy's utilising the Drèze framework. This paper will be discussed in section (iii) which considers conjectural and expectations models.

The Macroeconomic implications of this type of approach are well understood and are best expressed in the work of Muellbauer and Portes (1978), considered in some depth in section 2.1. The price adjustment mechanism proposed by Benassy requires very little modification of their model except that changes in government behaviour may have profound effects upon perceived demand curves, and consequent effects upon prices. Placebo effects may then be significant. However since this price adjustment mechanism itself is unsatisfactory any macroeconomic implications it has are uninteresting.

(ii) <u>Kinked Demand and Supply Curves and Approaches Arising from Discontinuities</u>

Sweezy (1939) first suggested that the cause of price rigidities might be due to the demand curve of a firm in an oligopolistic industry being kinked at the prevailing market price. The kink causes a discontinuity in the marginal revenue curve and consequently a marginal cost curve passing through the discontinuity could move within a certain parameter range without disturbing price or output. Sweezy argued that the demand curve faced by the oligopolist is less elastic below the prevailing market price than above it. Rival oligopolists were supposed to defend their market shares by following price cuts that any of their number made, but to accept an increased market share by not responding when one raises its price. The familiar picture figure (5.2.1) illustrates this:

Fig. (5.2.1)



p* is the prevailing price, and will not be disturbed by small perturbations in MC

It is not immediately obvious why firms should be concerned with their market share, unless this is their short-run strategy to maximize expected discounted future profits in some form of inter-temporal game.

Negishi (1961, 74, 79) derives a general equilibrium model based upon kinked demand curves, where the reasoning behind the existence of a perceived kink differs from Sweezy's, and where the usual criteria of profit and utility maximization are in operation. Using a monopolistic competition paradigm Negishi argues that firm i perceives an inverse demand curve of the following form

$$p_i = p_i\ (y_i,\ \bar{p}_i,\ \bar{y}_i) \qquad (5.2.3)$$

which has the following properties

(1) $\bar{p}_i = p_i(\bar{y}_i,\ \bar{p}_i, \bar{y}_i)$ The perceived demand curve goes through the currently experienced price output pair $(\bar{p}_i, \bar{y}_i)$.

(2) $p_i = \bar{p}_i\ \forall\ y_i < \bar{y}_i$, $\partial p_i/\partial y_i < 0\ \forall\ y_i > \bar{y}_i$. The firm is a price taker facing a horizontal demand curve for all levels of output below its current output, but faces

a finitely elastic demand curve

for any output in excess of $\bar{y}_i$.

The argument is that if price rises above $\bar{p}_i$ all consumers who habitually purchase from the firm will observe the price rise and gain their supply elsewhere in the economy. If price falls below $\bar{p}_i$, this information will only be disseminated to some subset of the firms potential new customers and thus only a finite number will switch to it as their source of supply. The perceived inverse demand curve thus displays a kink at $(\bar{p}_i, \bar{y}_i)$.

Now let labour L_i, be the only variable input and the production function be written $y_i = F_i(L_i)$, the firmsmaximand may be written as (5.2.4).

$$\pi i = p_i(y_i, \bar{p}_i, \bar{y}_i)\ F_i(L_i) - wL_i \qquad (5.2.4)$$

Thus when the short-run expectations $y_i = \bar{y}_i$ and $p_i = \bar{p}_i$ are realised the first order conditions yield.

$$\left.\begin{aligned} &p_i(1-\varepsilon_i)\ F_i'(L_i) \leq w \\ &p_i\ F_i'(L_i) \geq w \end{aligned}\right) \qquad (5.2.5)$$

Implying

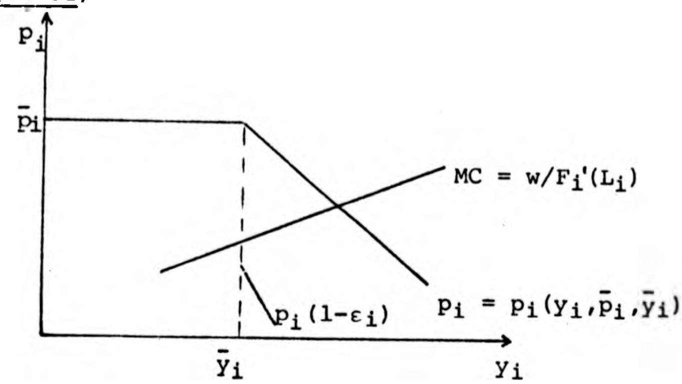
$$p_i \geq \frac{w}{F_i'(L_i)} > p_i(1 - \varepsilon_i) \qquad (5.2.6)$$

where ε_i is the R.H.S. elasticity of the inverse demand curve with respect to y_i.

(5.2.6) states that the first order conditions are satisfied when marginal cost lies between the price and marginal revenue, and has the implication that excess capacity exists and there is no incentive for the producer to adjust prices or quantity. Indeed small changes in effective demand will affect only output and will not disturb prices. Diagramatically

Negishi's analysis of the firms output/price decision is as figure (5.2.2)

Fig.(5.2.2)



It is argued that since consumers are upon their demand curves and facing no quantity rationing on the goods market at the prevailing market price $\bar{p}_i$, they too will have no incentive to attempt to adjust prices. This outcome Neglishi calls a 'Keynesian conjectural Equilibrium' for the firm.

In the treatment of the labour market it is argued that workers are either wholly unemployed receiving no income, or fully employed for an institutionally determined working week at the current wage rate $\bar{w}$. Homogeneous workers perceive probabalistic inverse labour demand curves as (5.2.7).

$$w = g(k,\bar{w},\bar{k}) \qquad (5.2.7)$$

where $0 \leq k \leq 1$ is the ratio of employed workers to total workers interpreted as the probability of employment.

The perceived labour demand curve has the following properties.

(1) $\bar{w} = g(\bar{k},\bar{w},\bar{k})$ The perceived inverse demand curve goes through the currently observed wage, employment probability pair $(\bar{w},\bar{k})$.

(2) $w = \bar{w} \ \forall \ k \leq \bar{k}$ $\quad$ $\partial w/\partial k < 0 \ \forall \ k > \bar{k}$ workers face a horizontal probabalistic inverse demand curve for all employment levels yielding a probability below $\bar{k}$, but face a finitely elastic demand curve for all probabilities above $\bar{k}$.

It is argued that workers perceive a finitely elastic demand curve to the right of $\bar{k}$ since they believe offering to work at an infinitesimally smaller wage to $\bar{w}$ will not guarantee them employment. This is so since firms are assumed not prepared to replace one worker by another at a marginally lower wage as this will effect moral and the productivity of other workers. Thus the perceived inverse demand curve displays a kink at $(\bar{w},\bar{k})$.

Subject to the perceived inverse labour demand curve, workers maximize the following utility function[4] (5.2.8).

$$ku(w,p) + (1-k)\bar{u} \qquad (5.2.8)$$

where $\bar{u}$ is the utility of being unemployed.

The first order conditions maximizing (5.2.8) subject to (5.2.7) by choice of k are

$$\left.\begin{aligned} &u(w,p)-\bar{u} \geq 0 \\ &u(w,p)-\bar{u}-\varepsilon' w(\partial u(w,p)/\partial w) \leq 0 \end{aligned}\right) \qquad (5.2.9)$$

where ε' is the R.H.S. elasticity of (5.2.7) with respect to k evaluated at $k=\bar{k}$.

The first condition in (5.2.9) states that the utility value of the wage should exceed the utility of leisure foregone and defines a minimum wage. The second condition defines the wage at which any marginal increase will set the probability of employment to zero, and thus defines the maximum wage.

If the conditions in (5.2.9) are satisfied workers will not reduce (or accept a reduction in) wages since the implied utility loss is not compensated by the expected utility gain arising from the increase in k. Workers are in a Keynesian conjectural equilibrium.

As in the case of the firm, the second inequality constraint in (5.2.8), if not strictly binding, implies changes in effective demand will be absorbed completely by changes in employment with no effect upon wages. It is also interesting to note that changes in the price level p will have no effect upon wages unless it causes one of the inequality constraints in (5.2.9) to be violated, but any change in the nominal wage that results is as Negishi remarks not merely due to the existence of unemployment.

As an explanation of price rigidities Negishi's arguments, especially the analysis of the firms perceived goods demand curve, are very persuasive. The price adjustment (non-adjustment) mechanism is consistant with the desirable properties (a), (b), (c) and (e) and may easily be made consistant with (d) by assuming that there are costs involved in disseminating information about price cuts. This would explain why the firm cutting its price cannot capture the whole market. The analysis appears a good candidate for a price adjustment mechanism, and may continue to be a fruitful line of inquiry. However there are some problems with the analysis; the initial price quantity pair on each market is historically given and not endogenous to the model, and the actual level of prices is arbitrary, being required only to satisfy the first order conditions (5.2.5) and (5.2.8). It is also assumed that in the Keynesian demand deficient equilibrium the model describes, demanders of goods do not perceive the possibility of obtaining the same quantities at lower prices, they are described as on their demand curves and are thus argued to have no incentive to try to adjust prices. This seems unreasonable, especially in the case of the labour market when high unemployment exists, firms will worry less about the productivity effects of employing lower wage workers when the unproductive can be easily and inexpensively replaced. This may not be an overriding criticism as the inclusion of hiring and training costs in the analysis would reduce the likelihood of firms turning over their labour force in periods of high unemployment.

In terms of providing a price endogenous microeconomic basis for macroeconomics, this analysis appears promising. It may well be compatible with the fix price approaches of Malinvaud (1977) Muellbauer and Portes (1978) and indeed with the suggestions made in chapter 2 and 3 of this thesis. The arbitrariness of prices presents no problem to the policy maker since initial prices are data in his calculations, however the persistence of inflation appears to be an inexplicable phenomenon within Negishi's current framework, but may perhaps be handled by introducing inflationary expectations into perceived demand curves.

Heal (1982) has also suggested how discontinuities may give rise to a price mechanism which displays several desirable properties. His approach is to hypothesise that over some range the production function of an economy may display increasing returns, giving a non-convex production set.

This implies that there exists a vector of relative prices at which the economy's goods supply correspondence is not single valued and the labour demand function is discontinuous. His arguments are technically complex and may be most easily described diagramatically as in figures (5.2.3) and (5.2.4).

Figure (5.2.3)

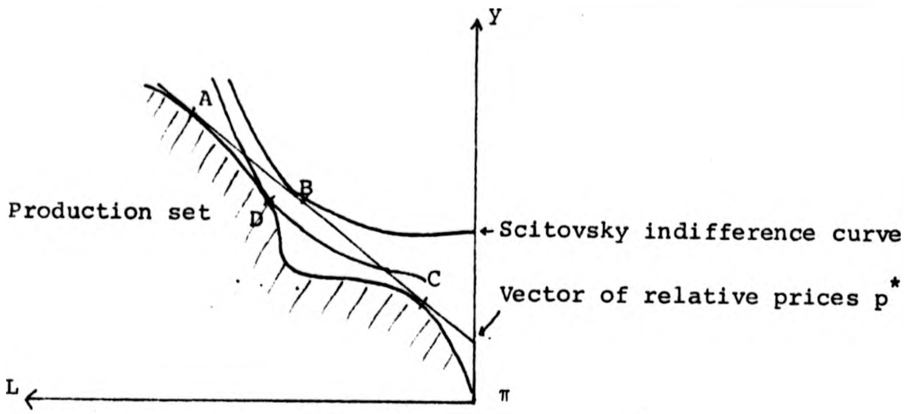
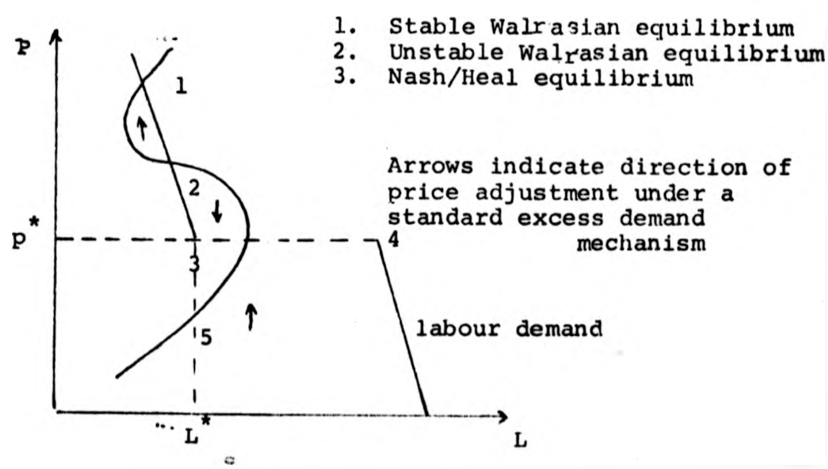


Figure (5.2.4)



In figure (5.2.3) the production set is drawn with increasing returns over a range of inputs. Utilising a Scitovsky indifference curve it is demonstrated that the first best solution lies at point D, however a vector of relative prices which decentralises this equilibrium cannot be found since they would yield negative profits. At prices p^* there are three candidates for an equilibrium, A, B and C. Point B defined by MRS = p^* lies outside the production set and is not feasible, the equilibrium has to be established at A, C or at a different vector of relative prices. Intuitively it can be seen that equilibrium will be established at point C, if relative prices are p^*, since given no forced trading A is dominated by B for consumers which is itself dominated by C for producers. Figure (5.2.4) demonstrates how the relative price vector p^* is established. The labour supply curve of consumers passes through the discontinuity in the labour demand curve ($A \equiv 4$, $C \equiv 3$). Since supply exceeds demand for a range of prices above p^* and is less than demand below p^* a notional excess demand formulation (or indeed an effective excess demand formulation using Benassy-Clower demands) will cause prices to return to the level of the discontinuity. Heal demonstrates that this implies a Nash equilibrium will be established at point $3 \equiv C$. This may be described as Keynesian since at the prevailing relative wage involuntary unemployment exists. It is also shown in the paper that the consumption demand of households will be consistent with firms output decisions and that the labour supply curve does in fact pass through the discontinuity. The labour supply curve has been drawn with a backward bending section to demonstrate that, since the economy must possess an odd number of equilibria, a stable Walrasian equilibrium occurs at 1 and an unstable one at 2.

This analysis was presented at the AUTE conference in March

1982 and not all the work is in its final form. Taken in this light any comments or criticisms should be considered tentative, and apologies are made to Heal if his ideas have been misconstrued.

Initial inspection of the analysis would suggest that its reliance upon an excess demand function to generate price adjustment is a weakness. However this may perhaps be rationalised as follows. Consider the behaviour of an individual unemployed worker who wishes to gain employment with a particular firm, it may appear that he can gain employment by offering to work at slightly below the going wage p^*. The firm would be willing to employ him as this represents an increase in its profits, he simply replaces another worker. This argument will be true for all firms and workers, so why is full employment not achieved at a wage determined by the labour supply curve, point 5 on figure (5.2.4)? The answer is that in the aggregate this is a false argument, if wages fall with no change in unemployment (voluntary has replaced involuntary) aggregate demand for output must fall and firms will choose to reduce nominal prices. Relative prices p^* will not be disturbed and the equilibrium will be unaltered. If firms and workers perceive that such a process will occur no attempt will be made to shade prices.[5] Neither individual firms nor workers will have any desire to change the relative price of labour.

Such an analysis would suggest that prices would tend to be fixed for goods whose production functions display increasing returns over some range and flexible for those goods whose production functions exhibit diminishing returns.[6]

The problem appears to be in explaining why prices should ever change in such a situation. The adjustment mechanism does not display our desirable property (e) indeed in a macroeconomic context it is difficult to see how this approach may be utilised especially if an examination of inflation is required.

(iii) <u>Conjectural and Expectations Approaches</u>

Many analyses which fall (loosely) within this class have similarities to the work of Negishi discussed in section (ii), it is debatable whether a distinction between the two is valid. However in the Negishi kinked demand curve approach agents are required to perceive a kink at the level of their current realisations, here conjectures or expectations may include kinks but it is the self-fulfilling nature of perceptions which is important.

Some of the most advanced work within the area is due to Hahn (1977, 1978). The analysis describes what is called an infinitesimally rational conjectural equilibrium (IRCE), and provides a method of price endogenisation within a Drèze type framework. It is assumed that agents perceive that they cannot trade in excess of a current constraint unless they adjust the prices at which they are prepared to trade. The argument goes as follows. Given a signal about current prices and quantity constraints

$$S_a = (p, \bar{z}^a, \underline{z}^a) \qquad (5.2.10)$$

agent a calculates his potential trade revisions on any market b as (5.2.11)

$$\left.\begin{aligned} \tilde{x}^a_b &= \max\,(0,\ \tilde{z}^a_b - \bar{z}^a_b) \\ \underset{\sim}{x}^a_b &= \min\,(0,\ \tilde{z}^a_b + \underline{z}^a_b) \end{aligned}\right\} \qquad (5.2.11)$$

The first part of (5.2.11) describes potential increases in transactions and the second decreases. ($\underline{Z}_b \in \mathbb{R}_+$).

The prices at which the agent conjectures that these revisions may be achieved define his conjecture function (5.2.12).

$$C^a(p, \tilde{x}^a, \underset{\sim}{x}^a) \tag{5.2.12}$$

C^a is increasing in both its second and third arguments.

Agents then maximize utility by choosing both price and quantity offered subject to the conjecture function (5.2.12). An IRCE is then a vector of signals $S = (p, \bar{Z}, \underline{Z})$ such that $\tilde{Z}^a$ trade offers, $\tilde{c}^a$ price offers, have the properties that

(1) $\sum_a \tilde{Z}^a = 0$ all markets clear

(2) $\tilde{c}^a = p \ \forall_a$ price realisations confirm conjectures

Hahn demonstrates that such an equilibrium exists, and that it may be consistent with both cases of one or neither side of a market being rationed, i.e. it admits the possibility of both Walrasian and non-Walrasian equilibria. A non-Walrasian equilibrium may exist because the trading possibilities which agents perceive depend upon the rates of change of other agents conjectural utility or profit functions, these second order terms may be incorrect even though the first order (slope or elasticity) terms are correct. For example if each agent perceives a linear conjecture tangential to the 'actual' function at the equilibrium price quantity realisation conjectures will be proven correct but the true trading possibilities will not be revealed.

As an explanation of price adjustment and determination Hahn's approach has a great deal to recommend it. It clearly has our desirable properties (a) and (c) in that price and quantity adjustment decisions are made simultaneously as part of individual agents maximization problems, and can explain both cases of fixed and flexible prices depending upon the shape of agents conjecture functions. Also the approach implicitly recognises that there is an opportunity cost to revising a trade in that the acquisition of an extra unit of 'stuff' requires that a different price must be paid for all units. The introduction of transactions costs into the structure would not appear to be too difficult an exercise as these could easily be included as one component in the conjectured price adjustment required to break a constraint.

The approach is not explicitly sequential and does not have our desirable property (b). This is in the nature of a Drèze type formulation where all constraints are known prior to trading, consequently the conjectures of prices associated with different levels of trade upon market i are based upon information about other markets which it is not reasonable for agents to have and $s = (p, \bar{Z}, \underline{Z})$ contains all prices and constraints. Interestingly the approach accords well with our desirable property (c), price and quantity adjustments are made upon the basis of verifiable information in that conjectures are correct in their first order terms and the information about the true shape of agents profit or utility functions is not verifiable since it is not transmitted.

Some problems do arise, firstly, why do agents upon the short side of a market, when they observe that others are rationed, not exploit their position by raising prices.[7] Secondly, what exactly determines an agents conjectural function, to assume it is given is just replacing exogenous prices by exogenous conjectures. Clearly conjectures should be related to past trading experiences and any information that may be gleaned from others about their potential behaviour. This has not yet been investigated.

To my knowledge there have been no successful macroeconomic analysis developed upon the basis of an IRCE, which is probably because the analysis, for all its nice properties, suffers from one crucial flaw. The fact that it does not explicitly model the sequential nature of trading. This would seem to be an essential feature of any microeconomic explanation of price and quantity adjustment that can adequately provide a basis for macroeconomics.

Expectations approaches, although embodying many of the properties of the conjectural approach, are an improvement in the vital respect that they assume trades are uncoupled over time. Drazen (1980) presents a model with endogenous flexible prices which displays quantity constraints. The analysis has the interesting features that quantity constraints may arise even when sellers are aware of their trading possibilities, and that demand fluctuations may result largely in quantity rather than price responses. Trading is sequential and takes the following natural form. The labour market opens first - in the morning and labour is purchased by firms and production takes place, then - in the afternoon - the goods market opens and firms sell to workers some proportion of their stocks. If supplies and demands do not match either quantities must adjust or agents upon the long side of the market must change prices. It is assumed that individual firms and workers perceive finitely sloped demand curves for their output and labour respectively. The arguments behind these assumptions may be examined by giving a brief sketch of the model.

Firms perceive that the demand curve they will face for their output takes the following form (5.2.13)

$$x(p) = \mu(p)\eta(p) \qquad (5.2.13)$$

where

$\eta(p)$ is the market demand curve

$\mu(p)$ is an arrival function

Since $\eta(p)$ has finite slope, $x(p)$ will also have finite slope if $\mu'(p)$ is not too large, so a firm cannot capture all of the market by a price cut or will not lose all its trade by a price increase. Drazen's discussion of $\mu'(p)$ is not rigorous however the following arguments may be put forward. Suppose workers/consumers can obtain little or no information about the prices of goods before visiting the market, all they know is the price that they paid upon the last visit, further let there be a utility cost to visiting a market.[8] Workers will select at random the trading post that they visit to obtain their supply, and on discovering that the price now differs from their previous trade in that good they have to decide whether to engage in some form of search activity which is costly or accept the new price; some simple marginal condition will determine their behaviour. Alternatively it could be argued that price information discriminates slowly amongst purchasers, and they accept any price quoted which is consistent with the state of their information. Drazen hints at both these types of rationalization for the $\mu(p)$ function but does not make the argument explicit.

Workers perceive that they face a finitely sloped labour demand curves of the form (5.2.14)

$$L(w) = k(w)\Lambda(w) \qquad (5.2.14)$$

where $k(w)$ is the proportion of employed workers in the total labour force

$\Lambda(w)$ is the market demand curve for labour.

$k(w)$ is interpreted by (homogeneous) workers as the probability of being employed at w. L(w) is assumed to have finite slope since a small reduction in w will not raise k(w) to one since workers know that a small wage cut will not induce firms to displace one worker by another since this may have a bad effect upon morale and the productivity of the remaining labour force. Alternatively there may be training costs associated with switching workers. Again the paper could be strengthened by making these arguments explicit.

Based upon their expected goods demand curves x(p) firms announce a labour demand $\lambda(w)$ which maximizes expected profit. Workers conjecture a consumption function c(p) and maximize their utility by choice of an optimum L^* which implies a w^*. Firms then produce according to a standard neoclassical production function (5.2.15)

$$y = f(L^*) \qquad (5.2.15)$$

(the output y may be used for sales or inventory stock). Workers receive income w^*L^*.

The goods market then opens, and given expected future variables, a demand determined equilibrium obtains if

$$\eta(p) = c(p, \lambda(w^*, \eta(p)), w^*) + Z/p \qquad (5.2.16)$$

where Z/p is constant profit income.

I.e. an equilibrium obtains if firms expectations are correct. If (5.2.16) does not obtain firms observe changes in their inventory stocks and revise their expected demand curves x(p) in the next round.

Drazen also argues that changes in demand tend to induce quantity rather than price responses, because the price derivatives of both goods and labour demand curves increase (decrease) as the level of demand increases (decreases). Roughly this is for the following reasons. Consumers utility functions are assumed to display decreasing relative risk aversion. A reduction in labour demand, a cut in income, implies that any increase in current consumption means that future consumption streams become more risky, at lower levels of income a greater price cut - risk premium - is required to induce this change in consumer behaviour. The goods demand curve shifts in and becomes steeper. Firms facing a fall in product demand can respond with inventory accumulation and by reducing their labour demand. It is shown that if the value of inventory is concave in inventories, then a worsening of sales prospects will require a larger wage reduction to induce any given employment increase. The labour demand curve moves inwards and becomes steeper. Quantities will tend to adjust rather than prices due to these slope changes in the labour and goods demand curves.

As a price adjustment mechanism this approach has several of our desirable properties, most importantly, (b), that price and quantity adjustments are explicitly related to the non-synchronized nature of trading. The main feature of the non-Walrasian equilibrium is that it is based upon the fulfilment of product market expectations. These are of course self fulfilling. The mechanism admits as extreme cases the possibility of both fixed and perfectly flexible prices, in the sense that a Walrasian equilibrium may be achieved and would satisfy (5.2.16). How much prices respond to changes in demand depends upon inventory holding costs and households attitude to risk. For example if households are highly risk averse at low levels of income price cuts will have a negligible effect upon goods demand, firms will choose over the relevant parameter range to fix the price of output. At high income levels firms will choose to adjust prices due to the high elasticity of demand for goods.

As a price adjustment/determination mechanism Drazen's analysis stands or falls upon the question of why demand curves are perceived as having finite slope and why the slope should increase as demand curves contract. The arguments that explain why demand curves have finite slope seem reasonably persuasive, there are clearly costs-associated with obtaining information when the system is out of equilibrium. It is not unreasonable to assume that workers engage in only a limited amount of search activity when looking for employment. It is also reasonable to assume that in the face of training, hiring and firing costs that firms will not be prepared to replace one worker by another unless a significant wage cut is offered. A non-Walrasian equilibrium based upon these arguments seem credible.

The arguments about household risk aversion and firms inventory carrying costs are also persuasive; it would be interesting to know over what sort of parameter ranges the impact of demand shifts will fall mainly upon prices and when they fall mainly upon quantities. The importance of these factors would depend upon those circumstances in which the theory predicts mainly prices or quantities adjust.

The approach suffers from some of the same shortcomings as the conjectural approach. Agents upon the short side of a market do not make use of the observations that quantity constraints exist to raise the prices for the goods they supply. This seems a particularly pertinent problem in an extremely demand deficient equilibrium where the demand curves would by Drazen's arguments be very steep. Would it not be in the firms interests to cut wages? Strangely, we might ask why don't firms raise prices in a depression, because if demand curves are highly price inelastic profits will rise since the sales constraint a firm faces will only tighten by a negligible amount.[9]

The macroeconomic implications of this analysis suggest that it may be a useful paradigm. The model has an unemployment equilibrium but also provides an explanation of price adjustment. In the simple form presented above this may not be regarded as an adequate explanation of inflation, however, if $\Lambda(w)$ the labour demand curve, $x(p)$ the expected goods demand curve and $c(p)$ the planned consumption function are based upon a common expectation of a price change then a non-Walrasian equilibrium of the Drazen sort may be consistant with inflation. Fiscal policy could be introduced in a meaningfully simple way, government expenditure would raise the price of output and reduce inventory stocks, causing firms to revise outwards their labour demand curves, with a subsequent increase in wages and employment. However it would be necessary to add asset markets to the model before any realistic policy conclusion could be drawn.

Several models conceptually similar to Drazen's treatment exist. Grandmont and Laroque (1976) examine a model based upon Drèze effective demands, in which price setting firms base output decisions upon expected product demand curves.

If firms have pessimistic expectation, unemployment arises in a manner similar to that discussed above. Varian (1977), in a paper discussed in chapter 2 section 1, also presents a similar argument in which firms form point expectations about output demand, and an equilibrium occurs when nominal wages and prices are static and expectations validated. Varian assumes that labour market transactions and production precede the product market transactions. Nominal prices and wages adjust according to effective excess demand functions, based upon Benassy type demands. The model does not examine the inventory stocks which accrue to producers when disequilibrium occurs due to deficient product demand, yet it states that firms are price setters. The analysis also does not explain who sets wages and how. Crucially Varian's price adjustment mechanism does not possess our desirable property (a) as price adjustments do not appear as part of any agents choice calculus.

On the whole the expectations approach seems to be a potentially fruitful line of research which may lead to a good microeconomic explanation of simultaneous price and quantity determination. It may not provide a complete microfoundation for macroeconomics analysis but it is perhaps part of the story. In non-contingent competitive markets this could be a good explanation of price adjustments, given that the question of how agents upon the short-side of markets affect prices is resolved.

(iv) Implicit Contracts

Deriving from the initial contributions of Baily (1974) and Azariades (1975) the implicit contract literature provides a rationale for endogenous price determination which has some desirable properties. The starting point for implicit contract models is the recognition that some form of 'locking-in' effect typically characterises the relationship between firms and workers. Workers are costly to train and learn firm specific skills.[10] This locking-in is known to both parties, who take it into consideration by bargaining about both current and future wages and employment at the beginning of their relationship.

The earlier contributions in this literature assume that risk neutral firms offer insurance in the form of a fixed wage rate to risk averse workers. In return workers accept a level of the fixed wage below the expected value of a state contingent wage. Firms carry all the risk that arises from uncertain demand and charge workers a risk premium of the difference between the fixed and expected wage rates. A brief sketch of Azariades model illustrates the argument.[11]

Let S be the set of states of the world where $s \in S$ is an element and p(s) is the mapping of states into a given firms prices. By offering a uniform contract to workers $\delta = \{w(s), L(s)\}$ the firm maximizes expected profit $\pi(\delta)$. The firm is indifferent between contracts which yield the same expected profit.

Workers are assumed to supply one unit of labour inelastically. Their utility when employed and unemployed will be $u[w(s)]$ and K respectively. The probability of a worker gaining employment if he enters a firms labour pool will depend upon L(s), the employment rule, and m, the number of other workers who enter the pool. We may define m as (5.2.17)

$$m = m(\lambda, (w(s), L(s))) \qquad (5.2.17)$$

where λ is the value of market alternatives to the workers.

Hence the probability of employment in any state s may be written as (5.2.18)

$$\xi(s) = \min\left[1, L(s)/m\right] \qquad (5.2.18)$$

The value to a worker of a given contract δ may thus be as (5.2.19)

$$V(\delta_1,m) = E_s\left[\xi u(w) + (1-\xi)k\right] \qquad (5.2.19)$$

clearly a contract will only be feasible if $V(\delta_1,m) = \lambda$ for some $m>0$ s.t $L(s) \leq m \quad \forall s$.

From this basic structure Azariades proves a very interesting result. Given any feasible variable wage contract δ_1 there exists a fixed wage contract δ_2 which dominates it in the sense that $V(\delta_1,m) = V(\delta_2,m)$ and $\pi(\delta_1) \leq \pi(\delta_2)$ where $\delta_1 = \{w(s), L(s)\}$ $\delta_2 = \{\hat{w},L(s)\}$, $\hat{w}$ is state invariant.

The proof is illustrative of the basic argument and goes as follows:

Let m satisfy $V(\delta_1,m) = \lambda$ (feasibility) and define $\delta_3 = \{\hat{w}-\varepsilon,L(s)\}$ where $\hat{w} = E(wL)/EL$. The contract δ_3 produces the same expected labour income for workers and expected profits for firms as does δ_1. Now

$$\Delta \equiv V(\delta_3,m) - V(\delta_1,m) = \frac{1}{m} EL\left[u(\hat{w}) - u(w)\right]$$

$$\geq EL(\hat{w}-w)u'(\hat{w}) \text{ by concavity of } u(\cdot) .$$

$$= u'(\hat{w})(\hat{w}EL - EwL) = 0$$

Thus $V(\delta_3,m) > V(\delta_1,m) = \lambda$, Hence $\exists\, \varepsilon>0$, s.t $\delta_2 = \{\hat{w}-\varepsilon ,L(s)\}$ satisfies $V(\delta_2,m) = V(\delta_1,m)$ and $\pi(\delta_2) \geq \pi(\delta_1)$ $\Box$

Workers are prepared to pay the risk premium ε to obtain insurance in terms of a fixed wage contract from the firm.

The fact that the theory provides a rationale for fixed wage rates does not imply that this analysis can be simply integrated with, for example, Dixit's (1976) model to provide an endogenous wage quantity constrained model. The implicit contract also specifies an employment rule $L(s)$. If the approach is to provide an explanation of an involuntary unemployment equilibrium the optimal contract must be such that $L(s) < m$ for some s. Azariades and Bailey both argued that the full employment contract would not be optimal, however Negishi (1979), Varian (1976) and Akerlof and Miyazaki (1980) have argued that the full employment contract is in fact optimal and may imply that in some states employment is <u>above</u> that which would be achieved in a spot market model.

Clearly an approach which gives price rigidities but also implies full employment is not of great interest. However more recent work, especially Grossman and Hart (1981a, 1981b) appears promising. Three basic modifications are made to the Baily-Azariades approach. Firstly it is assumed that both workers and firms are risk averse, secondly that firms may also pay a wage to laid-off workers, finally and most importantly it is assumed that only firms can observe the true state of the world s. There is asymmetric information since only firms know the marginal revenue product of workers.

Once both firms and workers are risk averse the optimal contract must specify the optimal degree of risk sharing between a worker and a firm. If the state of the world s was public information then an optimal contract would state that a worker should be employed in all states where his reservation wage R is exceeded by his marginal revenue product, and the wage paid in employment states $W_e(s)$ will give the optimal degree of risk sharing. The model will not explain either unemployment when $s \geq R$ (s is now taken to be the marginal revenue product of the worker), and cannot explain price rigidities since $W_e(s)$ has to be variable for optimal division of risk. However such a contract will be unenforcable if workers cannot observe the state s, since the firms will have an incentive to choose and announce that s_1 rather than s_2 has occurred, where $W_e(s_1) < W_e(s_2)$, given that both are employment states.

Moral hazard arising from asymmetric information means contracts cannot be made dependent upon s. Following some proposals by Hall and Lilien (1979), Grossman and Hart (1981a) consider the sort of contract that will be struck if the ex post level of employment is public knowledge. Hence the wage components of contracts may not be directly written to be contingent upon s, but they may be contingent upon employment. To understand how such an arrangement may yield fixed wages and unemployment in states in which the marginal revenue product of labour exceeds a given reservation wage R, consider the following argument due to Hart and Grossman. The bargain that a firm can offer to an individual worker is that he will receive wage W_e if he works for the institutionally determined period, and W_u if unemployed. Now let $h \equiv W_e - W_u$. There is no problem in ensuring that the firm achieves an optimal level of employment ex post since it will always employ the worker if $\bar{s} \geq h$ [where

$\tilde{s}$ is the actual realisation] and whatever the hiring rule it will announce an s which yields employment. An optimal contract, when only the firm knows $\tilde{s}$, involves choosing h and W_u to maximize (5.2.20)

$$\int_h^{\bar{s}} V(s-h-W_u)\,dG(s)+\int_{s_o}^{h} V(-W_u)\,dG(s) \qquad (5.2.20)$$

subject to (5.2.21)

$$\int_h^{\bar{s}} u(h+W_u-R)\,dG(s) + \int_{s_o}^{h} u(W_u)\,dG(s) \geq \bar{u} \qquad (5.2.21)$$

where

G(s) is the probability distribution over states with s_o and $\bar{s}$ being the 'worst' and 'best' states, lower and upper bounds on G(s). $\bar{u}$ is the utility the worker can achieve if he does not make a contract with the firm.

Employment will occur if $\tilde{s} \geq h$, hence if the optimal $h>R$ unemployment will occur in states where full employment would obtain on a neoclassical spot market. The basic idea can be illustrated as follows; let $h=R>s_o$, net income is thus constant in employment and unemployment states. The worker is indifferent between working and not working and thus is <u>locally</u> risk neutral at h=R. Expected net income and profit, from (5.2.20) and (5.2.21), may be added together to yield (5.2.22).

$$E\pi + EI = \int_h^{\bar{s}} (s-R)\,dG(s) \qquad (5.2.22)$$

(5.2.22) is maximized at h=R. The firm is bearing all the risk and an appropriate increase in risk sharing will increase utility. Consider the following; let h increase slightly with an appropriate fall in W_u such that $E\pi$ does not change. Since $s-(u+W_u) > -W_u$ in employment states, the change represents a transfer of the firms income from employment to unemployment states decreasing the riskiness of the expected profit stream. Since the firm is assumed risk averse it will accept a small cut in $E\pi$ in return for the reduction in risk. The workers EI rises and since he is locally risk neutral at $h=R$ he will also be made better off. Thus $h>R$ is Pareto superior to $h=R$. Because $h>R$ at the optimal contract and $\tilde{s}\geq h$ is required for employment then $\tilde{s}\geq h>R$ implies that unemployment of the worker may occur when the realisation of the workers marginal revenue product $\tilde{s}$ exceeds his reservation wage.[12] Unemployment will be involuntary in this sense.

If it is to be argued that firms will face costs if they pay identical workers different wages, the theory implies that a uniform wage will be paid to all workers in employment states. If the unemployment/lay-off wage is zero the contract implies selecting a $W_e=h$ to be paid in employment states.

As an explanation of wage rate determination between an individual firm and a group of homogeneous workers, implicit contracts struck under asymmetric information have several desirable features. Contracts are the consequence of maximizing behaviour by firms and workers. A contract which defines employment and wages as contingent upon the state of the world or employment as state contingent and remuneration linked to employment, has the essential characteristic that rules about prices and quantities are determined simultaneously.

The approach however suffers from several shortcomings.[13] No explanation of what the wage rate will actually be is presented, neither is there any consideration of how the optimal contract will change when the aggregate environment changes. It is also assumed that a perfect auction market exists for the output of the firm, explanation of price setting behaviour is also required here. The question of why firms obtain labour pools of a given size is not explained yet the theory relies upon some underlying argument about labour immobility as a rationale for the implicit contract. If workers are tied in to a firm permanently or $\bar{u}$ is very low why do not firms shift all risk to workers by offering the following contract $W_u=0$, $W_e=R=H$, $L=1$ if $s\geq R$, $L=0$ otherwise. This of course looks very familiar.

Further the way uncertainty is introduced into the Grossman and Hart analysis is doubtful. They assume that uncertainty is due to some random component in the production technology. The marginal physical product of workers varies over states. To suggest that only firms can observe changes in the MPP is somewhat peculiar. Workers on a production line know how fast the line is moving or how frequently it stops, they should be able to make a reasonable inference about the state of the world. Alternatively it could be argued that the demand curve firms face has some random component, which would seem to make it more reasonable to assume only firms observe the realisation $\tilde{s}$. An interesting question to ask might be how firms gain information upon the realisation $\tilde{s}$. If a wage is paid to all employed workers and output precedes the revelation of the true state through product market transactions how is employment chosen ex ante? The most immediately obvious answer is that firms observe not randomness in prices, but variations in quantities at given prices, experienced through inventory stock changes. Employment will be a consequence of the firms inventory position. Introducing inventories explicitly into an implicit contract model would allow the firm two channels for handling the random component in demand, state contingent contracts and the inventory holding decisions. It is thus suggested that the possibility of absorbing random demand shocks by inventory holding policies may make a risk averse firm, risk neutral in its attitude to the contracts it offers its workers. This may be an interesting line of investigation.[14]

It would perhaps be premature to judge the importance of implicit contracts as a wage adjustment and determination mechanism. Since the micro-models cannot be generalised by treating a worker and producer as representative agents, as is often done when considering potential microfoundations, few comments about the sort of macroeconomic analysis that will result can be made.

(v) <u>Explicit Contracts: Union Employer Bargaining</u>

Standard IS/LM macroeconomic models can explain the existence of an unemployment equilibrium which has the usual Keynesian features when the wage rate is rigid. Dixit (1976) has also shown that a 'Keynesian Temporary Equilibrium' arises under wage rigidity. The most straightforward way of explaining why prices do not adjust to clear the labour market is to use the observation that trade unions exist and play a role in the determination of wages. Fixed or imperfectly adjusting wage rates may then be explained either as a consequence of the institutional organisation of the labour market or as a result of optimising behaviour on the part of the union.

I.e. Wage and employment bargains may be struck periodically and must be adhered to for some institutionally given time period or they are continuously revised but do not give sufficient price adjustment for the labour market to clear.

The questions of why unions exist, what motivates their leaders, and under what conditions particular workers choose to join the union will not be examined. It will simply be assumed that unions are interested in maximizing some form of utility function the arguments in which are the wage rate and level of employment.

The problems of wage rate and employment determination on a unionised labour market is essentially a problem of bilateral monopoly. A monopolistic union facing a monopsonistic firm. Two basic approaches exist, demand determined solutions, and efficient solutions.

Demand determined solutions seek to select a point upon the firms labour demand curve. The easiest and one of the simplest solutions is due to Dunlop (1944) he assumes that the union maximizes the total wage bill (5.2.23)

$$\max_{w} U^u = wL(w) \tag{5.2.23}$$

hence
$$\frac{du^u}{dw} = L(w) + wL'(w) = 0 \tag{5.2.24}$$

which may be written

$$L(w)\ (1-E_{Lw}) = 0 \tag{5.2.25}$$

where $L(w)$ is the firms labour demand curve, $E_{Lw} = -wL'(w)/L(w)$ is the elasticity of demand for labour with respect to the wage rate.

The problem with such an approach is that it involves fixing employment at a level where marginal revenue is zero, with a result that the wage rate that would be achieved in the absence of the union is greater than the unionised wage if $E_{Lw}>1$. Figure (5.2.5) demonstrates this.

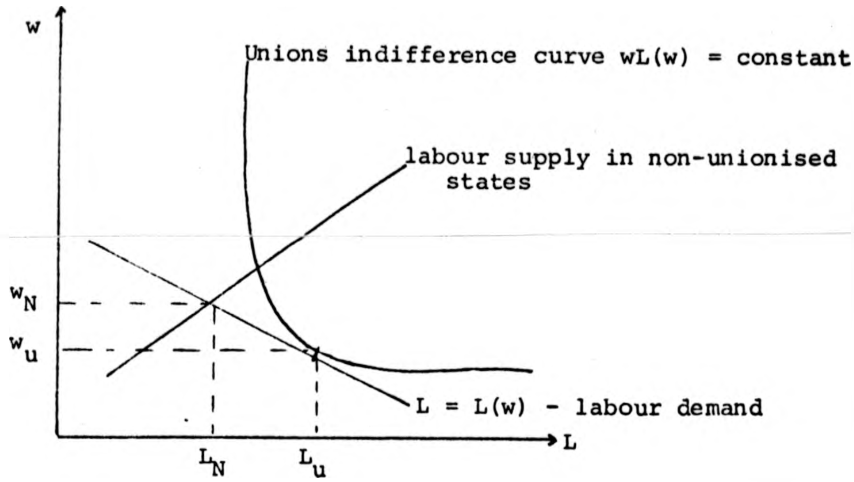
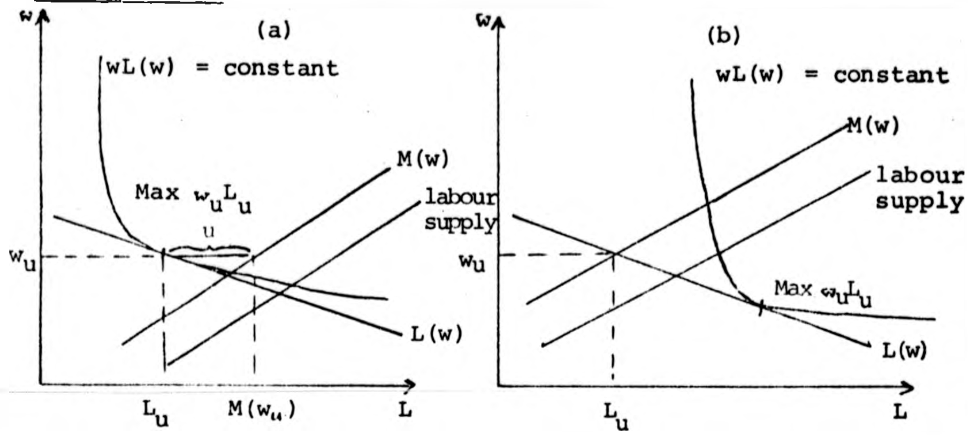


Figure (5.2.5)

In figure (5.2.5) $W_u L_u > w_N L_N$ but $w_N > w_u$: The wage bill is maximized at $w_u L_u$ the unions choice point on the firms labour demand curve, but the actual wage rate chosen lies below the non-unionised wage w_N.

Dunlop, aware of this problem, constrains the unions maximization problem by introducing a union membership function M(w) where M'(w)>0. This membership function is assumed to lie above the non-unionised labour supply curve for all wage rates. This then admits two possibilities as described by figure (5.2.6)(a) and (b).

Figure (5.2.6)

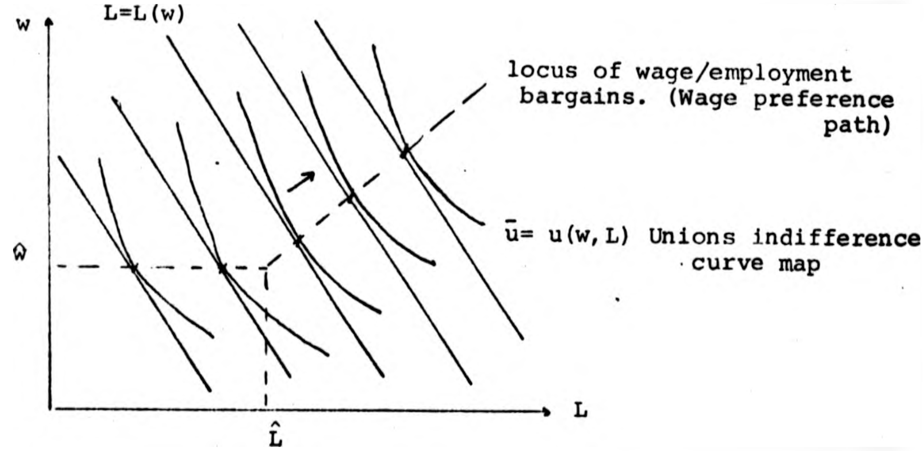


In (a) the wage bill maximizing pair (w_u, L_u) on the labour demand curve selects a lower level of employment than the supply of labour, $M(w_u)$ given by the unions membership function. Unemployment $M(w_u)-L_u$ occurs.

In (b) there is insufficient labour supply at the wage bill maximization point max $w_u L_u$ on L(w), and the wage is bid up to the intersection between the membership function of the union and the firms labour demand curve. Dunlop's approach is based upon the assumption that each worker has an identical opportunity of employment and that each has identical weight in the formulation of union policy. This simple representation of the bargaining problem suffers from a number of serious problems. Most importantly it does not explain how the membership function arises, which aspects of individual agents choice calculus cause them to join. It also does not explain why the particular maximand wL(w), wage bill maximization, is adopted. To generate this result from individual workers decisions requires: (i) That they are income maximizers (ii) that they are risk neutral. These are strong assumptions to make in general.

Using a more general form of the unions utility function Cartter (1959) argues that shifts in the labour demand curve, brought about by changes in the effective demand for the firms output, will cause both wages and employment to rise when demand rises, but only employment to fall when demand is reduced. The unions indifference map is such that it resists wage reductions. This is illustrated in figure (5.2.7):

Figure (5.2.7)



Cartter argues that given the initial wage employment bargain $(\hat{w},\hat{L})$ increases in demand, represented by outward shifts in the firms labour demand curve in figure (5.2.7), give rise to increases in both the wage rate and the level of employment, but for demand decreases only the level of employment changes. This implies that the indifference map of the union changes shape for all utility levels below the one associated with $(\hat{L},\hat{w})$. This explanation of what is supposed to be a stylised fact, unions resist wage reductions despite the employment consequences of such behaviour, is self-evidently inadequate. Suppose demand rises as in figure (5.2.7) wages and employment rise, what then happens when demand falls back to its initial levels. Unless preferences have changed, wages and employment both also return to their initial levels. Cartters argument only holds for the first initial movement away from $\hat{w},\hat{L}$.

Walters and Negishi (1980) propose a much more reasonable explanation of why unions resist wage reductions.[16] Unions are assumed to maximize the utility function (5.2.26).

$$U^u = wV(L(w)) \quad V(\cdot)>0, V'(\cdot)>0, V''(\cdot)<0 \qquad (5.2.26)$$

The introduction of the V(L(w)) function into the simple wage bill specification implies that those workers who are employed first, senior workers, have the most influence upon union policy. The condition for maximizing (5.2.26) is (5.2.27)

$$V(L(w))+wV'(L(w))L'(w) = \left[\frac{wL'(w)V(L(w))}{L(w)}\right]\left[E_{VL}-E_{wL}\right] \qquad (5.2.27)$$

where $E_{VL} = \frac{V'(L(w))L(w)}{V(L(w))}$ and $E_{wL} = \frac{1}{E_{Lw}} = -\frac{L(w)}{wL'(w)} > 0$

the firm maximizes profit as (5.2.28)

$$\text{Max } \pi = p(f(L))f(L)-wL \qquad (5.2.28)$$

where p(f(L)) the demand curve is perceived to have a kink such that $p'(f(L))^+ < p'(f(L))^-$, f(L) is the production function.

Profit maximization involves the conditions (5.2.29) and (5.2.30)

$$p'(f(L))^+(1-E_{pf}^-)f'(L) > w \qquad (5.2.29)$$

$$p(f(L))(1-E_{pf}^+)f'(L) < w \qquad (5.2.30)$$

where E_{pf}^- and E_{pf}^+ are the left and right hand price elasticities with respect to output.

In the light of (5.2.29) and (5.2.30) the labour demand curve rises as in figure (5.2.7) wages and employment rise, what then happens when demand falls back to its initial levels. Unless preferences have changed, wages and employment both also return to their initial levels. Cartters argument only holds for the first initial movement away from $\hat{w},\hat{L}$.

Walters and Negishi (1980) propose a much more reasonable explanation of why unions resist wage reductions.[16] Unions are assumed to maximize the utility function (5.2.26).

$$U^u = wV(L(w)) \quad V(\cdot)>0, V'(\cdot)>0, V''(\cdot)<0 \tag{5.2.26}$$

The introduction of the V(L(w)) function into the simple wage bill specification implies that those workers who are employed first, senior workers, have the most influence upon union policy. The condition for maximizing (5.2.26) is (5.2.27)

$$V(L(w))+wV'(L(w))L'(w) = \left[\frac{wL'(w)V(L(w))}{L(w)}\right]\left[E_{VL}-E_{wL}\right] \tag{5.2.27}$$

where $E_{VL} = \dfrac{V'(L(w))L(w)}{V(L(w))}$ and $E_{wL} = \dfrac{1}{E_{Lw}} = -\dfrac{L(w)}{wL'(w)} > 0$

the firm maximizes profit as (5.2.28)

$$\text{Max } \pi = p(f(L))f(L)-wL \tag{5.2.28}$$

where p(f(L)) the demand curve is perceived to have a kink such that $p'(f(L))^+ < p'(f(L))^-$, f(L) is the production function.

Profit maximization involves the conditions (5.2.29) and (5.2.30)

$$p(f(L))^+(1-E_{pf}^-)f'(L) > w \tag{5.2.29}$$

$$p(f(L))(1-E_{pf}^+)f'(L) < w \tag{5.2.30}$$

where E_{pf}^- and E_{pf}^+ are the left and right hand price elasticities with respect to output.

In the light of (5.2.29) and (5.2.30) the labour demand curve perceived by the union will probably have a vertical section at the current level of employment. The wage rate that maximizes $wV(L(w))$ will be the maximum value of w on the vertical section of the demand curve. A small cut in wages cannot improve the unions utility since it will have no employment effect. Since $L(w)$ is clearly zero at that w which maximizes $wV(L(w))$ the L.H.S. maximization condition is satisfied. To evaluate the R.H.S. condition we may simply rewrite (5.2.27) as (5.2.31)

$$\left[\frac{wV(L(w))}{L(w)}\ L'(w)^{+}\right]\left[E_{VL}-(1/E_{Lw}^{+})\right] \leq 0 \qquad (5.2.31)$$

where $L'(w)^{+}$ is the right-hand derivative

E_{Lw}^{+} the right-hand elasticity

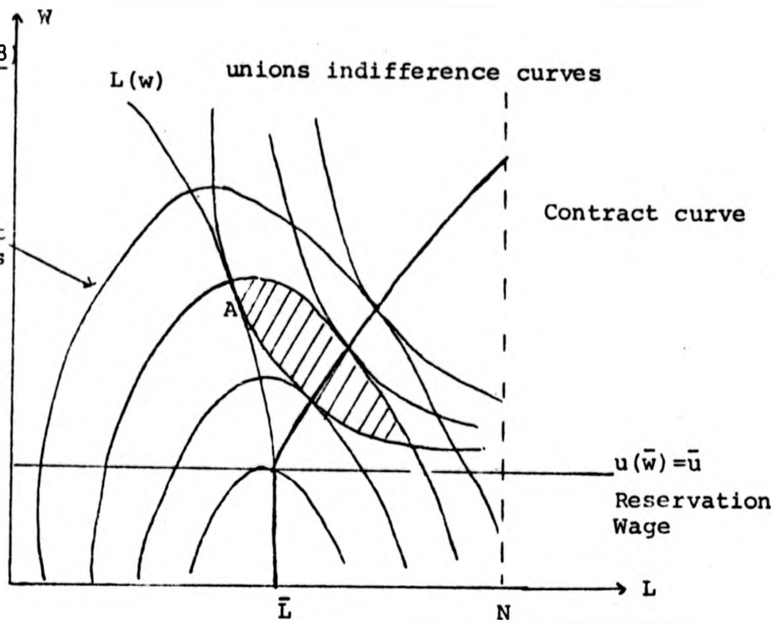
Consider now what happens when there is an increase in demand for the firms output. Two scenarios are possible depending upon the unions perceptions. (i) If the union perceives E_{Lw}^{+} is constant then for $f''(L)<0$ and if E_{pf} is perceived by employers as constant, The increased product demand will cause firms to raise employment L at a given w. Consequently the term $(E_{VL} - (1/E_{Lw}^{+}))$ in (5.2.31) is initially positive and then becomes zero, hence beyond this point all increases in demand are absorbed by changes in the wage rate. (5.2.29) implies that p must eventually rise once the change in L causes the condition to become satisfied by an equality. Initially the effect will be felt on employment but will then switch to changes in prices and wages. (ii) If the union perceives a constant right-hand slope $L'(w)^{+}$ rather than a constant elasticity E_{Lw}^{+} increases in both the wage rate and the level of employment must occur once (5.2.31) has achieved an equality through w changes.

For a reduction in product demand the argument is considerably simpler. A fall in demand will reduce employment with an unchanged product price and given wage rate as long as the R.H.S. profit maximization condition (5.2.30) holds.[16] The union will not of course react by reducing wages since this will gain no increase in employment. The full effect of the fall in product demand is felt on the level of employment.

Despite the improvements suggested by Walters and Negishi demand determined solutions to the bargaining problem do not seem to provide an acceptable explanation of wage rate determination. The arguments essentially rely upon the following process occurring, firms announce a labour demand curve which defines the profit maximizing level of employment at each wage rate. The union then selects a given wage employment combination on the labour demand curve which maximizes utility. There are several problems with such a process. If the firm learns about union preferences it will manipulate the outcome by announcing a labour demand curve the wage bill maximization point of which yields a greater profit than may be obtained on the true labour demand curve. The analysis also does not consider the possibility that the firm may be able to influence the point chosen by the union by threatening a lock-out. Finally it should be noted that demand determined solutions to the problem are always pareto inefficient if the level of employment enters the unions utility function.[17]

As Hall and Lilien (1979) point out an efficient bargain 'must specify the level of employment as well as the total compensation or average wage'. This can be easily illustrated by a simple diagram due to MacDonald and Solow (1981).

Fig. (5.2.8)



In figure (5.2.8) if we imagine that the unions indifference curves are the rectangular hyperbola as in Dunlop's analysis we see that the demand determined solution appears at A. Notice immediately that all wage employment pairs within the shaded lens are Pareto superior to A. The contract curve represents efficient combinations of w and L. In the context of an efficient bargaining solution the way in which wages and employment are determined and adjust will depend upon the shape and position of the contract curve, and the actual bargaining process adopted to pick the solution point upon the curve.

Given any particular product market structure the shape of the contract curve will depend upon the associated revenue function and unions preferences. Unions preferences should in some sense represent the preferences of its members. MacDonald and Solow suggest the following;

$$U^u = N^{-1}\{L(U(w)-R)+(N-L)U(w_u)\} \qquad (5.2.32)$$

where N is the total number of workers

R is the marginal disutility of labour

w_u unemployment or the opportunity cost wage

$U(w)$ and $U(w_u)$ represent the preferences of a representative worker.

The specification assumes the union is risk neutral and that senior workers have no extra say in union decision making. However this does seem a reasonable representation of union preferences and variants of this form will be adopted frequently in the next two chapters.

Actual bargaining solutions come in three basic forms. Game Theoretic as for example Nash (1950), which treat the problem as a fixed threat two persons non-zero sum bargaining game.[18] Arguments based upon the costs and benefits associated with strike duration deriving from Hicks (1932), and arguments about the propensity to fight based upon the early work of Zeuthen (1936). A good review of the various approaches may be found in de Menil (1971). The actual properties of bargaining solutions will be discussed in chapter 6.

The introduction of unions with monopoly power would seem to be an important explanation of wage rate determination. Solutions which yield efficient bargains are more desirable since they are consistant with the simultaneous determination of both wages and employment, as firms and unions maximize via the bargain. However, if the process is sufficiently costly then such agreements will only be struck periodically and it may then be argued that they provide a rationale for short-term wage rigidity.

The impact of disequilibrium upon bargaining and the macro-economic implications of unionization within the context of a disequilibrium model will be the subject matter of chapters 6 and 7.

In terms of explaining the behaviour of prices and wages in the macro economy it should perhaps be argued that each of the mechanisms discussed, with perhaps some reservations about the effective excess demand hypothesis as it stands, has some relevance. Combination of the expectation approach to output price determination and explicit contracts determining wages and employment might provide some interesting insights. As a consistent choice theoretic basis for price and quantity determination which may provide a rigorous microfoundation to macroeconomics the expectations approach currently looks most promising. The somewhat vague arguments associated with why price changes do not yield large quantity responses need to be examined in more detail, however arguments about the cost of search, speed or cost of information dissemination have at least the attraction of heuristic plausability.

FOOTNOTES

1. The work of Hahn and Negishi straddles several categories.

2. For a discussion of the Effective Excess Demand Hypothesis and references see Chapter 4.

3. Benassy (1980) notes that certain patterns of expectations formation may jeopardise the existence of an equilibrium of this type.

4. The specification assumes workers are risk neutral.

5. The absence of real balance effects may well be critical here as in Varian (1977).

6. Other sectors may not soak up the unemployment since the wage required may be below the shadow price of leisure.

7. Eric Maskin is acredited with having made this point first.

8. See Chapter 3.

9. This might suggest an explanation of stagflation.

10. See W.Oi (1962)

11. Bailey (1974) provides a very similar result in an inter-temporal context.

12. Grossman and Hart (1981) prove this result rigorously for the case of many workers.

13. Since the literature is still in its infancy one should not be too dismissive.

14. This is a problem to which the author hopes soon to give further attention.

15. Hart (1980) Considers a model of General Equilibrium with firms and unions monopolistically setting prices and wages respectively. However Hart does not manage to analyse the general case or explain the arbitrary division of labour and product markets adopted.

16. Negishi notes that this will always be the case if $E^{+}_{pf}>1$.

17. This was first pointed out by Leontief (1946).

18. See Luce and Raiffa (1957).

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6. WAGE AND EMPLOYMENT BARGAINING IN DISEQUILIBRIUM

6.1 Solutions to the Bargaining Problem

The idea that unionization by causing price rigidities may generate unemployment equilibria has been a recurring contention in conventional macroeconomics. The argument is usually that during periods when excess supply exists upon the labour market unions resist the wage cuts that would remove involuntary unemployment. This is because unions it is claimed represent their members who are typically drawn mainly from the employed members of the total labour force, and these workers place greater emphasis upon wages since seniority gives them a certain safety from redundancy. The unemployed have less voice in influencing union behaviour. This is the essence of the Walters-Negishi (1980) specification of the unions preferences discussed in the previous chapter. The utility which the union places upon the two objectives, wages and employment, at the margin will depend upon their marginal valuation by its actual members.

A well founded exposition of bargaining behaviour should not arbitrarily assign the control of different variables to different agents. Often it is assumed that the wage rate is determined by the union and employment by the firm.[1] Rational bargainers should detect and realise all possible pareto improvements by determining both wages and employment simultaneously as part of the bargain.[2] Given that the correct information is available the bargain should be efficient. Hall and Lilien (1979) and Malcomson (1982) discuss the ways in which efficient or approximately efficient bargains may be struck under uncertain supply and demand conditions.

The informational requirements and enforcability of fully contingent efficient contracts or bargains will not be examined here. It will be assumed that any bargain struck will be efficient at least at the moment it is determined. This chapter is concerned with the impact of disequilibrium upon the wage employment bargains. Chapter 7 will examine the macroeconomic implications of introducing bargaining into disequilibrium models.

To examine how changes in the disequilibrium situation effect the outcome of an efficient bargaining process we shall initially follow the path taken by Cartter (1959) and Fellner (1960). The bargaining problem consists of defining the locus of pareto efficient bargains and then selecting a particular wage employment pair on that locus.

A Simple Bargaining Model

The agents in this model are one firm and one union. The union operates a closed shop and labour is the only variable factor of production.

The firms demand curve and production function are jointly summarised by a revenue function as (6.1.1)

$$R = R(L) \qquad (6.1.1)$$

where (i) $R'(L) \gtreqless 0$ as $L \lesseqgtr L^*$

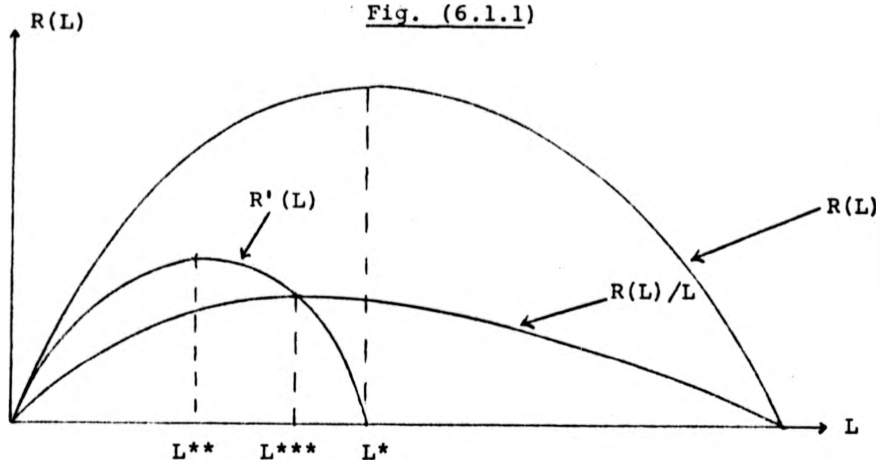
(ii) $R''(L) \gtreqless 0$ as $L \lesseqgtr L^{**}$

(iii) $R'''(L) < 0$ constant $\forall L$

(iv) $R'(L) = R(L)/L$ defines L^{***}

Diagramatically the revenue function is as figure (6.1.1).

Fig. (6.1.1)



Using (6.1.1) the profit function of the firm may now be written as (6.1.2)

$$\pi = R(L) - wL \qquad (6.1.2)$$

For any given isoprofit curve $\pi = \bar{\pi}$ the slope and curvature are derived from (6.1.2) as (6.1.3) and (6.1.4)

Slope of the isoprofit curve

$$\frac{dw}{dL} = \frac{R'(L)L - R(L) - \bar{\pi}}{L^2} = \frac{R'(L)-w}{L} \qquad (6.1.3)$$

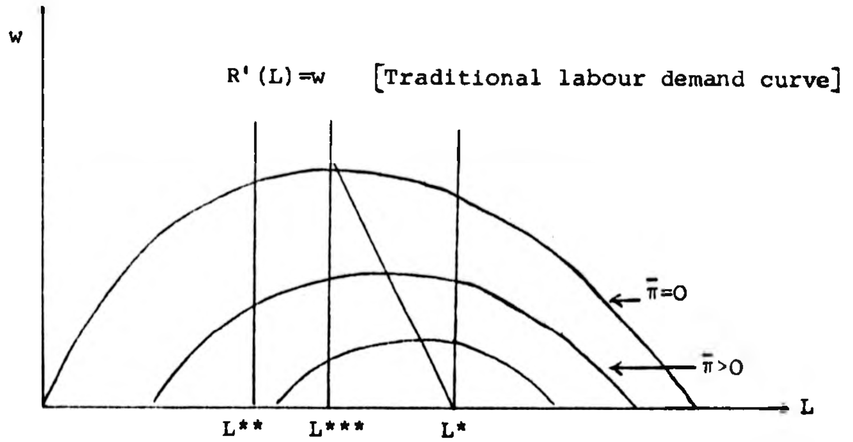
Curvature

$$\frac{d^2w}{dL^2} = \frac{1}{L^2}\left[R''(L)L + \frac{2}{L}\left[R(L) + \bar{\pi} - R'(L)L\right]\right] \qquad (6.1.4)$$

Hence $\frac{dw}{dL} < 0 \quad \forall\ L > L^{***} > 0$ and $\frac{d^2w}{dL^2} < 0\ \forall\ L > L^* > 0$[3]

Thus the firms isoprofit curves are as in figure (6.1.2)

Fig. (6.1.2)



In the bargaining process the firms objective is to get onto the highest isoprofit curve it can.

The union will be assumed to maximize a utility function over wages and employment as (6.1.5).

$$u = u(w,L) \qquad (6.1.5)$$

This is assumed quasi-concave and increasing in both arguments. The slope of any indifference curve will be

$$\frac{dw}{dL} = -\frac{uL}{uw} < 0 \text{ and curvature } \frac{d^2w}{dL^2} = \frac{u_{wL}u_L - u_{LL}u_w}{(u_w)^2} > 0$$

The unions indifference map is convex.

Using (6.1.3) and (6.1.5) the locus of pareto efficient bargaining points may be written as (6.1.6)

$$L = \frac{u_n}{u_L}\left[w-R'(L)\right] \qquad (6.1.6)$$

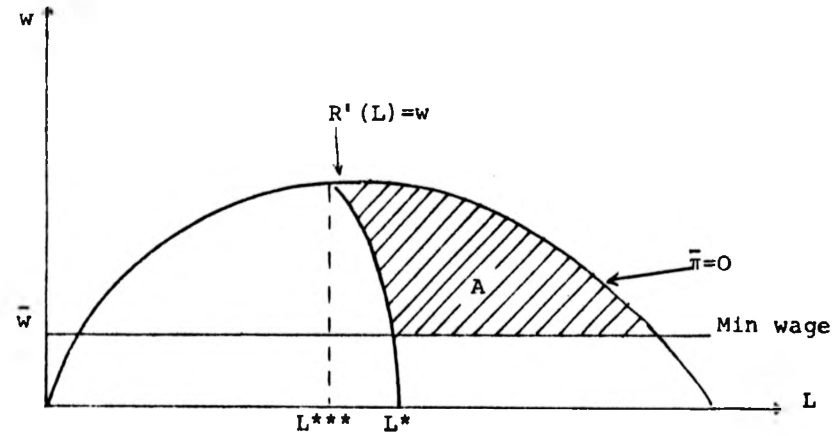
differentiating (6.1.6) gives (6.1.7)

$$\frac{dw}{dL} = \frac{u_{WL}\left[w-R'(L)\right] - u_w R''(L) - u_L - u_{LL}L}{u_{WW}\left[R'(L)-w\right] - u_w + Lu_{Lw}} \qquad (6.1.7)$$

Inspection of (6.1.7) reveals that in general the slope of the contract curve cannot be signed. McDonald and Solow (1981) claim that there is a strong presumption that the contract curve will have positive slope. However their analysis is based upon a less general specification of union preferences, implicitly they assume each worker has an equal say in union policy and each is equally effected by union behaviour. The expression (6.1.5) is completely general and allows for the possibility, suggested by Walters and Negishi (1980), of senior workers having a higher marginal evaluation on wage increments and lower evaluation on employment changes than junior workers, and also a proportionately larger say in union policy. In such a case a negatively sloped contract curve is more likely. For example if the unions utility function were specified as $u = w^{1-L/N}L^{L/N}$ then a sufficiently large initial labour pool N or small employment level L will ensure a negatively sloped contract curve. This specification has the property that senior workers place a higher marginal valuation upon wage increments.[4]

In the completely general case of bargaining, progress is limited to stating that the solution will lie in a particular region in the bargaining space. If it is assumed that (i) the firm will not accept a negative profit solution, (ii) the union will never accept a wage rate below that which will ensure its members employment elsewhere, (iii) employment will never be set such that $R'(L)>w$. These three assumptions allow the bargaining space to be divided up as figure (6.1.3). Clearly bargains will not be struck outside the region A.

Fig. (6.1.3)



To make further progress with the analysis so that it will be possible to examine the specific implications of disequilibrium for the bargain, it is necessary to give the model further structure.[5] Thus, let the unions utility function be Cobb-Douglas[6] and the firms revenue function be quadratic as (6.1.8) and (6.1.9)

$$u = w^{\alpha}L^{\beta} \qquad \alpha+\beta=1 \tag{6.1.8}$$

$$R(L)=bL+cL^{2} \qquad c<0 \tag{6.1.9}$$

The firms profit function may now be written

$$\pi = bL + cL^{2} - wL \tag{6.1.10}$$

Differentiating (6.1.8) and (6.1.10) totally and holding u and π constant defines the pareto efficient bargaining locus as (6.1.11)

$$w = \frac{b+2cL}{1-\beta/\alpha} = \frac{R'(L)}{1-\beta/\alpha} \tag{6.1.11}$$

the locus has slope

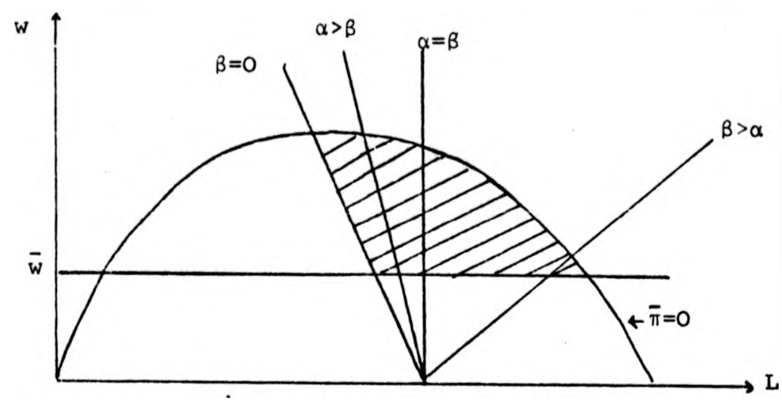
$$\frac{dw}{dL} = \frac{2c}{1-\beta/\alpha} = \frac{R''(L)}{1-\beta/\alpha} \gtreqless 0 \text{ as } \beta \gtreqless \alpha$$

and curvature

$$\frac{d^2w}{dL^2} = \frac{R'''(L)}{1-\beta/\alpha} = 0 \ \forall\ \alpha,\beta$$

Hence the efficiency loci are clearly rays originating from $w=0$, $L= -b/2c$ as in figure (6.1.4)

Fig. (6.1.4)



This specification has some very useful properties. Recalling that the coefficients α and β are elasticities and using the facing that the α=β efficiency locus is vertical, a simple interpretation of bargaining loci is possible. If the unions utility is more responsive to the level of employment then the locus of pareto efficient bargaining points has positive slope and is located to the right of the α=β vertical, which may be interpreted as the wage bill maximization specification of the unions utility function. If however the unions utility is more responsive to the wage rate then the contract curves have negative slope and lie between the α=β wage bill maximization locus and the β=0 wage rate maximization locus. This specification allows a simple categorization of loci and does not make the presumption that they have a positive slope.

It is now fruitful to examine some actual bargaining solutions.

The Union as a Monopolist

The union maximizes its utility subject to the firm achieving its opportunity cost level of profit A.

$$\text{Max } u = w^{\alpha}L^{\beta} \qquad (6.1.12)$$

$$\text{S.T. } R(L)-wL = A \qquad (6.1.13)$$

Maximizing (6.1.12) subject to (6.1.13) and rearranging yields a solution equation in L as (6.1.14)[7]

$$(1-\beta)\ R'(L)L + (2\beta-1)(R(L)-A) = 0 \qquad (6.1.14)$$

Solving (6.1.14) for L and substituting into (6.1.13) would give the solution value of w.

The comparative static properties of the solution may be obtained by totally differentiating (6.1.14) and rearranging to give (6.1.15)

$$\frac{dL}{dA} = \frac{(2\beta-1)}{\beta\ R'(L) + (1-\beta)R''(L)L} \qquad (6.1.15)$$

Using the quadratic form of the revenue function and (6.1.11) it can be shown that the comparative static effects of a change in the firms opportunity cost - fixed threat point - are as described in table (6.1.1)

Table (6.1.1)

	$\alpha>\beta$	$\beta>\alpha$	$\alpha=\beta$
dL/dA	+	-	0
dw/dA	-	-	-

Hence an increase (decrease) in the firms opportunity cost raises (lowers) employment if the unions utility elasticity with respect to wages is greater than that with respect to employment, has the opposite effect if the employment elasticity is the greater and has no effect upon employment if the two are equal. In each case the wage rate will fall.

The firms opportunity cost profit level A, is that which the owners could earn by using the given capital in its next best employment. If demand in the market for the firms next best output is rationed, then the comparative static results presented in table (6.1.1) may be interpreted in the following light. A tightening of the ration in the alternative market will reduce A, a relaxation of the ration will raise A, with the effects as reported in the table (6.1.1). Another interpretation might be that the firms opportunity cost is defined as the output that could be obtained from employing another group of workers with different specific skills who can thus obtain a different product from the capital. A change in the firms opportunity cost could then be associated with a change in the availability of that particular type of specifically skilled labour, i.e. a change in the labour supply constraint on a particular labour market.[8]

The Firm as a Monopsonist

The firm maximizes profit subject to the union achieving its opportunity cost level of utility. The problem becomes

$$\text{Max } \pi = R(L) - wL \qquad (6.1.16)$$

$$\text{S.T} \quad w^{\alpha} L^{\beta} = B \qquad (6.1.17)$$

where $B = \bar{w}^{\alpha} N^{\beta}$, i.e. $\bar{w}$ is the wage at which the total labour pool N may gain employment elsewhere.

Maximizing (6.1.16) subject to (6.1.17) and rearranging will yield a solution equation in L as (6.1.18).

$$R'(L) - B^{1/\alpha}(1-\beta/\alpha)L^{-\beta/\alpha} = 0 \qquad (6.1.18)$$

which may be solved for L, and the result would allow (6.1.17) to solve for w.

The comparative static properties of this solution are obtained by totally differentiating (6.1.18) and rearranging to give (6.1.19)

$$\frac{dL}{dB} = \frac{(^{1}/\alpha)B^{1/\alpha\ -1}\ [1-\beta/\alpha]L^{-\beta/\alpha}}{R''(L)+(\beta/\alpha)B^{1/\alpha}\ (1-\beta/\alpha)L^{-(\beta/\alpha+1)}} \tag{6.1.19}$$

Using the quadratic form of the revenue function and (6.1.11) it can be shown that the comparative static effects of a change in the unions opportunity cost are as in table (6.1.2).

Table (6.1.2)

	$\alpha>\beta$	$\beta>\alpha$	$\alpha=\beta$
dL/dB	-	+	0
dw/dB	+	+	+

Thus an increase (decrease) in the union opportunity cost raises (lowers) employment if its utility elasticity of employment is larger than that with respect to wages, has the converse effect if the wage elasticity is larger, and no employment effect if the two are equal. The effect upon the wage rate is positive whatever the elasticities.

An increase in the unions opportunity cost may be associated with either a rise in fixed wage rates in other sectors in which its members could gain employment, or an increase in unemployment benefit levels, or perhaps a relaxation of a labour demand constraint on an alternative labour market which raises the probabalistic value of the associated wage rate.

The Nash Solution

Since the Nash solution is used here and extensively in later chapters, its properties will be discussed in a little detail.

The technique proposed by Nash [1950] uses the von Neumann-Morgenstern concept of cardinal utility to provide a determinate solution to the classical 2 player simple bargaining game. The classical approach to this game assumes that the two players can achieve any payoff vector $v = (\bar{u},\bar{\pi})$ within the compact payoff space P of the game G. If the two bargainers cannot agree, then each receives a fixed conflict payoff $c = (B,A)$. However, if an agreement is reached, then the solution must satisfy the two classical rationality postulates, R1 and R2.

(R1) Individual Rationality: Any agreement must yield each player a payoff at least as good as the conflict situation.

Hence $\qquad \bar{u} \geq B$ and $\bar{\pi} \geq A$

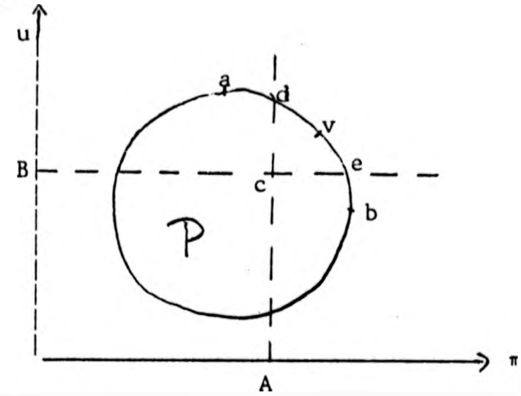
which implies the outcome must lie within the area cde in figure (6.1.5).

(R2) Joint Rationality: The agreement cannot be improved upon further for both players.

Hence $\qquad \nexists\ \overset{*}{v} = (\overset{*}{\bar{u}},\overset{*}{\bar{\pi}}) \in P$ s.t. $\overset{*}{\bar{u}} > \bar{u}$ and $\overset{*}{\bar{\pi}} > \bar{\pi}$

so the outcome must lie in the negotiation set of the game (see Luce and Raiffa [1957]). In figure (6.1.5) the negotiation set H of the game G is the boundary of P between points a and b.

Figure (6.1.5)



Hence, R1 and R2 together imply v must lie on the arc between d and e , termed the concession limits for the two players. However, the exact location of v on de is indeterminate.

To make v determinate, Nash proposed the following. The game is played in normal form, the players each make only one move independently and simultaneously. The move is to choose a real valued payoff demand. If the two demands are mutually compatible; $v = (u,\pi) \epsilon P$, then these are the solution to the game. If the demands are mutually incompatible; $v = (u,\pi) \not\epsilon P$, then each player receives only his conflict payoff; $v = (B,A)$. Nash agrues that the game will be resolved by the two players agreeing to equal payoffs, because neither player can rationally expect that a rational opponent will grant him better terms than he is willing to concede himself.

Formally the Nash solution satisfies four postulates, P1-P4.

(P1) Joint Efficiency: $v = (\bar{u},\bar{\pi}) \epsilon H$

(P2) Symmetry: The solution of a symmetric game lies on the line $\bar{u} = \bar{\pi}$ in the positive orthant.

(P3) Linear Invariance: If v is the solution to G and G* is the game obtained when one player's utility function is subjected to an order preserving linear transformation T . Then the solution $\overset{*}{v}$ to $\overset{*}{G}$ will be the image of v under the transformation T . $\overset{*}{v} = Tv$.

(P4) Independence of Irrelevant Alternatives: For the game G in payoff space P and solution v , if the payoff space is restricted to $P^* \subset P$ and $c = (B,A) \epsilon P^*$ and $v = (\bar{u},\bar{\pi}) \epsilon P^*$ then v is also the solution to G^* .

The solution to G , $v = (\bar{u},\bar{\pi})$ is the point satisfying $(\bar{u}-B)(\bar{\pi}-A) = \max\, [(u-B)(\pi-A)]$. Nash shows this to be a solution to both symmetric and asymmetric games in normal form.

In our simple model, the Nash maximand may be written as (6.1.20)

$$\text{Max Prod} = [w^{\alpha}L^{\beta}-B][R(L)-wL-A] \tag{6.1.20}$$

differentiating

$$\frac{d[\text{Prod}]}{dw} = \alpha w^{\alpha-1}L^{\beta}R(L) - (\alpha+1)w^{\alpha}L^{\beta+1} - \alpha A w^{\alpha-1} + BL = 0 \tag{6.1.21}$$

$$\frac{d[\text{Prod}]}{dL} = \beta w^{\alpha}L^{\beta-1}R(L) + w^{\alpha}L^{\beta}R'(L) - (\beta+1)w^{\alpha+1}L^{\beta}$$
$$- \beta A w^{\alpha}L^{\beta-1} - B\,R'(L) + Bw = 0 \tag{6.1.22}$$

Evaluating (6.1.21) and (6.1.22) at $A=B=0$ and using the quadratic form of the revenue function gives (6.1.23) and (6.1.24)

$$L = -\frac{b}{c}\frac{\left[\beta+1 - \left(\frac{1-\beta^2}{2-\beta}\right)\right]}{\left[\beta+2 - \left(\frac{1-\beta^2}{2-\beta}\right)\right]} \tag{6.1.23}$$

$$w = \left(\frac{\alpha}{\alpha+1}\right)\left[b - b\,\frac{\left[\beta+1 - \left(\frac{1-\beta}{2-\beta}\right)^2\right]}{\left[\beta+2 - \left(\frac{1-\beta}{2-\beta}\right)^2\right]}\right] \qquad (2.1.24)$$

(6.1.23) and (6.1.24) define the Nash solution.

Some messy algebra allows the comparative statics of the solution, when $A \neq B \neq 0$ to be signed as table (6.1.3)[9].

Table (6.1.3)

	$\alpha>\beta$	$\beta>\alpha$	$\alpha=\beta$
dL/dA	+	−	0
dL/dB	−	+	0
dw/dA	−	−	−
dw/dB	+	+	+

The interpretations that may be placed upon changes in the union and firms opportunity cost (conflict payoffs), B and A respectively, are the same as in the previous two cases.

The Nash solution in this simple model gives relatively straightforward results. However, two questions should perhaps be asked. Firstly is the problem of bilateral monopoly addressed here adequately described by the simple 2 player bargaining game? This form of game assumes that the threats points are exogenously given and the two players make independent simultaneous moves. In a real world bargaining situation an agent may be able to threaten a number of different penalties if his offer is rejected. A union may choose between a work to rule, a strike, or various forms of disruptive action. A firm may threaten variable period lockouts or closure. This may result in a variable threat game. The credibility of the threats will be important, if a bargainer can make a demand and at the same time make it impossible for himself to compromise, then providing the demand yields his opponent something better than the conflict payoff he will gain his demand. For example, the union leadership may promise its members it will settle for no less than a 10% pay rise, even though 5% may be acceptable, it may not be able to back down on 10% and will consequently win. In circumstances where both bargainers can reduce their own ability to compromise the timing of demands becomes crucial. Whoever demands first will be successful (Schelling [1960] discusses these possibilities at some length).

These arguments seem to place doubt upon the validity of the simple "fixed threat" bargaining game as a representation of bargaining situations. However, Harsanyi [1967, p. 187] states, "most empirical bargaining situations seem to have the nature of simple bargaining games rather than ultimatum games". This could well be because bargainers seem particularly inventive in slightly redefining the game to get around a commitment an opponent has managed to lock himself into. For example, a firm may concede a 10% pay demand but will then attempt to introduce productivity clauses into the agreement.

If we accept Harsanyi's statement, the second question arises. Are the Nash postulates P1-P4 reasonable characterizations of the features a solution should display? Postulate 1, joint efficiency, just states that the two bargainers exhaust all possible Pareto improving moves. This does not seem particularly contentious providing that there are no institutional constraints which prevent both the wage and the level of employment to be freely chosen. The symmetry postulate P2 is essentially another rationality postulate. Neither player believes the other will adopt a strategy less rational than his own. This postulate suggests neither player has an advantage in terms of bargaining power or skill. However, if two rational bargainers are playing the simple game in normal form, then there seems no role for bargaining skill, and bargaining power is captured in the specification of the conflict payoffs. Postulate 3 makes the solution independent of interpersonal comparisons of utility between the two players.

Interpersonal comparisons may be important for ethical reasons if the game is resolved by arbitration. Ethical judgment, however, involves imposing some measure of comparison on the two bargainers utilities, clearly neither rational bargainer would accept a measure which lowers his payoff.

The final postulate, independence of irrelevant alternatives, may be interpreted as that the solution should not be effected by excluding from the payoff space some potential agreement points that would not have been chosen. This postulate has been heaviliy criticized. Critics argue that the bargaining solution should reflect "bargaining power" or "fairness". For example, if the payoff to the firm was of a large order of magnitude in the monopsony solution, then because the firm could "win big" if a monopsony solution arose, then this should be reflected in higher payoffs for the firm in all other solutions.

Basically critics of the postulate claim unchosen solutions cannot be regarded as irrelevant. Taking this line of thinking to its logical conclusion Kalai and Smorodinsky (1975) abandon the use of the conflict payoff point, and divide the payoffs to the two bargainers in the same ratio as they would achieve in the monopsony and monopoly solutions. However, there does not seem any simple case for adopting this solution rather than the Nash.

Market Power Solution

One possible non-co-operative alternative to the Nash solution, the market power solution is obtained by taking a weighted average of the monopolist and monopsonist solutions.

Let $\emptyset$ represent the unions and $(1-\emptyset)$ the firms bargaining power.[10] Consider first the determination of the level of employment L from (6.1.14) and (6.1.18) we obtain (6.1.25)

$$\emptyset\left[R'(L)-B^{1/\alpha}(1-\beta/\alpha)L^{-\beta/\alpha}\right]+(1-\emptyset)\left[(1-\beta)R'(L)L+(2\beta-1)(R/L)-A)\right]=0 \tag{6.1.25}$$

Solving (6.1.24) for L and using the definition of efficiency (6.1.11) gives w.[11]

The comparative static effects of changes in the union and firms opportunity costs follow in the same manner as in the monopolist and monopsonist cases. Changes in the bargaining power parameter Ø move the solution along the appropriate efficiency ray as summarised in table (6.1.4).

Table (6.1.4)

	$\alpha>\beta$	$\beta>\alpha$	$\alpha=\beta$
dL/dA	+	-	0
dL/dB	-	+	0
dL/dØ	-	+	0
dw/dA	-	-	-
dw/dB	+	+	+
dw/dØ	+	+	+

Increases in Ø represent an increase in the unions bargaining power, as table (6.1.4) indicates changes in Ø effect the bargaining solution in the same manner as changes in the unions opportunity cost B. If for example an employment constraint in another sector where the union members may work is removed, this both varies the unions opportunity cost and its bargaining power.

6.2 The Impact of Disequilibrium Upon Bargains

Section 6.1 examined some solutions to the problem of bargaining upon the labour market. Attention was mainly focused upon the comparative static properties of the various solutions. Changes in the union or firms fixed threat points were interpreted as a consequence of changes in constraints in other sectors of the economy. This section considers how such changes, by effecting the firms inverse demand curve, may cause the position of the contract curve in bargaining space to change. This leads to several plausible scenarios in which the nominal wage varies little when product demand changes, but the employment level changes considerably. One of these explanations of a sticky nominal union wage has been replicated independently by McDonald and Solow (1981).

As previously the firm will be assumed to be a simple profit maximizer faced by a standard neoclassical production function

$$x=x(L) \qquad x'(L)>0 \quad ,x''(L)<0 \tag{6.2.1}$$

$$x'(0)=\infty \qquad x'(\infty)=0$$

The product market structure of the firm is described by the inverse demand function (6.2.2).

$$p=p(x(L),H) \quad p_1(\cdot)<0, \quad p_2(\cdot)>0 \tag{6.2.2}$$

where H represents the level of economic activity in other relevant markets. McDonald and Solow (M.S.) introduce H directly into a revenue function and interpret it as representing the trade cycle - this interpretation is not adopted here and it is shown that M.S.'s results do not necessarily follow in this analysis.

The firms profits may be written as (6.2.3)

$$\pi = p(x(L),H)\ x(L) - wL \qquad (6.2.3)$$

and hence we have the isoprofit curve (6.2.4) for $\pi = \bar{\pi} \geq 0$

$$w = \frac{1}{L}\left[\ p(x(L),H)\ x(L) - \bar{\pi}\right] \qquad (6.2.4)$$

The isoprofit curve will thus have slope as (6.2.5)

$$\frac{dw}{dL} = \frac{1}{L}\left[\frac{\partial p}{\partial x}\cdot\frac{\partial x}{\partial L},\ x(L) + \frac{\partial x}{\partial L}\cdot p(x(L),H)\right] - \frac{1}{L^2}\left[p(x(L),H)\,x(L) - \bar{\pi}\right]$$

$$= \frac{1}{L}\left[\frac{\partial p}{\partial x}\,\frac{\partial x}{\partial L}\cdot\ x(L) + p(x(L),H)\ \frac{\partial x}{\partial L} - w\right] \qquad (6.2.5)$$

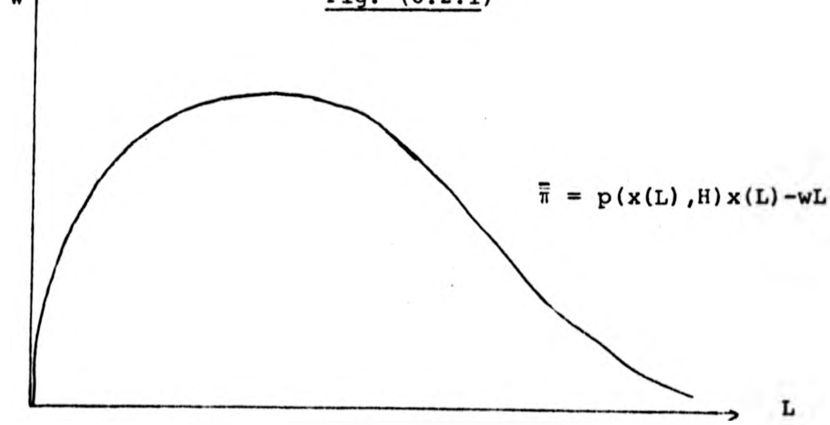
and curvature as (6.2.6)

$$\frac{d^2w}{dL^2} = \frac{1}{L}\left[\frac{\partial^2 x}{\partial L^2}\left(\frac{\partial p}{\partial x}\cdot\ x(L) + p(x(L),H)\right) + 2\ \frac{\partial p}{\partial x}\left(\frac{\partial x}{\partial L}\right)^2\right]$$

$$-\ \frac{2}{L^2}\ \frac{\partial x}{\partial L}\left[\ \frac{\partial p}{\partial x}\cdot x(L) + p(x(L),H)\ \right] + \frac{2}{L^3}\ \left[p(x(L),H)\,x(L) - \bar{\pi}\right] \quad (6.2.6)$$

Examination of (6.2.5) and (6.2.6) allows us to state that the isoprofit curves will be concave except for a convex tail as in figure (6.2.1)

Fig. (6.2.1)



Union behaviour may be described by a concave utility function as in section 6.1

$$u = u(w,L) \qquad (6.2.7)$$

where $u_w, u_L, u_{wL}, U_{Lw} > 0 \quad u_{ww}, u_{LL} < 0$

$$\left.\frac{dw}{dL}\right|_{u=\bar{u}} = -\frac{u_L}{u_w} \tag{6.2.8}$$

An efficient contract must now lie upon the contract curve obtained from (6.2.5) and (6.2.8)

$$\frac{1}{L}\left[\frac{\partial p}{\partial x}\cdot\frac{\partial x}{\partial L}\cdot x(L)+p(x(L),H)\frac{\partial x}{\partial L} - w\right] = -\frac{u_L}{u_w} \tag{6.2.9}$$

The contract curve (6.2.9) may possibly have negative slope if the union places a high marginal evaluation upon wages.[12]

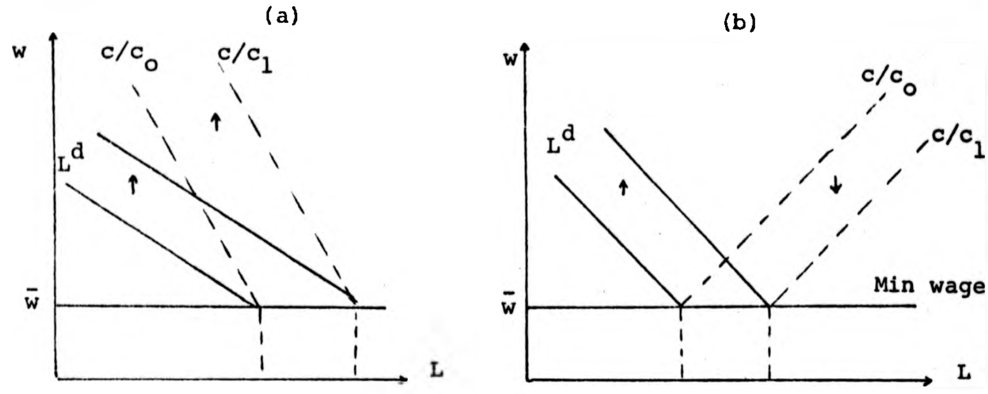
Our interest lies in examining how changes in other sectors of the economy, dH, which may be interpreted as rationing level or regime changes, effect the level of employment and wages. Most particularly, are there any scenarios in which there will be little tendency for wages to change? To consider the possibilities it is necessary to understand how both the position of the contract curve in bargaining space and the location of the solution upon the curve respond to potential changes in H.

Differentiating (6.2.9) holding L constant and assuming $p_{12}=0$ gives (6.2.10) and describes the movement of the contract curve.

$$\frac{\partial w}{\partial H} = \frac{\frac{\partial p}{\partial H}\cdot\frac{\partial x}{\partial L}}{[u_w{}^2 - u_{Lw}u_wL + u_{ww}u_LL\,]u_w{}^2} \tag{6.2.10}$$

Hence as with the slope of the contract curve the crucial argument is the unions marginal evaluation of wages, u_w. For small values of u_w the curve has positive slope and moves downwards when the firms product demand curve moves out. For large values of u_w the contract curve has negative slope and moves upwards with product demand increases. The possibilities are described in figure (6.2.2)(a) and (b).

Fig. (6.2.2)



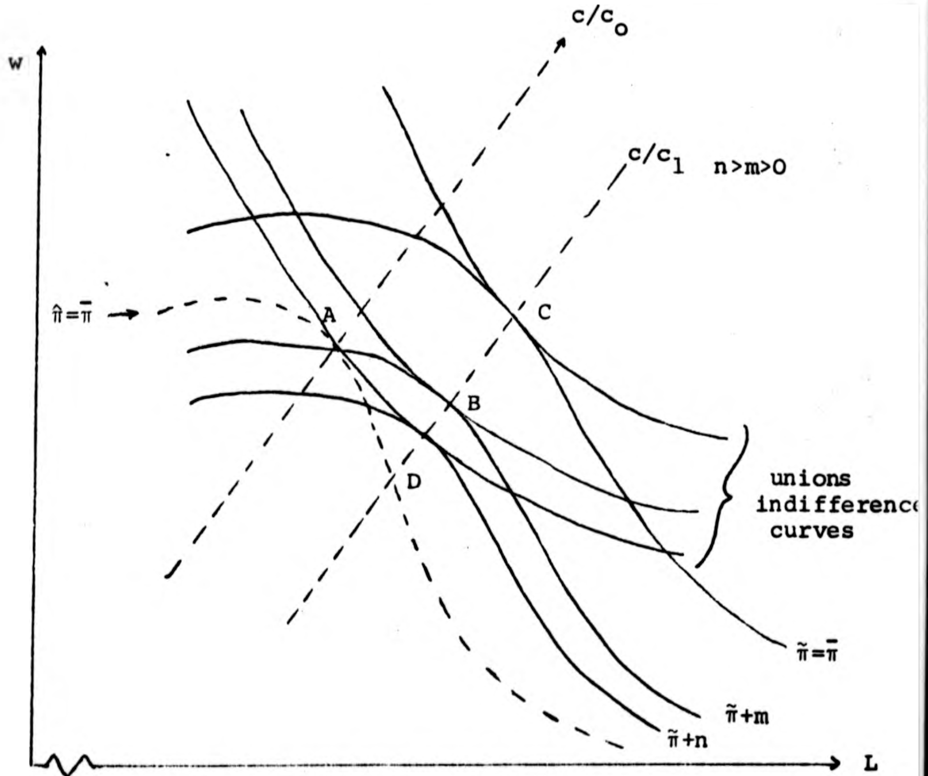
In figure (6.2.2)(a) u_w is assumed very large, the contract curve has negative slope and moves from c/c_o to c/c_1 as the firms product market demand conditions improve, $dH>0$. In (b), the more likely case and the one analysed by McDonald and Solow the contract curve moves from c/c_o to c/c_1 downwards.

To describe how wages and employment may change under different bargaining solutions consider figures (6.2.3) and (6.2.4).

Figure (6.2.3) describes the effect of a rise in the firms product demand upon various bargaining solutions. Point A in the initial bargaining solution that has been in force prior to the change upon the product market. A lies on the contract curve c/c_o which derives from the tangency points between the unions indifference map and a set of isoprofit curves associated with $\hat{\pi}$. An improvement in the product market generates a new set of isoprofit curves denoted $\tilde{\pi}$, our previous analysis implies that these will appear as depicted in the figure.[13]

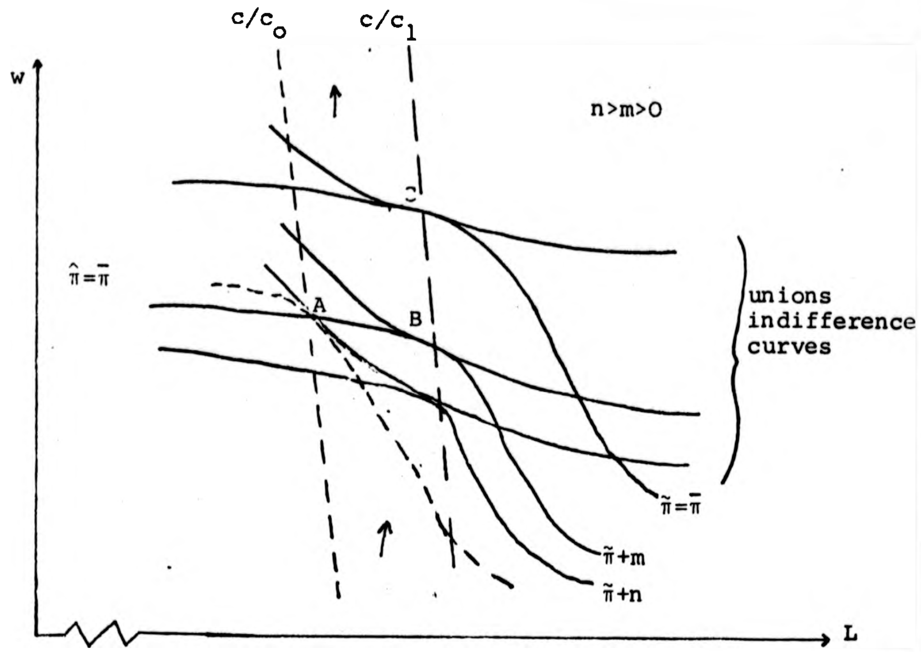
If the levels of utility and profit obtained at the initial point A are taken to be the union and firms opportunity costs then C may be interpreted as the new monopolistic solution and D the new monopsonistic solution. If the original opportunity cost/fixed threat points upon which A was struck are maintained then C is the new monopolistic solution if A was a monopolistic solution and D is the new monopsonistic solution if A was a monopsonistic solution. The point B is also of some interest here as the firm may be

Fig. (6.2.3)



described as obtaining all the extra reward associated with a product price rise at A whilst the union captures the total rewards from new pareto improvements. The same interpretation may be placed upon figure (6.2.4).

Figure (6.2.4)



Notice that if the contract curve has positive slope the monopolistic solution C is associated with a higher level of employment than the monopsonistic solution, and a higher wage rate. If the contract curve has negative slope the monopoly solution actually entails lower employment and higher wages than the monopsony case. Clearly the same statements can be made with reference to the market power solution since this is a linear combination of the two extremes.

Any change which effects the firms inverse demand curve may also effect the firm and unions opportunity costs and their relative bargaining power. There are several possibilities which could be examined here. As was found in examining bargaining solutions in section 6.1 clear statements about how wages and employment adjust cannot be made without giving the problem functional form. The choice of the correct functional specification is an interesting empirical question but not one that will be tackled here. Attention will be focused upon the various potential scenarios which the structure developed above generates when disequilibrium changes upon other markets are postulated. To simplify the arguments it will be assumed that the opportunity costs of the union and firm are not significantly inter-related. The argument that the firm achieves its opportunity cost activity by applying a different type of labour to its capital, and the union by having its members work in another sector with different capital, might support such an assumption.

Scenario 1: The sector in which the unions members have their alternative employment possibilities is characterised by a Repressed Inflation type situation: i.e. a shortage of both appropriate labour and goods.

If the supply constraints in the unions alternative sector tighten this will have three effects. The resultant spillovers on the product markets will cause the demand curve to shift out and consequently the contract curve to behave as in figure (6.2.3) or (6.2.4). The unions opportunity cost will rise since the probabilistic value of alternative employment will increase with effective excess labour demand. The unions bargaining power relative to that of the firm may also increase.[14] For the case where the contract curve has positive slope figure (6.2.5) demonstrates that for monopsonistic and market power solutions the three effects work together in raising employment but tend to offset each other in their effect upon wages.

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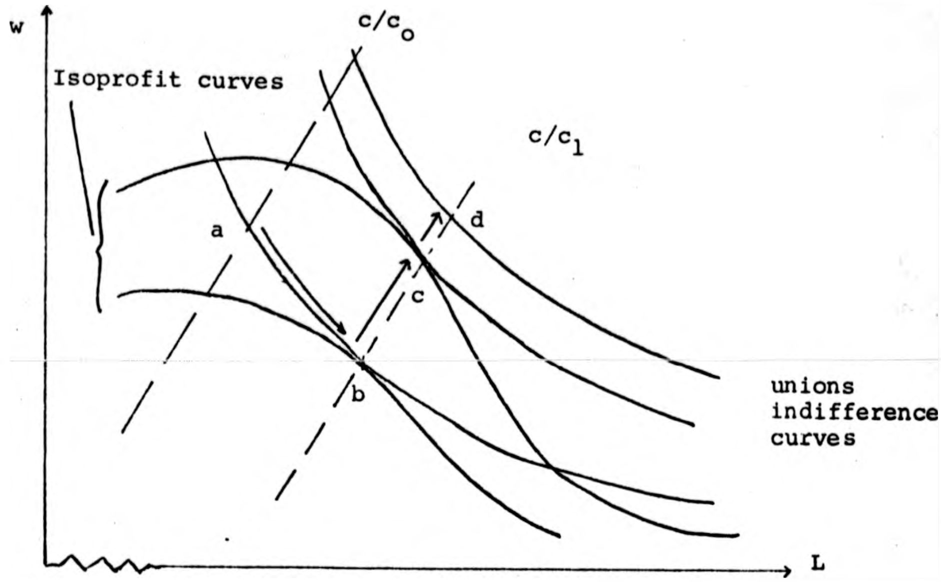
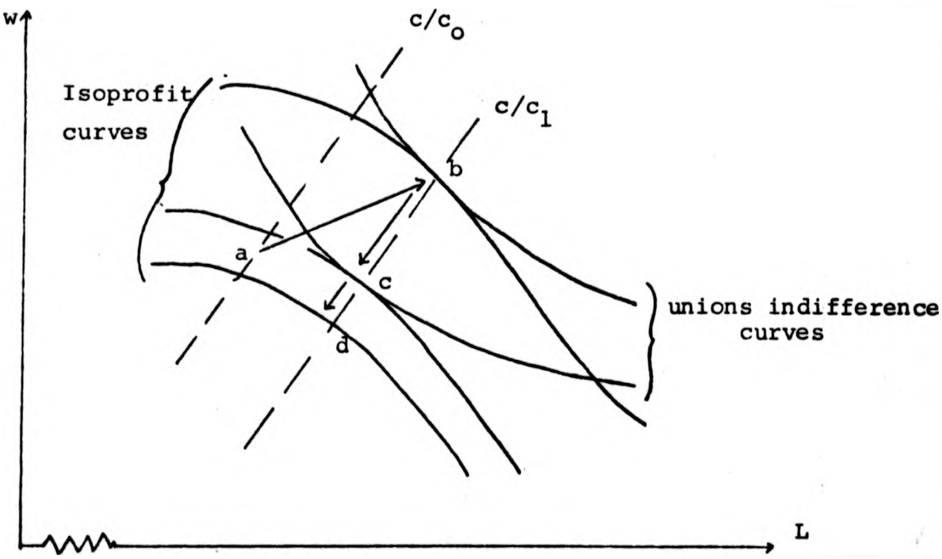
Figure (6.2.5)



The initial bargaining solution lies at point a in the bargaining space upon the contract curve c/c_o the change in constraint levels in the unions alternative employment sector, generates product demand spillovers and a new contract curve c/c_1. The movement to the new contract curve a-b lowers wages and raises employment. The rise in the unions opportunity cost pushes the monopsonistic solution up the contract curve b-c reinforcing the effect upon employment but offsetting the effect upon wages. If rather than a monopsonistic solution a market power solution arises then the increase in the unions bargaining power moves the solution further out along the contract curve, as for example c-d, further compounding the rise in employment but offsetting the fall in the wage rate.

Scenario 2: The sector in which the firm may employ its capital for an alternative productive use is characterised by a Keynesian unemployment type situation. Both the product and labour markets are demand determined. Relaxation of the constraints in this sector will increase income and hence demand for the good produced in the bargaining sector.[15] The firms opportunity cost and bargaining power will also rise. The case where the contract curve has positive slope and behaves as in figure (6.2.3) is again considered. Here there may be offsetting effects upon both wage and employment i.e. the monopolistic and bargaining power outcomes. Consider figure (6.2.6)

Figure (6.2.6)



a is the initial bargaining solution, located upon the contract curve c/c_o. The change in the firms inverse demand curve moves the contract curve to the right and the monopolistic solution to b, raising both employment and wages. The change in the firms opportunity costs means that its minimum profit level rises, causing the monopolistic solution to move back along the contract curve c/c_1 as indicated b-c, which has an offsetting effect upon both wages and employment. If a bargaining power solution arises the increased bargaining power of the firm pushes the solution from c-d, further reducing wages and employment and offsetting the changes originating from the demand function movement.

The two scenarios examined were chosen since they suggest possible explanations of sticky wages. The analytical structure can generate many other possibilities, and since there appear to be no overriding theoretical arguments which limit these possibilities empirical analysis may be required to decide which are the important and interesting cases.

One case that may be of potential interest occurs when a simple arbitration rule is applied to the bargaining problem. Let the rule be that the union receives a given proportion of total revenue as defined by (6.2.11)

$$wL = \lambda p(x(L),H)x(L) \qquad (6.2.11)$$

where $O<\lambda\leq 1$ constant

This may be termed an equity locus. It has slope as (6.2.12).

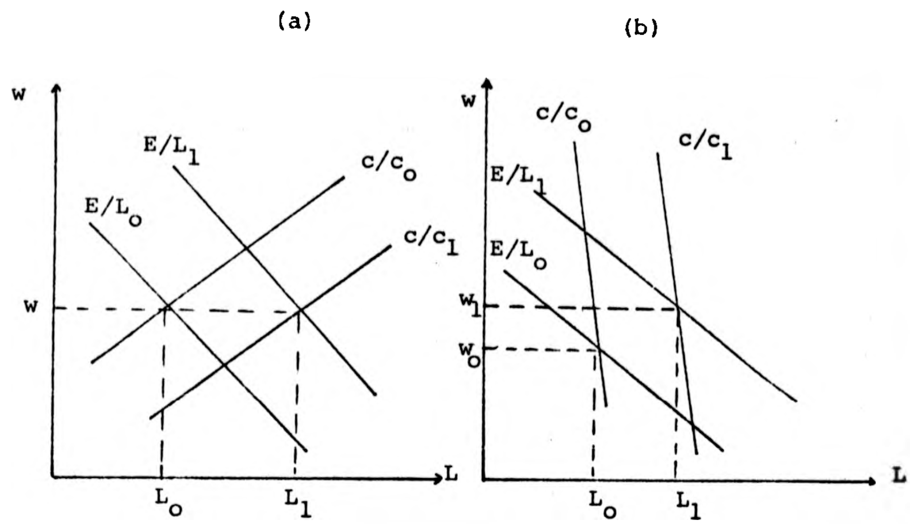
$$\frac{dw}{dL} = -\left[\frac{\lambda\left[\frac{\partial x}{\partial L}\left[\frac{\partial p}{\partial x}\cdot x(L) + p(x(L),H)\right]\right]+w}{L}\right] < 0 \qquad (6.2.12)$$

When demand for the firms output rises the whole equity locus moves upwards as indicated by (6.2.13)

$$\frac{\partial w}{\partial H} = \lambda\cdot\frac{\partial p}{\partial H}\cdot x(L) > 0 \qquad (6.2.13)$$

Combining this analysis of the behaviour of the equity locus with the contract curve demonstrates that the solution becomes as described in figure (6.2.7)(a) or (b)

Figure (6.2.7)



In both (a) and (b) in figure (6.2.7) as demand for the firms product rises the contract curve moves from $c/c_o - c/c_1$ and the equity locus from E/L_o to E/L_1.

In case (a) where the contract curve has positive slope the movement of the two curves produces offsetting effects upon the wage rate and complementary effects upon employment. In case (b) the effects upon wages and employment depend upon the relative magnitudes of the contract curve and equity locus shifts. Clearly wages or employment will rise, but not necessarily both.

Finally a brief remark about the bargaining behaviour of a sales constrained firm may be in order. Firstly it should be noted that a sales constraint only makes sense if the firm is a price taker. This implies that any workers employed in excess of those required to produce the given sales will yield the firm no further revenue, however it does not imply that these workers will not be employed. The analysis here diverges from that of McDonald and Solow who assume employment will not exceed that required to produce the given sales.

Let $\bar{x}$ be the given sales constraint, and define L_{max} to be the level of employment such that $\bar{x} = x(L)$. Care must now be taken in deriving the firms isoprofit curves.

(1) For $L<L_{max}$ the firms problem is defined by (6.2.14)

$$\max_{L} \pi = \bar{p}x(L)-wL \qquad (6.2.14)$$

An isoprofit curve valued at $\pi=\bar{\pi}$ has slope on (6.2.15)

$$\frac{dw}{dL} = \frac{\bar{p}\,\frac{\partial x}{\partial L}\,L + \bar{\pi} - \bar{p}x(L)}{L^2} \qquad (6.2.15)$$

(2) For $L>L_{max}$ the firms problem is defined by (6.2.16)

$$\max_{L} \pi = \bar{p}x(L_{max}) - wL \qquad (6.2.16)$$

here the isoprofit curve of value $\pi=\bar{\pi}$ has slope as (6.2.17)

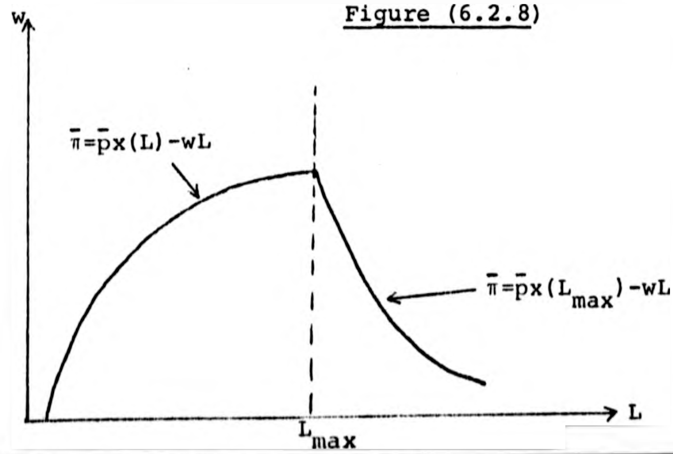
$$\frac{dw}{dL} = \frac{\bar{p}x(L_{max}) + \bar{\pi}}{L^2} < 0 \qquad (6.2.17)$$

and curvature

$$\frac{d^2w}{dL^2} = \frac{2\ \bar{p}x(L_{max}) - \bar{\pi}}{L^2} > \qquad (6.2.18)$$

Hence the firms isoprofit map is as figure (6.2.8)

Figure (6.2.8)



There seems a strong possibility here that the bargain will be struck at the employment level L_{max} associated with the isoprofit curve kink. However if the curvature of the unions indifference curves exceeds that of the isoprofit curve to the right of L_{max}, it is quite feasible for labour whose value marginal product is zero to be employed.

FOOTNOTES

1 See Addison and Siebert for a good basic review.

2. It may be argued that such bargains are not directly observed, however demarkation and manning agreements, 'featherbedding', will have employment effects which taken together with periodic wage negotiations may give approximately efficient bargains.

3 Since $\frac{d^3w}{dL^3} = R'''(L) + R''(L) - 2\frac{R''(L)}{w^2} - 6\left[R(L) + \bar{\pi} - R'(L)L\right]$

and $\frac{d^2w}{dL^2} = 0$ evaluated at $L = L^*$

4 Under this specification it may be shown that

$$\left.\frac{dw}{dL}\right|_{u=\bar{u}} = \frac{-(w+L)}{N-L} < 0$$

$$\left.\frac{d^2w}{dL^2}\right|_{u=\bar{u}} = \frac{-(N+w)}{(N-L)^2} < 0$$

The contract curve becomes:

$$\frac{R'(L)-w}{L} = -\frac{(w+L)}{N-L}$$

which represents points of interior maximum for the union if:

$$\frac{-(N+w)}{(N-L)^2} > \frac{1}{L^2}\left[R''(L)L + \frac{2}{L}\left[R(L) + \bar{\pi} - R'(L)L\right]\right] .$$

The slope of the contract curve is:

$$\frac{dw}{dL} = \frac{N\ R''(L) - L\ R''(L) - R'(L) + 2w + 2L}{N-2L} < 0$$

for N sufficiently large or L sufficiently small.

5 De Menil (1971) in an interesting analysis considers wage bill maximization using the Nash bargaining solution.

6 Dertouzos and Pencavel (1981) adopt a Stone-Geary form of the unions utility function, they do not consider efficient bargains but rather allow the union to select a point upon the firms labour demand curve.

7. Derivation is as follows:

substituting (6.1.13) into (6.1.12) gives:

$$u = \left[\frac{R(L)-A}{L}\right]^{\alpha} L^{\beta}$$

differentiating

$$\frac{du}{dL} = \alpha\left[\frac{R(L)-A}{L}\right]^{\alpha-1}\left[\frac{R'(L)L-R(L)+A}{L^2}\right]L^{\beta}+\beta\left[\frac{R(L)-A}{L}\right]^{\alpha} L^{\beta-1} = 0$$

dividing by $\left[\frac{R(L)-A}{L}\right]^{\alpha-1}$ and $L^{\beta-1}$ yields

$$\alpha\left[\frac{R'(L)L-R(L)+A}{L^2}\right] L + \beta\left[\frac{R(L)-A}{L}\right] = 0$$

Using $1-\alpha=\beta$ and rearranging gives:

$$(1-\beta)R'(L)L + (2\beta-1)(R(L)-A) = 0 \quad \text{as required}$$

8 The interpretations given are of course not exhaustive.

9. The most intuitively straight forward way of verifying the comparative static results is to evaluate the polar cases $\alpha=0$, $\alpha=\beta$ and $\beta=0$ and then use the linearity of the efficiency loci to make the generalization.

10. This solution will only be approximately efficient when the efficiency loci are non-linear.

11. Generally we have $(w^*,L^*)=(\emptyset(w_1)+(1-\emptyset)(w_2),\emptyset(L_1)+(1-\emptyset)(L_2))$ where subscripts 1 and 2 refer to monopsonistic and monopolistic levels of wages and employment respectively.

12. To obtain the slope of the contract curve totally differentiate (6.2.9) to give:

$$\frac{dw}{dL} = \frac{u_w^{\,2} + u_{Lw}Lu_w + u_{ww}\,u_L L}{\left[2\,\frac{\partial p}{\partial x}\left(\frac{\partial x}{\partial L}\right)^2 + \frac{\partial^2 x}{\partial L^2}\left[\frac{\partial p}{\partial x}\,x(L)+p(x(L),H)\right]\right]u_w^{\,2}} \gtreqless 0$$

which can be negative if u_w is sufficiently large.

13. The diagram is as drawn since by (6.2.10) the contract curve must move downwards if u_w is small, hence the tangency point with each indifference curve must lie to the right implying that the isoprofit map must be flatter as drawn.

14. This suggests that the solution is sensitive to both the distance between the two opportunity cost curves

and also their location in bargaining space.

15. Presuming the good is normal.

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7. MACROECONOMIC IMPLICATIONS OF BARGAINING

7.1 Simple Single Sector Analysis

In this section labour market bargaining is introduced into a simple Malinvaud (1977), Barro and Grossman (1971) type macroeconomic model. The purpose of this analysis is to examine how wage rate endogenization by bargaining effects the properties of a macroeconomic model, particularly the comparative static effects of fiscal policy. The model will also provide clarification of some conjectures made by McDonald and Solow (1981) that bargaining may explain sluggish real wage adjustment.

A very simple specification of the economy will be adopted. It will be assumed that a representative firm employs some proportion of N identical workers to produce a homogeneous output. This output is purchased by the government and both employed and unemployed workers. Unemployed workers consume by spending an unemployment benefit wage paid by the government. Workers each supply one unit of labour inelastically and are represented in the bargaining process by a single representative union. The bargaining process is continuous and an efficient outcome is always achieved. All members of the labour force belong to the union.

Two cases are studied, in model 1 the price of output is fixed. In model 2 price adjustment ensures that the product market always clears.

(1) Fixed Product Price

The Firm: The firm is assumed a strict profit maximizer whose maximand may be written as (7.1.1).

$$\max_{w,L} \quad \pi = px - wL = F(\tilde{L}) - wL \tag{7.1.1}$$

$$\text{S.t} \qquad \tilde{L} \leq L \leq N$$

$F(\tilde{L})$ is the production function, $p=1$ is the normalized output price. L number of workers on the payroll $\tilde{L}$ number of workers actually used in production. N total working population. (7.1.1) implies that the firm may hoard labour L^H as (7.1.2)

$$L^H = L - \tilde{L} \tag{7.1.2}$$

The Union: The union seeks to maximize the sum of the utilities of its members as (7.1.3)[1]

$$\max_{w,L} \quad V = LV(w) + (N-L)V(\hat{w}) \tag{7.1.3}$$

where $V(w)$ is the indirect utility function of an employed worker and $V(\hat{w})$ that of an unemployed worker receiving benefit $\hat{w}$ from the government. There is no specific utility or disutility attached to employment per se.

Workers: The N identical workers each supply one unit of labour and demand output as (7.1.4)

$$x = \begin{cases} x(w) & \text{if employed} \\ x(\hat{w}), & \text{otherwise} \end{cases} \tag{7.1.4}$$

Government: The government purchases output g, it has its demand satisfied before workers are supplied, pays unemployment benefit $\hat{w}$ and levies a 100% profits tax. In the short-run it finances any deficits by printing money.

As in the standard fix-price literature reviewed earlier, there are several feasible regimes which may be identified as arising in this structure. Each equilibrium will be characterised by the level of employment and labour hoarding on the labour market and whether the goods market is supply or demand determined. Which possibility actually arises depends upon the levels of the exogenous variables $\hat{w}$, g, p, and the particular bargaining solution adopted. There are eight possible outcomes as characterised by figure (7.1.1).

Figure (7.1.1)

	Full Employment		Unemployment	
	LH	NLH	LH	NLH
ESG	2	*	1	3
EDG	X	4	X	5

* This is clearly a borderline case between regimes 2 and 4.

ESG and EDG indicate excess supply and demand for goods. LH and NLH indicate labour hoarding and no labour hoarding. Combinations of excess goods demand and labour hoarding are not feasible under any efficient bargaining solution. Hence, [illegible]arding borderline cases, there are five regimes of inter[illegible] bargaining, these are indicated 1-5 on figure (7.1.1). To examine these the Nash bargaining solution is adopted.

Keynesian Unemployment and Labour Hoarding

Here the levels of the exogenous variables are such that the goods market is demand determined and unemployment together with labour hoarding occurs upon the labour market.

The Nash solution requires the two bargainers to maximize the product of their utility surpluses over their opportunity costs. In the case of the firm the opportunity cost is $\pi=0$ and for the union $V=NV(\hat{w})$. Hence the joint maximand of the bargainers may be written as (7.1.5)

$$\max_{w,L} \text{Prod} = [\bar{x}-wL]\, L\, [V(w)-V(\hat{w})] \qquad (7.1.5)$$

differentiating

$$\frac{\partial \text{ prod}}{\partial L} = \bar{x}-2wL = 0 \qquad (7.1.6)$$

$$\frac{\partial \text{ prod}}{\partial w} = \bar{x}-wL - L\left[\frac{V(w)-V(\hat{w})}{V_w(w)}\right] = 0 \qquad (7.1.7)$$

where $\bar{x}$ is demand on the product market (7.1.8)

$$\bar{x} = Lx(w) + (N-L)\,x(\hat{w}) + g \qquad (7.1.8)$$

Thus the three equations (7.1.6) (7.1.7) and (7.1.8) define an equilibrium triple $\{w,L,\bar{x}\}$ for a given $\hat{w}$,N,p and g.

Expression (7.1.6) indicates that the solution requires revenue to be split equally between the two bargainers, the Nash is equivalent to a fair-shares solution or an equity locus as discussed in McDonald and Solow.

Using (7.1.6) to remove $\bar{x}$ from (7.1.7) and rearranging gives a surprising result

$$\frac{V(w)-V(\hat{w})}{V_w(w)} = w \qquad (7.1.9)$$

The real wage upon this regime depends only upon the unemployment benefit rate, if this is constant then the real wage does not respond to changes in demand, so in some sense, it is invariant over part of the cycle. McDonald and Solow working in a partial equilibrium structure conjecture this result, here it has been demonstrated rigorously as occurring upon this regime, and as we shall see does not appear elsewhere. The intuition behind the fixed wage is interesting, it arises in all specifications of the unions utility function which assume the unions marginal utility of employment to be a constant. This is because maximization here requires that the wage be chosen such that its marginal utility is just equal to that of employment, and then the level of employment is chosen to achieve the best distribution of revenue between the union and firm. The product of the two bargainers utility surpluses will thus be maximized.

Given (7.1.9) the comparative static properties of the equilibria follow easily. Eliminating $\bar{x}$ from (7.1.8) using (7.1.6) gives (7.1.10)

$$L = \frac{g + Nx(\hat{w})}{2w - x(w) + x(\hat{w})} \qquad (7.1.10)$$

differentiating W.R.T. g

$$\frac{dL}{dg} = \frac{1}{2w - x(w) + x(\hat{w})} > 0 \qquad (7.1.11)$$

Since the x functions refer to demands by individual workers and 2w is twice an individual workers income the standard assumption MPC$\leq$1 ensures dL/dg>0. An increase in government expenditure raises employment, and since w is invariant with respect to g wdL/dg may be interpreted as the national income multiplier on this regime.

Since output is demand determined the effect of the increase in government expenditure may be obtained from (7.1.6) and (7.1.11) to give

$$\frac{d\bar{x}}{dg} = \frac{2w}{2w - x(w) + x(\hat{w})} > 0 \qquad (7.1.12)$$

output upon this regime increases with government expenditure.

The comparative static effects of changes in unemployment benefit also follow straightforwardly. Differentiating (7.1.9) with respect to unemployment benefit $\hat{w}$ gives (7.1.13)

$$\frac{dw}{d\hat{w}} = - \frac{V_{\hat{w}}(\hat{w})}{V_{ww}(w)w} > 0 \qquad (7.1.13)$$

a rise in unemployment benefit raises wages.

From (7.1.6) and (7.1.8)

$$\frac{dL}{d\hat{w}} = \frac{L[x_w(w)-2]\frac{dw}{d\hat{w}} + (N-L)x_{\hat{w}}(\hat{w})}{2w - x(w) + x(\hat{w})} \qquad (7.1.14)$$

the effect of an increase in unemployment benefit upon employment is ambiguous, since the first term in the numerator of (7.1.14) is negative and the second positive, however, the second term is small when there is close to full employment and $dL/d\hat{w}$ will then be negative.

(7.1.14) and (7.1.6) give

$$\frac{dx}{d\hat{w}} = \frac{2L[x(\hat{w})-x(w)+wx_{\hat{w}}(\hat{w})]\frac{dw}{d\hat{w}} + (N-L)x_{\hat{w}}(\hat{w})}{2w - x(w) + x(\hat{w})} \qquad (7.1.15)$$

a sufficient condition for (7.1.15) to be positive is $x(\hat{w}) + wx_{\hat{w}}(\hat{w}) > x(w)^2$.

Thus a rise in unemployment benefit rate raises wages, probably raises output, but may reduce total employment. This is because the benefit increase has a direct effect upon product demand and an indirect effect through raising the unions opportunity cost and, hence, its allocation of revenue in the bargaining process.

The effect of a change in either government policy variable upon labour hoarding will be ambiguous in sign. Labour hoarding L^H may be defined as (7.1.16)

$$L^H = L - \bar{L}(\bar{x}) \qquad (7.1.16)$$

where $\tilde{L}(\bar{x})$ is the inverse production function. Hence the effects upon labour hoarding of changes in unemployment benefit and government expenditure are ambiguous, as (7.1.17) and (7.1.18) demonstrate.

$$\frac{dL^H}{d\bar{W}} = \frac{dL}{d\bar{W}} - \frac{\partial\tilde{L}}{\partial\bar{x}}\ \frac{\partial\bar{x}}{\partial\bar{W}} \qquad (7.1.17)$$

$$\frac{dL^H}{dg} = \frac{dL}{dg} - \frac{\partial\tilde{L}}{\partial\bar{x}}\ \frac{\partial\bar{x}}{\partial g} \qquad (7.1.18)$$

These expressions cannot be signed without further specification of the inverse production function.

If government expenditure and unemployment benefit rise with the result that the bargaining process yields full employment with labour hoarding then a second Keynesian regime is possible.

Keynesian Full-Employment with Labour Hoarding

If the product market remains demand determined at high levels of activity then the bargain may specify that all workers are employed even though not all are required to produce the output demanded. Here revenue is sufficiently high that the bargain requires full employment to distribute it between the union and the firm in the way that will maximize the product of their utilities.

Since full employment is assumed the firms maximand must be rewritten as (7.1.19)

$$\max_{w} \quad \pi = \bar{x} - wN \tag{7.1.19}$$

The unions maximand becomes (7.1.20)

$$\max_{w} \quad V = V(w) - V(\hat{w}) \tag{7.1.20}$$

and output demand will be of the simple form (7.1.21)

$$\bar{x} = N\ x(w) + g \tag{7.1.21}$$

In this case the bargain has only to determine the employment wage. From (7.1.19) and (7.1.20) the Nash objective function depends only upon the wage rate and may be expressed as (7.1.22)

$$\max_{w} \text{ Prod} = \left[\bar{x} - wN\right]\left[V(w) - V(\hat{w})\right] \tag{7.1.22}$$

differentiating

$$\frac{\partial \text{ prod}}{\partial w} = \left[\bar{x} - wN\right] V_w(w) - N\left[V(w) - V(\hat{w})\right] = 0 \tag{7.1.23}$$

Equations (7.1.21) and (7.1.23) define an equilibrium pair $\{\bar{x}, w\}$.

Full employment has two immediate implications. Firstly, revenue redistribution between the two bargainers can only be achieved through wage rate adjustments. Secondly, changes in the unemployment benefit level cannot have direct demand effects.

From (7.1.21) and (7.1.23) a little manipulation yields a solution equation in w, (7.1.24)

$$N\left[\frac{V(w) - V(\hat{w})}{V_w(w)}\right] = Nx(w) + g - wN \tag{7.1.24}$$

Differentiating with respect to $\hat{w}$, and g gives the comparative static effects of changes in the governments two policy variables.

$$\frac{dw}{dg} = \left[N \left(\frac{V_w(w)^2 - V_{ww}(w)\left[V(w)-V(\hat{w})\right] \quad -x_w(w)}{V_w(w)^2} + 1 \right) \right]^{-1} > 0 \qquad (7.1.25)$$

$$\frac{dw}{d\hat{w}} = \frac{V_w(w)^2 - V_{ww}(w)\left[V(w)-V(\hat{w})\right]}{V_{\hat{w}}(\hat{w})\, V_w(w)} > 0 \qquad (7.1.26)$$

Thus both increases in government expenditure and the unemployment benefit rate raise the wage rate. Increased government expenditure raises revenue, some of which accrues to the union through the only possible channel, wage increases. A rise in unemployment benefit lowers the unions utility surplus and requires a redistribution of revenue away from the firm to restore optimality to the bargain.

The effect of an increase in government expenditure on output is obtained from (7.1.21) and (7.1.25)

$$\frac{d\bar{x}}{dg} = N\, x_w(w) \frac{dw}{dg} + 1 > 0 \qquad (7.1.27)$$

and the effect of a rise in unemployment benefit upon output follows from (7.1.21) and (7.1.26)

$$\frac{d\bar{x}}{d\hat{w}} = N\, x_w(w) \frac{dw}{d\hat{w}} > 0 \qquad (7.1.28)$$

Output clearly rises with both policy variables since both imply an income increase, and output is demand determined. On this regime the effects of government policies upon labour hoarding are clear as (7.1.29) and (7.1.30) demonstrate.

$$\frac{dL^H}{d\hat{w}} = - \frac{\partial \tilde{L}}{\partial \bar{x}} N \, x_w(w) \frac{dw}{d\hat{w}} < 0 \qquad (7.1.29)$$

$$\frac{dL^H}{dg} = - \frac{\partial \tilde{L}}{\partial \bar{x}} \left[N \, x_w(w) \frac{dw}{dg} + 1\right] < 0 \qquad (7.1.30)$$

With full employment and labour hoarding increased demand will reduce hoarding as shown.

There are two interesting implications of the analysis of this regime. Firstly, full employment can occur when there is a shortfall in effective demand. Secondly, by comparing this analysis with the other Keynesian regime it can be seen that wage variability depends crucially upon whether or not the system is at a full-employment equilibrium. Unemployment is required for wage rigidity.

If, starting in a Keynesian unemployment labour hoarding situation, demand increases had eradicated labour hoarding before full employment was reached, then what might be described as a semi-neoclassical regime would obtain.

Semi-Neoclassical

On this regime the production function determines the level of employment such that product demand may be satisfied, bargaining simply requires that w is chosen to maximize (7.1.31)

$$\max_{w} \quad \text{Prod} = [F(L)-wL]L[V(w)-V(\hat{w})] \qquad (7.1.31)$$

The first order condition is (7.1.32)

$$\frac{V(w)-V(\hat{w})}{V_w(w)} = \frac{F(L)-wL}{L} \qquad (7.1.32)$$

and upon this regime output equals demand as (7.1.33)

$$F(L) = Lx(w) + (N-L)x(\hat{w}) + g \tag{7.1.33}$$

(7.1.32) and (7.1.33) define an equilibrium pair {w,L} upon this regime.

To examine the comparative statics of this regime, (7.1.32) and (7.1.33) are totally differentiated and rearranged in matrix form to give (7.1.34)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dw \\ dL \end{bmatrix} = \begin{bmatrix} b_1 d\hat{w} \\ b_2 d\hat{w} + dg \end{bmatrix} \tag{7.1.34}$$

where

$$a_{11} = 2 - \frac{V_{ww}(w)[V(w)-V(\hat{w})]}{V_w(w)^2} > 0$$

$$a_{12} = -\left[\frac{[F_L(L)-w]L - F(L) + wL}{L^2}\right] > 0$$

$$a_{21} = -Lx_w(w) < 0$$

$$a_{22} = F_L(L) - x(w) + x(\hat{w}) \gtreqless 0$$

$$b_1 = \frac{V_{\hat{w}}(\hat{w})}{V_w(w)} > 0$$

$$b_2 = (N-L)x_{\hat{w}}(\hat{w}) > 0$$

The sign of the determinant of the system (7.1.34) depends upon the sign of the element a_{22}. Upon this regime it is easy to see that $a_{22} > 0$. Consider a small change in $\hat{w}$, $d\hat{w}$, which just increases employment by one worker, the extra output he produces will be $F_L(L)$, and the extra demand he expresses will be $x(w)-x(\hat{w})$. However, all other unemployed workers will now demand more output $[x(\hat{w}+d\hat{w}) - x(\hat{w})][N-L]$. Then $F_L(L) = x(w)-x(\hat{w}) + [x(\hat{w}+d\hat{w}) - x(\hat{w})][N-L]$ so $F_L(L) > x(w)-x(\hat{w})$ and $a_{22} > 0$. Then $\det[a] > 0$.

The comparative static effects of changes in government expenditure and unemployment benefit follow by Cramers rule. Consider first changes in government expenditure.

$$\frac{dw}{dg} = \frac{\begin{vmatrix} 0 & a_{12} \\ 1 & a_{22} \end{vmatrix}}{|a|} > 0 \qquad (7.1.35)$$

$$\frac{dL}{dg} = \frac{\begin{vmatrix} a_{11} & 0 \\ a_{12} & 1 \end{vmatrix}}{|a|} > 0 \qquad (7.1.36)$$

$$\frac{dx}{dg} = F_L(L)\,\frac{dL}{dg} > 0 \qquad (7.1.37)$$

Provided $a_{22} > 0$ increases in government expenditure raise wages output and employment. The effects of changes in unemployment benefit follow in the same manner.

$$\frac{dw}{d\hat{w}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{|a|} < 0 \text{ if } b_1a_{22} < b_2a_{12} \qquad (7.1.38)$$

$$\frac{dL}{d\hat{w}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{|a|} > 0 \qquad (7.1.39)$$

$$\frac{dx}{d\hat{w}} = F_L(L)\frac{dL}{d\hat{w}} > 0 \qquad (7.1.40)$$

Hence, increases in unemployment benefit unambiguously raise employment and output due to the demand effects, the effect upon the wage rate is ambiguous, however, when $N = L$ then $b_2 \approx 0$ i.e., when the system is close to full employment, then $dw/d\hat{w} > 0$, or when there is a lot of unemployment $dw/d\hat{w} < 0$.

Further expansion of demand would cause supply to constrain the product market at full employment output, giving a Repressed Inflation regime.

Repressed Inflation

With no labour hoarding and full employment the product market will be supply determined and workers will be rationed for their goods demand.

Hence the firms maximand must be rewritten as (7.1.41)

$$\underset{w}{\text{Max}}\ \pi = F(N) - wN \qquad (7.1.41)$$

and the unions maximand must also be rewritten to describe the goods market rationing faced by its members.

$$\text{Max } V = N\left[V(w, x/N) - V(\hat{w})\right] \qquad (7.1.42)$$

since workers are identical the assumption of a simple proportional rationing scheme seems appropriate.[3]

Total output supply to consumers will be full employment output less government purchases as (7.1.43)

$$x = F(N) - g \qquad (7.1.43)$$

(7.1.43) has the immediate implication that $dx = -dg$, any increase in government purchases reduces output available for consumers.

The joint maximand of the two bargainers may now be written as (7.1.44)

$$\max_{w} \text{Prod} = [F(N)-wN]N[V(w,x/N) - V(\hat{w})] \qquad (7.1.44)$$

maximizing (7.1.44)

$$\frac{\partial \text{ prod}}{\partial w} = [F(N)-wN]N\ V_w(w,x/N)-N^2[V(w,x/N)-V(\hat{w})] = 0 \qquad (7.1.45)$$

hence

$$[F(N)-wN]V_w(w,x/N)-N[V(w,x/N)-V(\hat{w})] = 0 \qquad (7.1.46)$$

Given that the regime obtains the equilibrium is defined by any value of w which satisfies (7.1.46).

Changes in government expenditure effect the goods ration faced by individual workers and hence the marginal value of the employment wage. Using the fact that $dx = -dg$ we may differentiate (7.1.46) to obtain the comparative static effect of a government expenditure change (7.1.47)[4]

$$\frac{dw}{dg} = -\frac{dw}{dx} = -\frac{1}{N}\left(\frac{V_x(w,x/n) - [F(N)-wN]\ V_{wx}(w,x/N)}{[F(N)-wN]V_{ww}(w,x/N)-2NV_w(w,x/N)}\right) > 0 \qquad (7.1.47)$$

As the marginal utility of the wage falls with a tightening of the goods ration the bargain raises the wage rate to compensate the union for some of the utility loss.

Changes in the unemployment benefit level effect the unions utility surplus and effect the wage rate via the bargain as (7.1.48).

$$\frac{dw}{d\hat{w}} = \frac{-NV_{\hat{w}}(\hat{w})}{[F(N)-wN]V_{ww}(w,x/N)-2NV_w(w,x/N)} > 0 \qquad (7.1.48)$$

The wage rate rises to achieve the necessary revenue redistribution between the two bargaining agents.

On this regime any increase in government expenditure or unemployment benefit will only give rise to wage inflation. A further regime is possible, when government expenditure is relatively high, the product market supply determined, and the marginal product of labour low, then the bargain may yield unemployment and goods rationing. This may be termed a Neoclassical regime.

Neoclassical Regime

With no labour hoarding, unemployment and rationing of consumers upon the goods market the Neoclassical regime may seem an unlikely possibility. Its existence is a consequence of the bargaining solution. The failure of the system to employ more workers when there is excess demand for their output, is possible because although increases in employment raise the unions utility they will cut the firms profit significantly if wages are high and the marginal product of labour is low. Maximization of the product of the two bargainers utility surpluses may prevent increases in employment. There will of course be no labour hoarding.

On such a regime the firms maximand is as (7.1.49)

$$\max_{w,L} \pi = F(L)-wL \tag{7.1.49}$$

and the unions maximand becomes (7.1.50)

$$\max_{w,L} V = LV(w,\bar{n}) + (N-L)\ V(\hat{w}) \tag{7.1.50}$$

where $\bar{n}$ is defined as output available for consumption by an individual worker, the goods ration of an employed worker. It is assumed that unemployed workers have a low output demand and experience no rationing upon the product market. Hence $\bar{n}$ may be defined as (7.1.51).

$$\bar{n} = \frac{F(L) - g - (N-L)x(\hat{w})}{L} \tag{7.1.51}$$

The Nash bargaining maximand may now be defined as (7.1.52)

$$\max_{w,L} \text{Prod} = [F(L)-wL]L[V(w,\bar{n}) - V(\hat{w})] \tag{7.1.52}$$

The first order conditions for maximization are:

$$\frac{\partial \text{ Prod}}{\partial w} = [F(L)-wL]LV_w(w,\bar{n}) - L^2[V(w,\bar{n})-V(\hat{w})]=0 \tag{7.1.53}$$

$$\frac{\partial \text{ Prod}}{\partial L} = [F(L)-2wL + F_L(L)L][V(w,\bar{n})-V(\hat{w})]=0 \tag{7.1.54}$$

Equations (7.1.51), (7.1.53) and (7.1.54) define a Neoclassical equilibrium triple $\{w,L,\bar{n}\}$.

The comparative statics of this regime are somewhat more complex than the previous four, however the system may be simplified by noting that (7.1.54) implies w and L must be chosen such that $F(L)-2wL+F_L(L)L = 0$ [5], replacing (7.1.54) by this expression, and rearranging the other two conditions the regime can be described by (7.1.55-57).

$$F(L)-wL = L\left[\frac{V(w,\bar{n})-V(\hat{w})}{V_w(w,\bar{n})}\right] \tag{7.1.55}$$

$$F(L)-2wL+F_L(L)L = 0 \tag{7.1.56}$$

$$L\bar{n} = F(L)-g-(N-L)x(\hat{w}) \tag{7.1.57}$$

To examine the comparative statics of the regime the system (7.1.55-57) is totally differentiated and rewritten in matrix form to give (7.1.58)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dw \\ dL \\ d\bar{n} \end{bmatrix} = \begin{bmatrix} -[\dfrac{V_{\hat{w}}(\hat{w})L}{V_w(w,\bar{n})}]d\hat{w} \\ 0 \\ -(N-L)x_{\hat{w}}(\hat{w})d\hat{w}-dg \end{bmatrix} \tag{7.1.58}$$

where

$$a_{11} = -L\left[1 + \left[\frac{V_w(w,\bar{n})^2-V_{ww}(w,\bar{n})(V(w,\bar{n})-V(\hat{w}))}{V_w(w,\bar{n})^2}\right]\right] < 0$$

$$a_{12} = F_L(L) - w - \left[\frac{V(w,\bar{n})-V(\hat{w})}{V_w(w,\bar{n})}\right] < 0$$

$$a_{13} = L\left[\frac{V_{\bar{n}}(w,\bar{n})V_w(w,\bar{n})-V_{w\bar{n}}(w,\bar{n})[V(w,\bar{n})-V(\hat{w})]}{V_w(w,\bar{n})^2}\right] \gtrless 0$$

$$a_{21} = -2L < 0$$

$$a_{22} = 2F_L(L) - 2w + F_{LL}(L)L < 0$$

$$a_{23} = 0$$

$$a_{31} = 0$$

$$a_{32} = \bar{n} - F_L(L) - x(\bar{w}) \gtrless 0$$

$$a_{33} = L > 0$$

Assuming $a_{13}<0$ and $\det[a]>0$ [6] the comparative static effects of changes in government expenditure and unemployment benefit follow by Cramers rule. Consider changes in government expenditure.

$$\frac{dw}{dg} = -\frac{\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}}{|a|} > 0 \qquad (7.1.59)$$

$$\frac{dL}{dg} = -\frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{|a|} > 0 \qquad (7.1.60)$$

$$\frac{d\bar{n}}{dg} = -\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{|a|} \gtrless 0 \qquad (7.1.61)$$

An increase in government expenditure reduces output available to workers, the goods ration tightens. The marginal value of the wage for employed workers and hence the union falls, which suggests as indicated by (7.1.59) and (7.1.60) that wages and employment must rise to achieve the correct sharing of the utility loss between the union and firm.[7] Although (7.1.61) cannot be signed it would seem reasonable to argue that the direct effect of the increase in government expenditure tightening the consumers goods ration offsets any increase in output arising from the employment increase required to obtain the optimal sharing of utility loss between the two bargainers.

The comparative static effects of a change in the unemployment benefit rate are obtained in the same manner, writing b_1 and b_2 for the coefficients of $d\bar{w}$ gives

$$\frac{dw}{d\bar{w}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ o & a_{22} & a_{23} \\ b_2 & a_{32} & a_{33} \end{vmatrix}}{|a|} > 0 \qquad (7.1.62)$$

$$\frac{dL}{d\bar{w}} = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & o & a_{23} \\ a_{31} & b_2 & a_{33} \end{vmatrix}}{|a|} < 0 \qquad (7.1.63)$$

$$\frac{d\bar{n}}{d\bar{w}} = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & o \\ a_{31} & a_{32} & b_2 \end{vmatrix}}{|a|} < 0 \text{ if } |a_{11}a_{22}b_2+a_{21}a_{32}b_1|>|a_{21}a_{12}b_2) \qquad (7.1.64)$$

The effect of a rise in the unemployment benefit rate reduces the unions utility surplus. A wage rate rise and an employment level fall as (7.1.62) and (7.1.63) are required to achieve the correct redistribution of the loss between the two bargainers. The ambiguity of the effect of the benefit increase upon the goods ration arises because only employed workers are assumed to have sufficient income to encounter rationing, thus although there is clearly less output available for employed workers to consume, there are also fewer employed workers and their share of available goods will rise.

These are summarized in table (7.1.1).

Table (7.1.1)

	REGIME				
	K^u	K^F	S	R|I	N
$\frac{dL}{d\hat{w}}$	*		+		-[1]
$\frac{dx}{d\hat{w}}$	*	+	+		-[1]
$\frac{dw}{d\hat{w}}$	+	+	*	+	+[1]
$\frac{dL^H}{d\hat{w}}$	*	-			
$\frac{dL}{dg}$	+		+		+[1]
$\frac{dx}{dg}$	+	+	+		+[1]
$\frac{dw}{dg}$	0	+	-	*	+[1]
$\frac{dL^H}{dg}$	*	-			

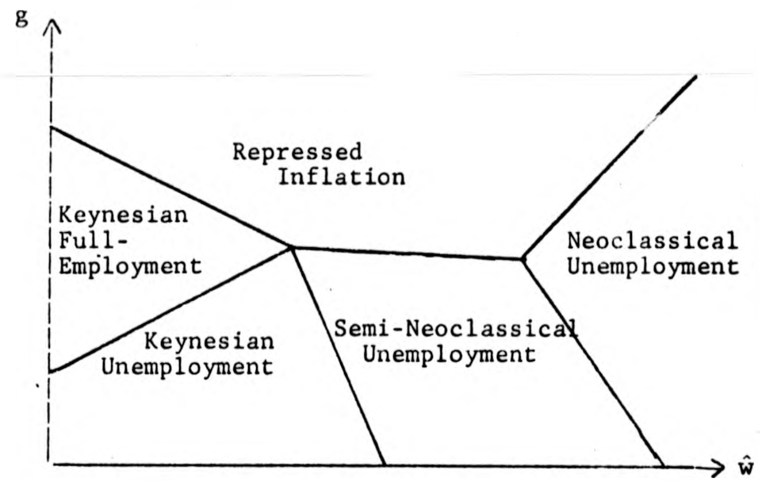
*See text for conditions for signing these expressions.

[1] $V_{\bar{n}}(w,\bar{n})V_w(w,\bar{n}) - V_{w\bar{n}}(w,\bar{n})[V(w,\bar{n}) - V(\hat{w})] < 0$ and

$\bar{n} - F_L(L) - x(\hat{w}) > 0$ are necessary for these results.

Using the comparative static properties of the model and the regime definitions, the regimes may be represented diagramatically in w,g space as figure (7.1.2).

Figure (7.1.2)



The model displays several interesting properties. Increases in government expenditure raise both employment and output upon the Keynesian regime without giving wage inflation. Upon the Keynesian full employment regime demand increases generated by either an increase in government expenditure or unemployment benefit give rise to wage inflation together with an output increase obtained by a reduction in labour hoarding. On the repressed inflation regime no output effects are possible since there is full employment, both government expenditure and unemployment benefit increases, yield wage inflation. On the semi-Neoclassical regime, government expenditure increases raise output and employment but wages actually fall as a consequence of the bargaining solution. Unemployment benefit increases raise both output and employment, but have an ambiguous effect upon wages. On the Neoclassical regime government expenditure increases raise output, employment and wages, however an upward revision of unemployment benefit reduces output and employment but causes wage inflation

(2) <u>Flexible Product Price</u>

If price adjustment clears the product market as in Dixit (1976) and Bliss (1974), then the unions utility function must be rewritten as (7.1.65)

$$\max_{w,L} V = LV(w,p) + (N-L)\ V(\hat{\omega},p) \tag{7.1.65}$$

and the firms maximand as (7.1.66)

$$\max_{w,L} \pi = pF(L)-wL \tag{7.1.66}$$

it will be assumed that the firm is a price taker and that price is determined by the product market equilibrium condition (7.1.67).

$$x = L\ x(w,p) + (N-L)\ x(\hat{\omega},p) + g = F(L) \tag{7.1.67}$$

Since the firm cannot be demand constrained and is a price taker all labour employed will be used in production of output. No labour hoarding can occur. An equilibrium may be defined by (7.1.67) and the solution to the bargaining problem. Again adopting the Nash bargaining solution, the joint maximand may be written as (7.1.68)

$$\max_{w,L} \text{Prod} = [px-wL]L[V(w,p)-V(\hat{w},p)] \tag{7.1.68}$$

maximizing (7.1.68)

$$\frac{\partial \text{ prod}}{\partial w} = px-wL-L\left[\frac{V(w,p)-V(\hat{w},p)}{V_w(w,p)}\right] = 0 \tag{7.1.69}$$

$$\frac{\partial \text{ prod}}{\partial L} = px-2wL = 0 \tag{7.1.70}$$

rearranging (7.1.69) and (7.1.70)

$$pF(L)-wL = L\left[\frac{V(w,p)-V(\hat{w},p)}{V_w(w,p)}\right] \tag{7.1.71}$$

$$pF(L) = 2wL \tag{7.1.72}$$

Thus (7.1.67) (7.1.71) and (7.1.72) define an equilibrium n-tuple $\{w,p,L,x\}$. To obtain the comparative static properties of the model we totally differentiate the equilibrium conditions to give (7.1.73).

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} dL \\ dw \\ dp \end{bmatrix} = \begin{bmatrix} b_1 d\hat{w} \\ 0 \\ b_3 d\hat{w} - dg \end{bmatrix} \tag{7.1.73}$$

where

$$a_{11} = pF_L(L) - w - \left[\frac{V(w,p)-V(\hat{w},p)}{V_w(w,p)}\right] < 0$$

$$a_{12} = -L - \left[1-\frac{V_{ww}(w,p)\,[V(w,p)-V(\hat{w},p)]}{V_w(w,p)^2}\right] < 0$$

$$a_{13} = F(L)-\left[\frac{[V_{\bar{n}}(w,p)-V_p(w,p)]\,V_w(w,p)-V_{wp}(w,p)\,[V(w,p)-V(\hat{w},p)]}{V_w(w,p)^2}\right] \gtreqless 0$$

$$a_{21} = pF_L(L) - 2w < 0$$

$$a_{22} = -2L < 0$$

$$a_{23} = F(L) > 0$$

$$a_{31} = x(w,p) - x(\hat{w},p) - F_L(L) \gtreqless 0$$

$$a_{32} = x_w(w,p)L > 0$$

$$a_{33} = L\, x_p(w,p) + (N-L)x_p(\hat{w},p) > 0$$

$$b_1 = \frac{V_{\hat{w}}(\hat{w})}{V_w(w,p)} < 0$$

$$b_3 = -(N-L)x_{\hat{w}}(\hat{w},p) < 0$$

Assuming a_{13}, $a_{31} < 0$ then det $[a] > 0$ if $|a_{11}a_{22}a_{33}| <$ $a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{21}a_{12}a_{33} - a_{32}a_{23}a_{11}$: inspection suggests that this will not be a strong assumption to make.

Using Cramer's rule the comparative statics of the model follow.

$$\frac{dL}{dg} = -\frac{\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}}{|a|} > 0 \qquad (7.1.74)$$

$$\frac{dx}{dg} = -F_L(L)\frac{dL}{dg} > 0 \qquad (7.1.75)$$

$$\frac{dw}{dg} = -\frac{\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}}{|a|} > 0 \qquad (7.1.76)$$

$$\frac{dp}{dg} = -\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{|a|} \gtreqless 0 \text{ iff } a_{11}a_{22} \lesseqgtr a_{21}a_{12} \qquad (7.1.77)$$

Thus an increase in government expenditure raises employment and output and gives rise to wage inflation. The ambiguity of the effect of an increase in government expenditure upon the price level arises since employment and hence output supply increase for two reasons, firstly in response to the demand increase, and secondly to redistribute utility between the two bargainers. The large output increase may give rise to an eventual fall in price upon the product market, the possibility that fiscal expansions will be deflationary is an interesting consequence of introducing wage/employment bargaining. This may be highlighted by totally differentiating (7.1.72) and dividing by dg to give:

$$\frac{dp}{dg} = \frac{2L}{F(L)}\frac{dw}{dg} + \left[\frac{2w - pF_L(L)}{F(L)}\right]\frac{dL}{dg} \tag{7.1.78}$$

which suggests that fiscal expansion will have a price deflationary effect when the initial equilibrium is characterised by a low level of employment, wages and output. The comparative static effects of a change in unemployment benefit follow by the same method.

$$\frac{dL}{d\bar{w}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|a|} > 0 \tag{7.1.79}$$

$$\frac{dx}{d\bar{w}} = F_L(L)\frac{dL}{d\bar{w}} > 0 \tag{7.1.80}$$

$$\frac{dw}{d\bar{w}} = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|a|} > 0 \quad \text{iff } |a_{21}b_3a_{13} - b_3a_{23}a_{11}| < b_1a_{23}a_{31} + a_{21}b_1a_{33} \tag{7.1.81}$$

$$\frac{dp}{d\bar{w}} = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|a|} > 0 \quad \text{iff } |a_{11}a_{22}b_3| < a_{21}a_{22}b_1 - a_{21}a_{22}b_1 - a_{21}a_{12}b_3 \tag{7.1.82}$$

The increase in unemployment benefit raises employment and output unambiguously, its impact on wages and prices depends essentially upon the bargain. If the rise in employment implied in (7.1.79) is large, because of both the demand effect of a change in $\hat{\omega}$ and a requirement for a redistribution of the extra revenue between the two bargainers, then a fall in the employed wage may be required for this optimal revenue distribution. Large output increases may reduce the price of output because employment rises due to both demand and 'bargaining' effects.

7.2 A Two Sector Flex-Price Bargaining Model

The preceding section examined the introduction of bargaining into simple Malinvaud (1977) and Dixit (1976) type disequilibrium models. In this section the flexible price model will be extended to include a second production sector. The important characteristic of the second sector will be that it hires labour upon a competitive non-unionised labour market.

In this more complex world the informational requirements and costs of bargaining are higher, consequently it may be argued that the process occurs at discrete intervals rather than continuously. This assumption, which has the additional advantage of improving the tractability of the analysis, will be made.[8] Bargains are struck upon the basis of expectations and revised periodically in the light of experience. Interest will thus be focused upon the impact of government policy under a given wage employment bargain, and the way in which bargains will be revised when expectations are proven incorrect.

Initially two further simplifying assumptions will be made. Both labour and capital will be assumed inter-sectorally immobile in the short run.[9]

(1) The Unionised Production Sector

In this sector a given labour pool of workers, who inelastically supply one unit of labour each, are represented by a typical union in wage employment bargaining with a representative firm. Since the price of output is assumed sufficiently flexible to always clear the output market and since individual firms are price takers all labour that the bargain specifies should be employed will actually be productive. There will be no labour hoarding.

The Firm: Is a strict profit maximizer whose miximand may be written as (7.2.1)

$$\max_{w_1, L_1} \ \pi_1 = p_1 F(L_1) - w_1 L_1 \tag{7.2.1}$$

$$\text{S.t} \quad L_1 \leq N$$

where N is the total labour pool of the unionised sector.

The Union: The union seeks to maximize the sum of its members utilities as (7.2.2)

$$\max_{w_1, L_1} \ V = L_1\, V(w_1, p_1, p_2) + (N - L_1)\, V(\hat{\omega}, p_1, p_2) \tag{7.2.2}$$

Since labour is immobile in the short-run workers may be either employed in the sector, receiving the wage w_1 or unemployed receiving unemployment benefit $\hat{\omega}$. The unions maximand is independent of the wage rate of the competative sector.

<u>Workers</u>: Each of the N workers supply one unit of labour inelastically at all wage rates which yield higher utility than the unemployment benefit level. They purchase output from both production sectors as (7.2.3) and (7.2.4).

$$x_1 = \begin{cases} x_1(w_1, p_1, p_2) & \text{if employed} \\ x_1(\hat{w}, p_1, p_2) & \text{otherwise} \end{cases} \qquad (7.2.3)$$

$$x_2 = \begin{cases} x_2(w_1, p_1, p_2) & \text{if employed} \\ x_2(\hat{w}, p_1, p_2) & \text{otherwise} \end{cases} \qquad (7.2.4)$$

<u>Government</u>: Purchases output from both production sectors, g_1 and g_2, and has first claim on output. It pays unemployed workers benefit $\hat{w}$ and levies a 100% profits tax. If its budget is in deficit it finances its excess expenditure by printing money.

The determination of the level of employment and the wage rate in the unionised sector is achieved through bargaining. Again adopting the Nash solution, the maximand is as (7.2.5)

$$\text{Max Prod} = \left[V(w, p_1, p_2) - V(\hat{w}, p_1, p_2)\right] L\left[p_1 F(L_1) - w_1 L_1\right] \qquad (7.2.5)$$

With a little simplification, the first order conditions of (7.2.5) may be written as (7.2.6) and (7.2.7).

$$\frac{\partial \text{ Prod}}{\partial w} = p_1 F(L_1) - w_1 L_1 - \left[\frac{V(w_1, p_1, p_2) - V(\hat{w}, p_1, p_2)}{V_{w_1}(w_1, p_1, p_2)}\right] L = 0 \qquad (7.2.6)$$

$$\frac{\partial \text{ Prod}}{\partial L_1} = p_1\left[F(L_1) + F_{L1}(L_1)\right] - w_1(1 + L_1) = 0 \qquad (7.2.7)$$

(7.2.6) and (7.2.7) determine w_1 and L_1 for given values of w_2, p_1 and p_2; since it is assumed that the bargain is struck periodically it will be efficient while the values of w_2, p_1 and p_2 upon which it was based persist. Two interpretations are possible here, either the firm and union are myopic in which case w_2, p_1, and p_2 are the current values of prices, or the bargainers predict these prices and base the bargain upon expectations. For simplicity it will be assumed both bargainers share the same expectations.

(2) <u>The Non-Unionised Production Sector</u>

In this sector there is no bargaining. A representative competative firm treats both wages and prices as parametric and employs some or all of the R workers in the labour pool so as to maximize its profit.

<u>The Firm</u>: Maximizes its profit subject to a labour supply constraint as (7.2.8)

$$\max_{L_2} \pi_2 = p_2 G(L_2) - w_2 L_2 \tag{7.2.8}$$

$$\text{S.t} \quad L_2 \leq R$$

From which we obtain a labour demand function (7.2.9)

$$L_2 = L_2(w_2, p_2) \tag{7.2.9}$$

and a goods supply function given by the production function.[10]

$$x_2 = G(L_2) \tag{7.2.10}$$

<u>Workers</u>: Maximize the utility gained from the consumption of output from the two production sectors. Giving rise to demands (7.2.11) and (7.2.12)

$$x_1 = \begin{cases} x_1(w_2, p_1, p_2) & \text{if employed} \\ x_1(\hat{w}, p_1, p_2) & \text{otherwise} \end{cases} \tag{7.2.11}$$

$$x_2 = \begin{cases} x_2(w_2, p_1, p_2) & \text{if employed} \\ x_2(\hat{w}, p_1, p_2) & \text{otherwise} \end{cases} \tag{7.2.12}$$

The analysis of the two production sectors may now be combined to examine the behavioural properties of the model. Two equilibrium concepts may perhaps be identified in this context.

A short-run equilibrium in which w_1 and L_1 are fixed by the bargain and other variables adjust to establish an equilibrium which will persist only as long as the period between bargains being struck if the bargain is based upon incorrect expectations. A medium-run equilibrium arises if expectations upon which the bargain is based prove correct.

The scenario which will be studied below is as follows. It will be assumed that the economy is initially in a medium-run equilibrium, the government may thus use its policy instruments to attempt to achieve policy objectives. The bargain may not adjust until it is next due for renegotiation, consequently the short-run impact of government policy effects only variables outside the bargain. Once the bargain is again restruck the change in the economic environment gives rise to an adjustment of wages and employment in the unionised sector and the medium-run effects of government policy measures are felt.

A medium-run equilibrium in the model may be described by equations (7.2.13-17).

$$F(L_1)=L_1x_1(w_1,p_1,p_2)+L_2x_1(w_2,p_1,p_2)+(N+R-L_1-L_2)x_1(\hat{w}_1p_1,p_2)+g_1 \tag{7.2.13}$$

$$G(L_2)=L_1x_2(w_1,p_1,p_2)+L_2x_2(w_2,p_1,p_2)+(N+R-L_1-L_2)x_2(\hat{w},p_1,p_2)+g_2 \tag{7.2.14}$$

$$L_2(w_2,p_2) \leq R \tag{7.2.15}$$

$$p_1\left[F(L_1)+F_{L_1}(L_1)\right] - w_1(1+L_1) = 0 \tag{7.2.16}$$

$$p_1F(L_1) - w_1L_1 - \left[\frac{V(w_1,p_1,p_2)-V(\hat{w},p_1,p_2)}{V_{w_1}(w_1,p_1,p_2)}\right] L = 0 \tag{7.2.17}$$

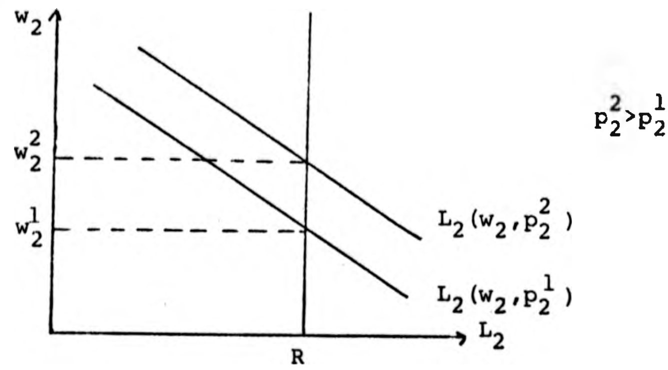
Expression (7.2.13) and (7.2.14) are the product market equilibrium conditions for the two sectors outputs, (7.2.16) and (7.2.17) are the distribution and efficiency conditions from the Nash bargaining solution. Condition (7.2.15) states that the labour market in the non-unionised sector will either be demand determined or will achieve full employment R. This suggests there are four medium-run equilibrium configurations characterised by the level of employment in each of the two sectors. The possibilities are:

(1)	$L_1 = N$, $L_2 = R$	Full employment in both sectors
(2)	$L_1 = N$, $L_2 < R$	Unemployment in the non-unionised sector
(3)	$L_1 = N$, $L_2 = R$	Unemployment in the unionised sector
(4)	$L_1 < N$, $L_2 < R$	Unemployment in both sectors.

There is no role for government policy in (1) since in this case all resources are fully utilised.[11] In the short-run there are thus two possibilities $L_2=R$, $L_1<N$ and $L_2<R$ combined with either $L_1=N$ or $L_1<N$ since both w_1 and L_1 are fixed in the short-run.

If full employment obtains upon the competitive labour market, then both sectors of the economy are capacity constrained. If the government raises either its purchases of the good or unemployment benefit this will simply bid up the prices of the two goods and raise the competative wage as indicated in figure (7.2.1)

Figure (7.2.1)

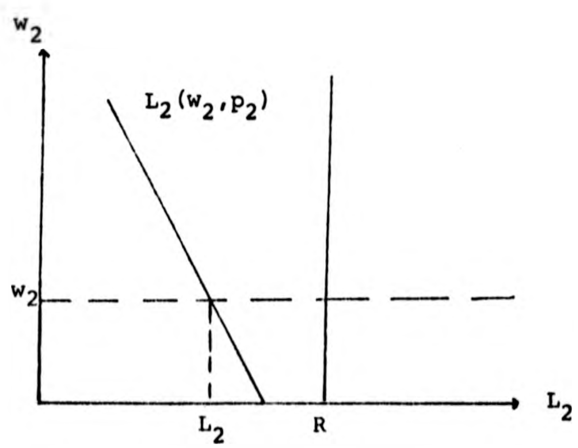


A more interesting case arises when there is unemployment upon the competative labour market, which represents a case of market failure since there does not exist an equilibrium competative wage. Define the minimum wage which will induce workers to supply labour as (7.2.18)

$$w_2 = \hat{w} \tag{7.2.18}$$

The competitive labour market may thus be described by figure (7.2.2).

Figure (7.2.2)



The intuition behind this possibility is that the wage employment bargain of the unionised sector provides very low levels of both wages and employment. The level of effective demand in the economy may fail to bid up the price of the good produced in the non-unionised sector, which may subsequently lead to a failure of the competitive labour market. The necessary government policy to remove this unemployment must give rise to an increase in the price p_2 to move the labour demand curve rightwards. An increase in government purchases g_2 is the most obvious measure, however an increase in g_1 may also be very effective if the two goods are close substitutes. Determination of the correct expenditure policy to remove unemployment $R-L_2$ will depend upon the shapes of the demand and Engel curves of the three groups of consuming workers. Adjustment of the level of unemployment benefit may also be used to remove unemployment. In the scenario suggested by figure (7.2.2) no cut in $\hat{w}$ can remove all the unemployment and indeed may increase it if the demand effects cause a large leftwards shift in the labour demand curve. A rise in unemployment benefit will increase demand for the good, bidding up its price, and may produce a sufficiently large rightward shift in the labour demand curve to remove the unemployment. This is particularly likely if there is unemployment in the unionised sector, since the increase in the unemployment benefit rate will increase income by $d\bar{w}(N-L_1)$ which will have purely expansionary effects upon p_2 and hence L_2.

The medium run effects of government policy will occur when the bargain is restruck to incorporate the changes that have occurred in p_1, p_2 and $\hat{w}$. Totally differentiating (7.2.16) and (7.2.12) and rearranging in matrix form yields (7.2.18).

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dw_1 \\ dL_1 \end{bmatrix} = \begin{bmatrix} b_1 dp_1 \\ b_{21} dp_1 + b_{22} dp_2 + b_{23}\, d\hat{w} \end{bmatrix} \quad (7.2.18)$$

where

$$a_{11} = -(1+L_1) < 0$$

$$a_{12} = p\left[F_{L_1}(L_1) + F_{L_1L_1}(L_1)\right] > 0$$

$$a_{21} = -L- \left[\frac{V_{w_1}(w_1,p_1,p_2)^2 - V_{w_1w_1}(w_1,p_1,p_2)\left[V(w_1,p_1,p_2)\right.}{V_{w_1}(w_1,p_1,p_2)^2}\right.$$

$$\left. \left. - V(w_1,p_1,p_2)\right]\right] < 0$$

$$a_{22} = -w_1 - \left[\frac{V(w_1,p_1,p_2) - V(\hat{w},p_1,p_2)}{V_{w_1}(w_1,p_1,p_2)}\right] < 0$$

$$b_1 = -\left[F(L_1) + F_{L_1}(L_1)\right] < 0$$

$$b_{21} = -F(L_1) + \left[\frac{\left[V_{p_1}(w_1,p_1,p_2) - V_{p_1}(\hat{w},p_1,p_2)\right]V_{w_1}(w_1,p_1,p_2)}{V_{w_1}(w_1,p_1,p_2)^2}\right.$$

$$\left. - \frac{V_{w_1p_1}(w_1,p_1,p_2)\left[V(w_1,p_1,p_2) - V(\hat{w},p_1,p_2)\right]}{V_{w_1}(w_1,p_1,p_2)^2}\right]$$

$$b_{22} = \frac{\left[V_{p_2}(w_1,p_1,p_2) - V_{p_2}(\hat{w}_1 p_1,p_2)\right]V_{w_1}(w_1,p_1,p_2)}{V_{w_1}(w_1,p_1,p_2)^2}$$

$$- \frac{V_{w_1p_2}(w_1,p_1,p_2)\left[V(w_1,p_1,p_2) - V(\hat{w},p_1,p_2)\right]}{V_{w_1}(w_1,p_1,p_2)^2} > 0$$

$$b_{23} = -L\left[\frac{V_{\hat{w}}(\hat{w},p_1,p_2)}{V_{w_1}(w_1,p_1,p_2)}\right] < 0$$

Hence the determinant of the system $|a|>0$.

If we assume $b_{21}<0$ and that the government has pursued an expansionary policy raising both expenditure and unemployment benefit since the last bargain was struck, such that dp_1 and dp_2 are positive, then the influences upon the bargain and hence the medium run characteristics of the model are as follows:

$$\frac{dw_1}{dp_1} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_{21} & a_{22} \end{vmatrix}}{|a|} > 0 \tag{7.2.19}$$

$$\frac{dL_1}{dp_1} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_{21} \end{vmatrix}}{|a|} > 0 \text{ iff } a_{11}b_{21} > |a_{21}b_1| \tag{7.2.20}$$

$$\frac{dw_1}{dp_2} = \frac{-b_{22}\,a_{12}}{|a|} < 0 \tag{7.2.21}$$

$$\frac{dL_1}{dp_2} = \frac{b_{22}\,a_{11}}{|a|} < 0 \tag{7.2.22}$$

$$\frac{dw_1}{d\hat{w}} = \frac{-b_{23}\,a_{12}}{|a|} > 0 \tag{7.2.23}$$

$$\frac{dL_1}{d\hat{w}} = \frac{b_{23}\,a_{11}}{|a|} > 0 \tag{7.2.24}$$

Although they do not tell a clear story the comparative static results do provide some interesting insights into the medium run effects of government policy in this type of economy. The comparative static effects upon wages and employment in the unionised sector of a rise in the price, p_2, may appear somewhat strange. The rationale behind this result is that an increase in p_2 actually increases the unions utility surplus, because it reduces the value of $V(\hat{w},p_1,p_2)$

more than $v(w_1, p_1, p_2)$, thus w_1 and L_1 fall to redistribute some of this gain to the firm. The effect of a rise in the unemployment benefit rate is most interesting in the short-run, the analysis suggests that if unemployment exists in the non-unionised sector a rise in benefit reduces unemployment due to its demand effects. In the medium run the rise in unemployment benefit reduces the unions utility surplus and thus a rise in the level of employment and wage rate are required to share the loss correctly between the union and firm, thus unemployment falls in the unionised sector.

In the case where there is unemployment in the unionised sector, but the non-unionised sector is employing all R members of its labour pool, there is a role for government policy. The short-run effects of government expenditure and unemployment benefit increases will be purely inflationary, however their medium run effect upon the bargain may lead to a rise in both output and employment; a little inflation, particularly a rise in p_1, is a good thing in this circumstance.

The basic implication of this analysis is that the existence of a unionised sector within a simple macroeconomic model does introduce rigidities in the form of a fixed level of wages and employment in the short-run. However in the medium run the bargain struck in the unionised sector adjusts in response to changes in the environment. Government expenditure and unemployment benefit levels should be chosen in the light of both their short and medium run consequences. The distributional effects of fiscal expansions upon the prices of the goods produced in the two sectors should perhaps be given close scrutiny as (7.2.19-22) suggest.

7.3 A Two Sector Disequilibrium Model with Bilateral Monopoly in One Sector

In this section a model similar to that examined in 7.2 is analysed, the major difference being that here the fix-price method is adopted. All prices will be assumed exogenously fixed in the short-run, except the price of labour in the unionised sector, which will be determined by bilateral bargaining. The main purpose of this analysis is to examine how the existence of a wage employment bargain will modify the characteristics of the models rationing regimes, and to examine the comparative static effects of changes in such a bargain.

There are three types of decision makers in the economy firms, workers and unions, again there are two production sectors one of which draws its labour from a unionised labour pool, the other obtains workers on a competative labour market. A representative firm and union formulation will be adopted. Workers will not be described by a representative individual since it is assumed each supplies one unit of labour inelastically as it is desired that the analysis considers unemployment rather than underemployment.

(1) The Unionised Production Sector

In the unionised sector it will be assumed that the firms output price is exogenously fixed and that there is a given labour pool of N identical workers.[12]

The Firm: Maximizes profit through periodic wage employment bargains agreed with the union its maximand may thus be written as (7.3.1).

$$\max_{w_1, L_1} \pi_1 = p_1x_1 - w_1L_1 = F(\tilde{L}_1) - w_1L_1 \qquad (7.3.1)$$

$$\text{S.t} \qquad \tilde{L}_1 \leq L_1 \leq N$$

$F(\tilde{L}_1)$ is the concave production function, $\tilde{L}_1$ the number of workers used in productive activity. L_1 the number of workers on the payrole. N is the total labour pool. p_1 is normalised to unity.

Workers: The N workers belonging to the labour pool each maximize an indirect utility function as described by (7.3.2)

$$\text{Max } V = \begin{cases} V(w_1 | p_1, p_2) & \text{when employed} \\ V(\hat{w} | p_1, p_2) & \text{otherwise} \end{cases} \qquad (7.3.2)$$

where $\hat{w}$ is again the unemployment benefit rate.

Hence the total demands for the two consumption goods by workers in this labour pool are as (7.3.3) and (7.3.4)

$$x_1 = L_1x_1(w_1) + (N-L_1)x_1(\hat{w}) \qquad (7.3.3)$$

$$x_2 = L_1x_2(w_1) + (N-L_1)x_2(\hat{w}) \qquad (7.3.4)$$

The Union: Bargains with the firm to attempt to raise employment and the sector wage, so as to maximize the sum of its members utilities.

$$V = L_1V(w_1) + (N-L_1)\ V(\hat{w}) \qquad (7.3.5)$$

The bargain struck between the union and firm must be efficient and satisfy some distribution rule. The Nash solution may be taken as illustrative, although other distribution rules such as the fair shares solution suggested in McDonald and Solow (1981) could equally well be adopted with similar results. The Nash maximand is written as (7.3.6)

$$\max_{w_1, L_1} \text{Prod} = \left[F(\tilde{L}_1) - w_1L_1\right] L_1 \left[V(w_1) - V(\hat{w})\right] \qquad (7.3.6)$$

The first order conditions for maximization are (7.3.7) and (7.3.8)

$$\frac{\partial\ \text{prod}}{\partial L_1} = F(\tilde{L}_1) - 2wL + F_{\tilde{L}_1}(\tilde{L}_1)\frac{\partial \tilde{L}_1}{\partial L_1} = 0 \qquad (7.3.7)$$

$$\frac{\partial\ \text{prod}}{\partial w_1} = F(\tilde{L}_1) - w_1 L_1 - L_1\left[\frac{V(w_1)-V(\hat{w})}{V_{w_1}(w_1)}\right] = 0 \qquad (7.3.8)$$

Condition (7.3.7) is the distribution rule and (7.3.8) the efficiency condition the term $\partial\tilde{L}_1/\partial L_1$ takes a value of unity when the bargain specifies no labour hoarding and zero otherwise.

(2) The Non-Unionised Sector

In this sector the prices of both output and labour are exogenously determined. There is a given labour pool of R identical workers.

The Firm: Is a price taker on both labour and output markets and hires workers to maximize profit in the standard neo-classical manner.

$$\underset{L_2}{\text{Max}}\ \pi_2 = p_2 x_2 - w_2 L_2 = G(L_2) - w_2 L_2 \qquad (7.3.9)$$

$$\text{S.t} \quad L_2 \leq R$$

where $G(L_2)$ is the concave production function, the output price p_2 is normalised to unity.

Thus since the only constraints the firm faces are given by the production function and the size of the competative labour pool, the firms notional supplies and demands may be written as (7.3.10) and (7.3.11).

$$L_2 = L_2(w_2, p_2) \qquad (7.3.10)$$

$$x_2 = \min \begin{cases} x_2(w_2, p_2) \\ G(R) \end{cases} \qquad (7.3.11)$$

Workers: The workers in this sector maximize utility as described by the indirect utility function (7.3.12).

$$V = \begin{cases} V(w_2) & \text{if employed} \\ V(\hat{w}) & \text{otherwise} \end{cases} \qquad (7.3.12)$$

associated with these indirect utilities are the aggregate product demand functions (7.3.13) and (7.3.14).

$$x_1 = L_2\, x_1(w_2) + (R-L_2)\, x_1(\hat{w}) \qquad (7.3.13)$$

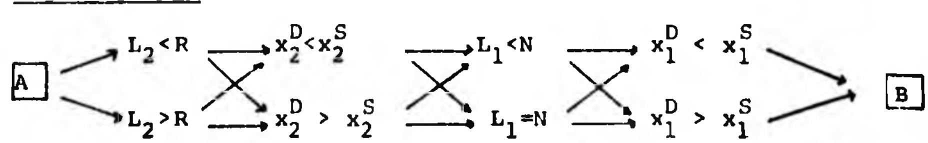
$$x_2 = L_2\, x_2(w_2) + (R-L_2)\, x_2(\hat{w}) \qquad (7.3.14)$$

Government: Purchases output from both production sectors, g_1 and g_2, and has its demand satisfied before workers are supplied. It pays the unemployment benefit rate $\hat{w}$ to all unemployed workers and levies a 100% profits tax.[13] It finances any deficit by printing money.

(3) Constraint Combinations, Aggregate Effective Supplies and Demands

The preceding sections examine how the various agents in the economy solve their notional maximization problems when faced by a vector of fixed prices. The notional supplies and demands will only be mutually consistent if the fix-price vector is at its market clearing constellation, typically supplies and demands will not match and the usual fix-price picture will emerge, here however the bargain in the unionised sector will produce some new possibilities. Figure (7.3.1) describes the possible constraint combinations.

Figure (7.3.1)



As figure (7.3.1) demonstrates there are 16 'routes' from A to B and thus 16 possible constraint combinations or disequilibrium regimes, superscripts D and S indicate demand and supply respectively.

Aggregate effective demands and supplies will thus depend upon both prices and constraints. On each regime a different combination of constraints impinges upon economic agents maximization problems, yielding different forms of the supply and demand functions. The responses of agents to quantity constraints depends upon the structure of the sector in which they are operating.

Aggregate effective supplies and demands may now be defined as follows.

The representative firm in the non-unionised sector will always be upon its production function and will:

$$\text{Demand Labour} \quad L_2 = \min \begin{cases} G^{-1}(x_2^D) \\ L_2(w_2, p_2) \end{cases} \tag{7.3.15}$$

$$\text{Supply Output} \quad x_2^S = \min \begin{cases} x_2(w_2, p_2) \\ G(R) \end{cases} \tag{7.3.16}$$

The representative firm in the unionised sector has its level of employment and wage rate predetermined by the bargain. The firm is thus either constrained in its output by the amount of labour the bargain specifies or hoards labour if demand falls short of that which its employees could produce.[14] The amount of labour it hoards is described by (7.3.17).

$$L_H = \max \begin{cases} L_1 - F^{-1}(x_1^D) \\ 0 \end{cases} \tag{7.3.17}$$

The total effective demands for the outputs of the two sectors are obtained by summing the demands expressed by the workers employed in the two sectors, the demands by the unemployed and the government as (7.3.18) and (7.3.19).[15]

$$x_1^D = L_1x_1(w_1, x_2^S) + L_2x_1(w_2, x_2^S) + (K-L_1-L_2)x_1(\hat{w}) + g_1 \quad (7.3.18)$$

$$x_2^D = L_1x_2(w_1, x_1^S) + L_2x_2(w_2, x_1^S) + (K-L_1-L_2)x_2(\hat{w}) + g_2 \quad (7.3.19)$$

The inclusion of the terms x_1^S and x_2^S recognises that agents demand for one good will be effected by a spillover effect if they are rationed in their purchases of the other. It is assumed that unemployed workers receive insufficient income to encounter a constraint and that the government is never rationed.[16] K is the total working population, K=R+N.

The potential regimes of the economy may now be described by a set of minimum conditions as utilised by Muellbauer and Portes (1978).

<u>Unionised Sector</u>

$$x_1^S = F(L_1) \qquad \text{given by the bargain} \quad (7.3.20)$$

$$x_1^D = L_1x_1(w_1, x_2^S) + L_2x_1(w_2, x_2^S) + (K-L_1-L_2)x_1(\hat{w}) + g_1 \quad (7.3.21)$$

$$x_1 = \min \left(\begin{array}{l} x_1^S \\ \\ x_1^D \end{array} \right. \quad (7.3.22)$$

$$L_1 = L_1 \qquad \text{given by the bargain} \quad (7.3.23)$$

<u>Non-Unionised Sector</u>

$$x_2^S = \min \left(\begin{array}{l} x_2^S(w_2, p_2) \\ G(R) \end{array} \right. \quad (7.3.24)$$

$$x_2^D = L_1x_2(w_1, x_1^S) + L_2x_2(w_2, x_1^S) + (K-L_1-L_2)x_2(\hat{w}) + g_2 \quad (7.3.25)$$

$$x_2 = \min \left(\begin{array}{l} x_2^S \\ \\ x_2^D \end{array} \right. \quad (7.3.26)$$

$$L_2^S = R \quad (7.3.27)$$

$$L_2^D = \min\begin{cases} G^{-1}(x_2^D) & \text{if } x_2^D < x_2^S \\ L_2^D(w_2, p_2) & \text{otherwise} \end{cases} \quad (7.3.28)$$

$$L_2 = \min\begin{cases} L_2^S \\ L_2^D \end{cases} \quad (7.3.29)$$

Thus any short run equilibrium in this economy may be characterised by the wage employment bargain and the short-side market clearing rules as described by (7.3.20)-(7.3.29).

(4) Constraint Regimes and Their Comparative Static Properties

In the economy studied here there are 16 different constraint regimes but an exhaustive exposition of each is not provided. Attention is focused upon five examples which demonstrate the most interesting effects which the introduction of bargaining has upon the models comparative static properties.

Keynesian Unemployment: A shortfall in effective demand for the output of both sectors combined with unemployment of workers in both labour pools.

The following inequalities hold: $L_1<N$, $L_2<R$, $x_1^D<x_1^S$ $x_2^D<x_2^S$

Hence the effective demands of the regime may be written as (7.3.30)-(7.3.32)

$$x_1^D = L_1x_1(w_1)+G^{-1}(x_2^D)x_1(w_2)+[K-L_1-G^{-1}(x_2^D)]x_1(\hat{w})+g_1 \quad (7.3.30)$$

$$x_2^D = L_1x_2(w_1)+G^{-1}(x_2^D)x_2(w_2)+[K-L_1-G^{-1}(x_2^D)]x_2(\hat{w})+g_2 \quad (7.3.31)$$

$$L_2^D = G^{-1}(x_2^D) \quad (7.3.32)$$

and labour hoarding arises in the unionised sector as (7.3.33)

$$L^H = L_1-F^{-1}(x_1^D) \quad (7.3.33)$$

Thus the representative firm in the unionised sector faces a shortfall in demand for output, x_1^D, but cannot sack workers because of the wage/employment bargain. Any employees over the number the firm requires to produce the output demanded are simply hoarded, L^H. The bargain prevents the short-run effects of a demand shortfall upon employment occurring. The firm in the non-unionised sector also faces a demand shortfall, and thus reduces its workforce accordingly.

Since the novel component of this model is the addition of a unionised production sector attention will be focused upon how this changes the comparative static effects of changes in government policy, and how changes in the bargain effect the other variables.

For a given bargain the effect of an increase in government purchases of output from the unionised sector raise output and reduce labour hoarding as described by (7.3.34) and (7.3.35).

$$\frac{dx_1^D}{dg_1} = 1 \tag{7.3.34}$$

$$\frac{dL^H}{dg_1} = -F^{-1'}(x_1^D)\,\frac{dx_1^D}{dg_1} = -F^{-1'}(x_1^D) \tag{7.3.35}$$

Since the only effect of an increase in government purchases in this sector is to induce the firm to use hoarded labour productively,[17] no extra wage income is generated and workers demands for goods do not change, there are no multiplier effects. An increase in government purchases from the non-unionised sector has multiplier effects, increasing output and employment in the non-unionised sector, and raising output and reducing labour hoarding in the unionised sector as (7.3.36)-(7.3.39)

$$\frac{dx_2^D}{dg_2} = \frac{1}{1 - [x_2(w_L) - x_2(\hat{w})]\,G^{-1'}(x_2^D)} \tag{7.3.36}$$

$$\frac{dL_2^D}{dg_2} = G^{-1'}(x_2^D)\,\frac{dx_2^D}{dg_2} \tag{7.3.37}$$

$$\frac{dx_1^D}{dg_2} = [x_1(w_2)-x_1(\hat{w})]G^{-1'}(x_2^D)\frac{dx_2^D}{dg_2} \tag{7.3.38}$$

$$\frac{dL^H}{dg_2} = -F^{-1'}(x_1^D)\frac{dx_1^D}{dg_2} \tag{7.3.39}$$

(7.3.36) is the non-unionised sector demand multiplier. If the government raises the unemployment benefit rate this increases output in both sectors and also raises employment in the non-unionised sector whilst reducing labour hoarding in the unionised production sector as (7.3.40)-(7.3.43)

$$\frac{dx_2^D}{d\hat{w}} = \frac{[K-L_1-G^{-1}(x_2^D)]x_2'(\hat{w})}{1-[x_2(w_2)-x_2(\hat{w})]G^{-1'}(x_2^D)} \tag{7.3.40}$$

$$\frac{dL_2^D}{d\hat{w}} = G^{-1'}(x_2^D)\frac{dx_2^D}{d\hat{w}} \tag{7.3.41}$$

$$\frac{dx_1^D}{d\hat{w}} = [K-L_1-G^{-1}(x_2^D)]x_1'(\hat{w})+[x_1(w_2)-x_1(\hat{w})]G^{-1'}(x_2^D)\frac{dx_2^D}{d\hat{w}} \tag{7.3.42}$$

$$\frac{dL^H}{d\hat{w}} = -F^{-1'}(x_1^D)\frac{dx_1^D}{d\hat{w}} \tag{7.3.43}$$

A change in the wage employment bargain, holding government expenditure and unemployment benefit constant, will effect output employment and labour hoarding as described by (7.3.44)-(7.3.47)

$$dx_2^D = \frac{[x_2(w_1)-x_2(\hat{w})]\,dL_1 + L_1x_2'(w_1)dw_1}{1-[x_2(w_2)-x_2(\hat{w})]G^{-1'}(x_2^D)} \tag{7.3.44}$$

$$dL_2^D = G^{-1'}(x_2^D)dx_2^D \tag{7.3.45}$$

$$dx_1^D = [x_1(w_1)-x_1(\hat{w})]dL_1 + L_1x_1'(w_1)dw_1 + G^{-1'}(x_2^D)[x_1(w_2)-x_1(\hat{w})]dx_2^D \tag{7.3.46}$$

$$dL^H = dL_1 - F^{-1'}(x_1^D)dx_1^D \tag{7.3.47}$$

Thus if both goods are normal and the change in the bargain involves the wage rate and level of employment in the unionized sector moving in the same direction, then an improvement in the unions position will raise employment and output in both sectors, the effect upon labour hoarding is unclear.

Keynesian Unemployment in the Non-Unionised Sector, Unemployment in the Unionised Sector: A shortfall in effective demand for the good produced in the non-unionised sector and a shortfall in supply of the good produced in the unionised sector.

The following inequalities hold: $L_1<N$, $L_2<R$, $x_1^D>x_1^S$, $x_2^D<x_1^S$,

Hence the effective demands and supplies of the regime may be written as (7.3.48)-(7.3.50)

$$x_1^S = F(L_1) \tag{7.3.48}$$

$$x_2^D = L_1 x_2(w_1,\bar{x}_1)+G^{-1}(x_2^D)x_2(w_2,\bar{x}_1)+(K-L_1-G^{-1}(x_2^D)\ )x_2(\hat{w})+g_2 \tag{7.3.49}$$

$$L_2^D = G^{-1}(x_2^D) \tag{7.3.50}$$

where

$$\bar{x}_1 = \frac{F(L_1) - g_1 - [K-L_1-G^{-1}(x_2^D)]x_1(\hat{w})}{L_1 + L_2} \tag{7.3.51}$$

(7.3.51) describes a rationing scheme where the demands of the unemployed and government are satisfied and the remaining output is divided equally between employed workers.

To obtain the comparative static properties of government expenditure increases differentiate (7.3.49)-(7.3.51) holding w_1, L_1, and $\hat{w}$ constant to give (7.3.52)-(7.3.54)

$$dx_2^D = \frac{\left[L_1 x_{2,\bar{x}_1}(w_1,\bar{x}_1) + L_2 x_{2,\bar{x}_1}(w_2,\bar{x}_1)\right] dx_1 + dg_2}{1 + G^{-1'}(x_2^D)\left[x_2(\hat{W}) - x_2(w_2,\bar{x}_1)\right]} \qquad (7.3.52)$$

$$dL_2^D = G^{-1'}(x_2^D)\; dx_2^D \qquad (7.3.53)$$

$$d\bar{x}_1 = -\left[\frac{dg_1 - G^{-1'}(x_2^D)\left[x_1(\hat{W}) - \bar{x}_1\right] dx_2^D}{L_1 + L_2}\right] \qquad (7.3.54)$$

Thus since $x_{2,\bar{x}_1}(w_1,\bar{x}_1)$ and $x_{2,\bar{x}_1}(w_2,\bar{x}_1)$ are both negative[18] an increase in government expenditure upon either good will increase output and employment in the non-unionised sector. An increase in government purchases of the good produced in the unionised sector tightens the ration on that good faced by employed workers making them switch expenditure to the other good, hence raising demand and thus output and employment. A rise in unemployment benefit has similar effects as described by (7.3.55)-(7.3.57)

$$dx_2^D = \frac{\left[K - L_1 - G^{-1}(x_2^D)\right] x_2'(\hat{W})\, d\hat{W} + \left[L_1 x_{2,\bar{x}_1}(w_1,\bar{x}_1) + L_2 x_{2,\bar{x}_1}(w_2,\bar{x}_1)\right] d\bar{x}_1}{1 + G^{-1'}(x_2^D)\left[x_2(\hat{W}) - x_2(w_2,\bar{x}_1)\right]} \qquad (7.3.55)$$

$$dL_2^D = G^{-1'}(x_2^D)\, dx_2^D \qquad (7.3.56)$$

$$d\bar{x}_1 = -\left[\frac{\left[K - L_1 - G^{-1}(x_2^D)\right] x_1'(\hat{W})\, d\hat{W} - G^{-1'}(x_2^D)\left[x_1(\hat{W}) - \bar{x}_1\right] dx_2^D}{L_1 + L_2^D}\right] \qquad (7.3.57)$$

Clearly output and employment in the non-unionised sector rise when unemployment benefit rises, due to the increase in demand for the sectors output arising from the increased income of the unemployed and from the demand 'switching' effect due to the tightening of the goods ration faced by employed workers.

A change in the wage employment bargain involving a rise in both employment and wages in the unionised sector has the effects as (7.3.58)-(7.3.60)

$$dx_2^D = \frac{[x_2(w_2,\bar{x}_1)-x_2(\hat{w})]\,dL_1+[L_1 x_{2,\bar{x}_1}(w_1,\bar{x}_1)+G^{-1}(x_2^D)\,x_{2,\bar{x}_1}(w_2,\bar{x}_1)]\,dx_1}{1+G^{-1'}(x_2^D)\,[x_2(\hat{w})-x_2(w_2,\bar{x}_1)]} \tag{7.3.58}$$

$$dL_2^D = G^{-1'}(x_2^D)\,dx_2^D \tag{7.3.59}$$

$$d\bar{x}_1 = \frac{[F'(L_1)-\bar{x}_1+x_1(\hat{w})]\,dL_1+G^{-1'}(x_2^D)\,[x_1(\hat{w})-\bar{x}_1]\,dx_2^D}{L_1+L_2} \tag{7.3.60}$$

The effect of an increase in the wage rate and level of employment in the unionised sector upon the level of output and employment in the non-unionised sector, and hence overall employment, depends crucially upon the change in the ration $d\bar{x}_1$. Increased employment gives greater output as described by the production function, which may raise the level of the ration leading to a switching of demand away from the non-unionised sector good. If the direct demand effects upon x_2 raising the level of demand for this good outweigh the switching of demand arising from the change in the ration level then output and total employment will rise. If the demand switching effect dominates then the effect upon total employment of an increase in both components of the bargain is ambiguous. However if the ration actually tightens, due to the goods being distributed between a greater number of employed workers, then total output and employment unambiguously increase.

Full Employment in the Unionised Sector, Keynesian Unemployment in the Non-Unionised Sector: Here there is a shortfall in demand for the output of both production sectors. There is unemployment in the non-unionised labour pool, but in the unionised sector a full employment contract is in force and the demand shortfall gives rise to labour hoarding.

The rationing regime is characterised by the following inequalities. $L_1=N$, $L_2<R$, $X_1^D<x_1^S$, $x_2^D<x_2^S$.

Hence the effective demands of the regime may be written

$$x_1^D = N\ x_1(w_1)+G^{-1}(x_2^D)x_1(w_2) + \left[R-G^{-1}(x_2^D)\right]x_1(\hat{w})+g_1 \tag{7.3.61}$$

$$x_2^D = N\ x_2(w_1)+G^{-1}(x_2^D)x_2(w_2) + \left[R-G^{-1}(x_2^D)\right]x_2(\hat{w})+g_2 \tag{7.3.62}$$

$$L_2^D = G^{-1}(x_2^D) \tag{7.3.63}$$

and there is labour hoarding as defined by (7.3.64)

$$L^H = N-F^{-1}(x_1^D) \tag{7.3.64}$$

Clearly the comparative static effects of changes in government expenditure and unemployment benefit rates will be the same upon this regime as on the Keynesian unemployment regime. Increases in g_2 and $\hat{w}$ raise output and employment in the non-unionised sector and lower labour hoarding. An increase in g_1 simply reduces labour hoarding.

The effect of a change in the bargain here differs from that on the Keynesian regime since any improvement in the unions utility must be achieved by an increase in the wage rate w_1. The comparative static effects of such a change are described by (7.3.65)-(7.3.68)

$$dx_1^D = N\ x_1'(w_1)dw_1 + G^{-1'}(x_2^D)\left[x_1(w_2)-x_1(\hat{w})\right]dx_2^D \tag{7.3.65}$$

$$dx_2^D = \frac{N\ x_2'(w_1)\ dw_1}{1 + G^{-1'}(x_2^D)\left[x_2(\hat{w}) - x_2(w_2)\right]} \tag{7.3.66}$$

$$dL_2^D = G^{-1'}(x_2^D)\ dx_2^D \tag{7.3.67}$$

$$dL^H = -F^{-1'}(x_1^D)dx_1^D \tag{7.3.68}$$

An increase in the union wage in this case raises output and employment in the non-unionised sector and reduces labour hoarding in the unionised sector.

Classical Unemployment in the Non-Unionised Sector, Keynesian Unemployment in the Unionised Sector

In this constraint regime the fixed prices are such that the firm in the non-unionised sector desires to neither meet all demand nor employ its total labour pool. In the unionised sector there is a goods demand shortfall and the bargain specifies a level of employment less than the total available labour.

The regime is characterised by the following inequalities,
$L_1<N, \quad L_2<R, \quad x_1^D<x_1^S \quad\quad x_2^D>x_2^S$.

Where the effective supplies and demands are given by (7.3.69)-(7.3.71)

$$x_2^S = x_2^S\,(w_2,p_2) \tag{7.3.69}$$

$$L_2^D = L_2^D\,(w_2,p_2) \tag{7.3.70}$$

$$x_1^D = L_1\, x_1\,(w_1,\bar{x}_2)+L_2^D x_1\,(w_2,\bar{x}_2)+\left[K-L_1-L_2^D\right]x_1(\hat{w})+g_1 \tag{7.3.71}$$

The ration of the good $\bar{x}_2$ faced by employed workers is defined by (7.3.72)

$$\bar{x}_2 = \frac{x_2^S(w_2,p_2) - g_2 - \left[K-L_1-L_2^D\right]x_2(\hat{w})}{L_1 + L_2^D} \tag{7.3.72}$$

There will be labour hoarding in the unionised sector as (7.3.73)

$$L^H = L_1 - F^{-1}(x_1^D) \tag{7.3.73}$$

Only changes in relative prices effect output and employment in the non-unionised sector, the level of employment in the unionised sector is given by the bargain. Thus the only effects that changes in government expenditure and unemployment benefit may have will be upon output and labour hoarding in the unionised sector as described by (7.3.74)-(7.3.79).

$$dx_1^D = [L_1 x_{1,\bar{x}_2}(w_1,\bar{x}_2) + L_2^D x_{1,\bar{x}_2}(w_2,\bar{x}_2)]d\bar{x}_2 + dg \qquad (7.3.74)$$

$$d\bar{x}_2 = \frac{-dg_2}{L_1 + L_2^D} \qquad (7.3.75)$$

$$dL^H = -F^{-1'}(x_1^D)dx_1^D \qquad (7.3.76)$$

Thus an increase in government purchases raises demand for the good produced in the unionised sector and reduces labour hoarding. In the case of g_1 the effect is direct, in the case of g_2 this occurs because the ration on the employed workers $\bar{x}_2$ tightens, switching demand to the unionised sectors good.

$$dx_1^D = [L_1 x_{1,\bar{x}_2}(w_1,\bar{x}_2) + L_2^D x_{1,\bar{x}_2}(w_2,\bar{x}_2)]d\bar{x}_2 + [K-L_1-L_2^D]\, x_1'(\hat{\omega})d\hat{\omega} \qquad (7.3.77)$$

$$d\bar{x}_2 = \frac{-[K-L_1-L_2^D]x_2'(\hat{\omega})d\hat{\omega}}{L_1 + L_2^D} \qquad (7.3.78)$$

$$dL^H = -F^{-1'}(x_1^D)dx_1^D \qquad (7.3.79)$$

An increase in unemployment benefit only effects output and labour hoarding in the unionised sector.

A revision of the bargain involving an increase in wages and employment will again only have repercussions in the unionised sector as (7.3.80)-(7.3.82)

$$dx_1^D = [x_1(w_1,\bar{x}_2)-x_1(\hat{w})]\ dL_1 + L_1 x_{1,w_1}(w_1,\bar{x}_2)dw_1$$

$$+ [L_1 x_{1,\bar{x}_2}(w_1,\bar{x}_2) + L_2^D\ x_{1,\bar{x}_2}(w_2,\bar{x}_2)]d\bar{x}_2 \qquad (7.3.80)$$

$$d\bar{x}_2 = \frac{[x_2(\hat{w}) - \bar{x}_2]}{L_1 + L_2^D}\ dL_1 \qquad (7.3.81)$$

$$dL^H = dL_1 - F^{-1'}(x_1^D)\ dx_1^D \qquad (7.3.82)$$

Employment rises as defined by the bargain, labour hoarding will fall if the demand effects of increasing w_1 and L_1 raise the amount of labour used productively by more than the bargain specifies total employment must rise.

<u>Full Employment in the Unionised Sector, Repressed Inflation in the Non-Unionised Sector</u>: Producers in the non-unionised sector are constrained by the size of the labour pool and cannot meet the demand for their output. The bargain specifies full employment in the unionised sector despite a shortfall in demand for its output.

The regime is characterised by the following inequalities:
$L_1=N,\quad L_2=R,\quad x_1^D < x_1^S\ ,\quad x_2^D > x_2^S$

The effective supplies and demands in this case are (7.3.83)-(7.3.84)

$$x_1^D = N\ x_1(w_1,\bar{x}_2) + R\ x_1\ (w_2,\bar{x}_2) + g_1 \qquad (7.3.83)$$

$$x_2^S = G(R) \qquad (7.3.84)$$

where the rationing on the good $\bar{x}_2$ is defined by (7.3.85)

$$\bar{x}_2 = \frac{G(R)-g_2}{K} \qquad (7.3.85)$$

labour hoarding may be defined by (7.3.86)

$$L^H = N - F^{-1}(x_1^D) \qquad (7.3.86)$$

The only role for government policy upon this regime is to reduce labour hoarding as described by (7.3.87)-(7.3.89)

$$d\bar{x}_2 = \frac{-dg_2}{K} \tag{7.3.87}$$

$$dx_1^D = \left[N\ x_{1,\bar{x}_2}(w_1,\bar{x}_2) + R\ x_{2,\bar{x}_2}(w_2,\bar{x}_2)\right] d\bar{x}_2 + dg_1 \tag{7.3.88}$$

$$dL^H = -F^{-1'}(x_1^D)\,dx_1^D \tag{7.3.89}$$

An increase in government expenditure on either good will reduce labour hoarding. Any change in the bargain will entail only a change in the wage rate, a rise in which will reduce labour hoarding as (7.3.90) and (7.3.91)

$$dx_1^D = N\ x_{1,w_1}(w_1,\bar{x}_2)\,dw_1 \tag{7.3.90}$$

$$dL^H = -F^{-1'}(x_1^D)\,dx_1^D \tag{7.3.91}$$

A listing and brief characterisation of the constraint regimes not discussed in this section may be found in the appendix to this chapter.

The regimes examined demonstrate how the wage employment bargain will modify the comparative static properties of a two sector fix-price model. In the short- run the bargain places an upper limit upon the amount of output that may be produced in the unionised sector, and also it makes the level of wage income in the sector insensitive to changes in other sectors. The bargain provides a rationale for labour hoarding and consequently may throw some light on how the average productivity of labour varies.

In the analysis presented in this section a simple two sector general equilibrium model has been examined when four basic market inperfections are introduced.

Firstly, prices do not adjust sufficiently rapidly to equilibriate supplies and demands. This creates the usual consequences of rationing and spillover effects. In the face of such an imperfection, government expenditure policies have been shown by Muellbauer and Portes (1978) for example, to be able to raise output and employment as required. Here there are further imperfections.

Secondly, there is the problem of labour immobility. In the short-run workers belong to the labour pool of a particular production sector. If unemployment exists in a particular labour pool but not in others then policies designed to alleviate the problem need to have the correct distributional characteristics or the problem may be exacerbated.

Thirdly the labour supply of an individual worker is inelastic or institutionally fixed implying, together with labour immobility, that if excess demand exists for the sectors output policies such as payroll subsidies cannot induce an increase in labour supply and output.

Finally, fourthly and most importantly, trade unions are active in one sector of the model, with the consequence that the wage rate and level of employment in the unionised sector are determined by a bargaining solution to the problem of bilateral monopoly. Employment in this sector is not immediately responsive to changes in government expenditure, which must work through the bargaining process changing either the efficiency or distributive conditions of the bargain.

These, arguably realistic, market imperfections produce many new fix-price regimes which suggest that government demand management policies should be designed on a sector specific basis rather than in aggregate.

In conclusion it is suggested that unionisation of some labour pools and the introduction of bargaining is an acceptable method of endogenizing some wage rates. Unionisation cannot simply be 'tacked on' to a fixed-price model but has several behavioural consequences some of which have been investigated in preceding sections.

FOOTNOTES

1. By adopting this form of the unions utility function it is being implicitly assumed that hiring is random.

2. The necessary condition is

$$2L[x(\hat{w}) - x(w) + wx_{\hat{w}}(\hat{w})]\frac{dw}{d\hat{w}} + (N-L)x_{\hat{w}}(\hat{w}) > 0$$

3. As has been pointed out a proportional rationing scheme is manipulable and may lead to overbidding of demands by agents. See Drazen (1980) for a discussion.

4. The necessary condition is

$$V_x(w,x/N) > [F(N)-wN]V_{wx}(w,x/N)$$

5. The other possibility here is $V(w,\bar{n}) = V(\bar{w})$ however substitution of this condition into the equilibrium conditions demonstrates that this cannot occur.

6. The condition for det[a]>0 is $a_{11}a_{22}>a_{21}a_{12}$

7. The a_{13} term is important here, if $V_{w\bar{n}}(w,\bar{n})$ is large then the unions utility loss when the ration tightens is also large and the wage and level of employment must rise as argued.

8. It might be argued that there are lags and costs involved in obtaining the information to 'check' the bargain, and it is for these reasons that it is only renegotiated periodically.

9. We may introduce labour mobility by rewriting the unions objective function as:

$$V = \alpha\left(\frac{L_1}{N+\alpha(R-L_2)}\right) V(w_1,p_1,p_2) + \alpha\left[\frac{L_2}{R+\alpha(N-L_1)}\right] V(\hat{w}_2,p_1,p_2)$$

$$+ \left[1 - \frac{L_1}{N+\alpha(R-L_2)} - \frac{\alpha L_2}{R+\alpha(N-L_1)}\right] V(\hat{w}_1 p_1,p_2)$$

where α is that proportion of the working population who are mobile in any period.

10. The added complication of inventories is not introduced here despite its potential interest.

11. Unless government policy is designed to have distributional effects.

12. The union operates a closed shop.

13. This assumption is made for convenience, alternatively we could assume either that the MPC out of profit income is less than that out of wage income, or that profits are distributed at the end of the period. Neither would qualitatively effect the results.

14. If overtime were worked the firm would have a little more flexibility.

15. It will be assumed that the effective demand functions are continuous, this seems reasonable given K is large.

16. It is assumed that the government, being a large purchaser, is given priority by firms and is never rationed.

17. This will effect the level of the firms profit, however this will have no demand effect in the short-run. The role of profit in such models is discussed by Malinvaud (1981).

18. See Neary and Roberts (1980) or Cornes (1979).

APPENDIX TO CHAPTER 7

LISTING OF CONSTRAINT REGIMES UNDISCUSSED IN SECTION 7.3

Full Classical Unemployment Excess supply on both labour markets, excess demand on both goods markets.

Constraints: $L_1<N$, $L_2<R$, $x_1^D>x_1^S$, $x_2^D>x_2^S$

Effective Supplies and Demands

$$x_1^S = F(L_1)$$

$$x_2^S = x_2^S\ (w_2,p_2)$$

$$L_2^D = L_2^D\ (w_2,p_2)$$

$$L^H = 0$$

Full Repressed Inflation: Full employment in both production sectors and excess effective demand for both goods.

Constraints: $L_1 = N$, $L_2 = R$, $x_1^D > x_1^S$, $x_2^D > x_2^S$

The effective supplies may be written:

$$x_1^S = F(N)$$

$$x_2^S = G(R)$$

Keynesian Unemployment in the Non-Unionised Sector, Repressed Inflation in the Unionised Sector

Excess demand for the good produced in the unionised sector, full employment in that sector. Excess supply of the good produced in the non-unionised sector and unemployment.

Constraints: $L_1 = N$, $L_2 < R$, $x_1^D > x_1^S$, $x_2^D < x_2^S$

$$x_1^S = F(N)$$

$$x_2^D = N\ x_2(w_1,\bar{x}_1) + F^{-1}(x_2^D)\,x_2(w_2,\bar{x}_1) + [R-F^{-1}(x_2^D)]\,x_2(\hat{w}) + g_2$$

$$L_2^D = F^{-1}(x_2^D)$$

where $$\bar{x}_1 = \frac{F(N) - [R-F^{-1}(x_2^D)]\ x_2(\hat{w})-g_1}{N + L_2^D}$$

Classical Unemployment in the Non-Unionised Sector, Full Employment in the Unionised Sector The firm in the non-unionised sector is on the short-side of both markets. In the unionised sector the bargain specifies full employment, but there is some labour hoarding due to a demand shortfall.

Constraints: $L_1 = N,\quad L_2^D<R,\quad x_1^D<x_1^S,\quad x_2^D > x_2^S$

Effective Supplies and Demands

$$x_1^D = N\ x_1(w_1,\bar{x}_2) + L_2^D(w_2,p_2)x_1(w_2,\bar{x}_2) + [R-L_2^D(w_2,p_2)]\,x_1(\hat{w})+g_1$$

$$x_2^S = x_2^S\ (w_2,p_2)$$

$$L_2^D = L_2^D\ (w_2,p_2)$$

$$L^H = N - F^{-1}(x_1^D)$$

where

$$\bar{x}_2 = \frac{x_2^S\ (w_2,p_2) - [R-L_2^D(w_2,p_2)]\,x_2(\hat{w})-g_2}{N + L_2^D}$$

Classical Unemployment in the Non-Unionised Sector, Repressed Inflation in the Unionised Sector. Workers are constrained on both output markets. Full employment obtains on the unionised labour market, the firm is on the short-side of the non-unionised labour market.

Constaints: $L_1 = N,\quad L_2^D < R,\quad x_1^D > x_1^S,\quad x_2^D > x_2^S$.

Effective Supplies and Demands

$$x_1^S = F(N)$$

$$x_2^S = x_2^S\ (w_2,p_2)$$

$$L_2^D = L_2^D\ (w_2,p_2)$$

Underconsumption in the Non-Unionised Sector: Keynesian Unemployment in the Unionised Sector: Available labour just produces the output demanded in the non-unionised sector, clearly a boundary case. A demand shortfall gives labour hoarding and unemployment on the unionised sector.

Constraints: $L_1\ N,\quad L_2 = R,\quad x_1^D < x_1^S,\quad x_2^D = x_2^S$

Effective supplies and demands

$$x_1^D = L_1 x_1(w_1) + R x_1(w_2) + (N-L_1) x_1(\hat{w}) + g_1$$

$$x_2^D = L_1 x_2(w_1) + R x_2(w_2) + (N-L_1) x_2(\hat{w}) + g_2 = G(R)$$

$$L_2^D = G^{-1}(x_2^D) = R$$

$$L^H = L_1 - F^{-1}(x_1^D)$$

Underconsumption in the Non-Unionised Sector: Unemployment in the unionised sector and an excess of demand for output from the unionised sector. Again a boundary case in the non-unionised sector. Workers are rationed for the output of the unionised sector, since the bargain provides insufficient labour to achieve the required level of production.

Constraints: $L_1 < N,\quad L_2 = R,\quad x_1^D > x_1^S,\quad x_2^D = x_2^S$

Effective Supplies and Demands

$$x_1^S = F(L_1)$$

$$x_2^D = L_1 \cdot x_2(w_1, \bar{x}_1) + R\, x_2(w_2, \bar{x}_1) + (N-L_1) x_2(\hat{w}) + g_2 = G(R)$$

$$L_2^D = G^{-1}(x_2^D) = R$$

$$\bar{x}_1 = \frac{F(L_1) - g_1 - (N-L_1)\, x_1(\hat{w})}{R+L_1}$$

Full Underconsumption The firm in the non-unionised sector is on its production function. There is full employment in the unionised sector together with labour hoarding arising from a demand shortfall.

Constraints: $L_1 = N, \quad L_2 = R, \quad x_1^D < x_1^S, \quad x_2^D = x_2^S$

Effective Supplies and Demands

$$x_1^D = N\, x_1(w_1) + R\, x_1(w_2) + g_1$$

$$x_2^D = N\, x_2(w_1) + R\, x_2(w_2) + g_2 = G(R)$$

$$L_2^D = G^{-1}(x_1^D) = R$$

$$L^H = N - F^{-1}(x_1^D)$$

Underconsumption in the Non-Unionised Sector, Repressed Inflation in the Unionised Sector. The firm in the non-unionised sector is on its production function, demand exceeds full employment output in the unionised sector.

Constraints: $L_1 = N, \quad L_2 = R, \quad x_1^D > x_1^S, \quad x_2^D = x_2^S$

Effective Supplies and Demands

$$x_2^D = N\, x_2(w_1, \bar{x}_1) + R\, x_2(w_2, \bar{x}_1) + g_2 = G(R)$$

$$L_2^D = G^{-1}(x_2^D) = R$$

$$x_1^S = F(N)$$

where $\bar{x}_1 = \dfrac{F(N) - g_1}{K}$

Repressed Inflation in the Non-Unionised Sector: Keynesian Unemployment in the Unionised Sector. The firm in the non-unionised sector cannot meet demand despite employing the whole of its labour pool. The firm in the unionised sector faces a shortfall in demand for output.

Constraints: $L_1 < N, \quad L_2 = R, \quad x_1^D < x_1^S, \quad x_2^D > x_2^S$

Effective Supplies and Demands

$$x_1^D = L_1\ x_1(w_1,\bar{x}_2) + R\ x_1(w_2,\bar{x}_2) + (N-L_1)\ x_1(\hat{w}) + g_1$$

$$x_2^S = G(R)$$

$$L^H = L_1 - F^{-1}(x_1^D)$$

where $\bar{x}_2 = \dfrac{G(R) - g_2 - (N-L_1)\ x_2(\hat{w})}{L_1 + R}$

Repressed Inflation in the Non-Unionised Sector, Unemployment and excess product demand in the unionised sector The level of employment agreed in the bargain constrains unionised sector output.

Constraints: $\dot{L}_1 < N$, $L_2 = R$, $X_2^D > x_1^S$, $x_2^D > x_2^S$

Effective Demands and Supplies

$$x_1^S = F(L_1)$$

$$x_2^S = G(R)$$

No labour hoarding.

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8. CONCLUSIONS

8.1 Summary and Results

The main aim of this thesis has been to review and make some contributions to disequilibrium macroeconomic theory. Two themes have dominated this work, the role of expectations in disequilibrium models and the implications of introducing bargaining into a disequilibrium framework.

In chapters 2.3 and 4 the roles of expectations in Neo-Keynesian macroeconomic models were investigated. It was argued that the literature can be divided into two sections, non-Walrasian models such as Varian [1977] or Heller and Starr [1978] where equilibria are based upon self-fulfilling expectations, and models such as Malinvaud [1977], Muellbauer and Portes [1978] or Barro and Grossman [1971] which rely upon price rigidities to give the (temporary) equilibria. The three chapters concentrate mainly upon fixed price models since these appear to have the more immediate macroeconomic policy implications. The original contributions in each of these chapters arise from a simple objection to the way fix-price models such as Malinvaud are conceptually formulated.

Fix-price analysts argue that there is no agent within an economy whose role is to organize perfect price adjustments (the Walrasian auctioneer) and as a consequence any equilibria must be established through the adjustment of quantities. This seems a reasonable proposition. There is no tatonnement upon prices. Therefore, not all desired transactions can be completed and quantity adjustment consequently occurs. However it is then argued that at given prices a tatonnement process on quantities operating according to some given rationing scheme establishes an equilibrium as defined either by Benassy [1975] or Dreze [1975]. The assumption of a perfect, frictionless quantity tatonnement process in this context seems particularly unpalatable and chapters 2 and 3 of this thesis modify this assumption, whilst chapter 4 investigates the consequences of replacing it with sequential trading at fixed-prices. Once the assumption of a perfect frictionless quantity tatonnement is abandoned or modified then quantity constraint expectations within the current period become important. This seems to have been recognized at least partially by Benassy [1975, appendix] and Honkapohja and Ito [1980].

In chapter 2 two significant modifications are made to the simple basic fix-price model. First it is assumed that at the start of each market period agents hold subjectively certain expectations of the levels of trades they may carry out on the various markets. If these expectations are less than their desired trades these are regarded as rations and are used to calculate desired transactions upon unrationed markets, i.e. there is a notional spillover effect. Further it is then assumed that once made market offers may only be revised downwards either due to productive adjustment costs or an inability to find another agent willing to recalculate all his trades so as to complete the other side of the desired transaction. With this mechanism a treatment with several interesting properties is obtained. What is termed the Keynesian expectational regime is investigated in depth, with results upon the expectational repressed inflation and classical regime following in a similar manner. The regime is termed expectational Keynesian since it is the worker/consumers expectation of a shortfall of demand for his labor which causes him to reduce his goods purchases. The effects of change in government policy instruments, goods purchases, lump sum transfers/taxes on firms or consumers, have been shown to have different effects in one period and over a succession of periods and also to have placebo effects if "announced" at the start of the current period. Perhaps the most interesting is the impact of government expenditure policy in the short-run on the expectational Keynesian regime. If an expenditure increase is carried out unannounced it raises employment and output but does not raise consumption until the next period when expectations adjust. If however an expenditure increase is announced, in the form of the government giving an undertaking to purchase sufficient output to maintain full-employment, then workers will not anticipate any rationing upon the labor market and their goods demand alone will be sufficient to establish a full employment equilibria. The government then has no need to purchase any output. Similar results obtain with the government reducing expenditure upon the repressed inflation regime. Also interesting is the way the equilibria adjust over a succession of periods in response to expectations and money balance adjustments. If government policies are held constant and the system is initially in an expectational Keynesian unemployment equilibrium, then expectations and money balances adjustments will push the system out along the production function with increasing levels of output, employment and consumption in successive periods until an orthodox Keynesian type unemployment equilibrium is established. The economy in an expectational regime does display a self-adjustment mechanism, but towards a standard fix-price equilibria, not towards full Walrasian equilibrium.

On first inspection the assumptions made in this section may seem strong, however the analysis is intended to demonstrate the implications of imperfect quantity adjustment and the effect of current constraint expectations upon current market transactions offers. These ideas were carried one stage further in chapter 3 where adjustment costs were introduced explicitly into the quantity adjustment process. In section 3.2 adjustment costs take the form of resources consumed in the adjustment process, firms and consumers are aware of these adjustment costs but have to state initial transactions demands before the true state of the world, as characterized by the quantity constraints they face, is known. Consequently initial transaction demands are calculated upon the basis of maximizing Von Neumann-Morgenstern objective functions. These offers maximize expected utility from trade net of expected adjustment costs. On learning the true state of the world, as characterized by their sets of feasible trades, agents then adjust optimally away from their initial transactions demands. Again only "Keynesian" cases were examined in detail, however analysis of the repressed inflation cases follows immediately. Several interesting features were shown to arise. The calculation of an agents initial transaction offer is complex since it is chosen simultaneously with the level of transactions that the agent wishes to complete once the state of the model has been revealed, the planned adjustment away from the initial transaction offers. The initial transactions offer could be generated by three different demand functions and could switch between them with changes in the models parameters. An interesting point which arises is that when the worker is optimistic about his labour sales, he places a high probability upon being unconstrained, then further increases in optimism will cause his choice labour supply and goods demand to fall. He chooses more leisure. The comparative static properties of this treatment are also of some interest, and it was shown, as perhaps might have been anticipated, that several comparative static effects are similar to those in the model developed in chapter 2. However it is again noticeable that the system is generally less responsive in terms of output and employment to changes in government expenditure, for example, than standard treatments such as Malinvaud [1977].

In the fourth chapter of this thesis the role of expectations in tatonnement and non-tatonnement models was discussed, and it was suggested that the correct method of describing how a fix-price equilibrium is established should be by some sequential trading process. In section 4.2 a simple model of a sequential adjustment process was developed. It was argued that in such a treatment expectations are of crucial importance since workers and firms, unable to trade upon both markets simultaneously, must base transactions upon one market on their expectations of trade possibilities on the other. Keynesian and Repressed inflation trading sequences are modeled, each of which was able to take one of two forms depending upon the decision making process in the sequence. If trade effectively takes place first upon the goods market then firms sell inventory to consumers who purchase from their current money stocks in anticipation of the income they may realize from labour sales at the next stage of the sequence. Firms, being on the short side of the labour market, will have no problems in purchasing sufficient labour to rebuild stocks. This describes one possible Keynesian trading sequence, the dynamics of which arise from workers not knowing the firms production function and consequently not correctly anticipating the level of employment which results when they transact upon that market in the next instance. If anticipations are incorrect involuntary money stock decumulation and expectations adjustment provide the dynamics of the system. Alternatively, if the decision making process is such that the labour market effectively meets first, then firms must decide how much labour to purchase in anticipation of the goods they will be able to sell when that market opens. Since firms do not know workers preferences these anticipations may be incorrect, and firms expectations and inventory stock adjustment give rise to the system's dynamics.

Similar analysis was carried out for the repressed inflation regions under the two alternative decision making processes; upon this regime it was the firms expectations of labour availability and households exectations of goods availability which were crucial in defining the dyanmics.

The stability properties of these dynamic systems were examined and it was found that on both regimes either stable, cyclical or saddle point type behavior was possible. These results are more complex than the conclusion drawn by Böhm [1978] and Honkapohja and Ito [1980]. Hence it was argued that in the stable cases the comparative statics exercises of calculating the Keynesian demand and repressed inflation supply multipliers were valid. In the cyclical and saddle point cases it was shown that government expenditures and money supply (repeated lump sum transfer) rules could be derived which would stabilize the system. An interesting result arose in the Keynesian cyclical cases. It was found that the simple reduction of government expenditure would stabilize the system, this gave a trade off in this regime between the expansionary effects of increases in government spending and the possible instability it might cause.

The stability properties of the quantity adjustment process are also important when considering subsequent price adjustment. A unstable quantity adjustment process would invalidate the use of the effective excess demand hypothesis to define price adjustments.

In chapters 5, 6 and 7 the second theme of the thesis, the determination of prices, was considered. In chapter 5 the various approaches that have been proposed in the literature for endogenizing prices in disequilibrium (or non-Walrasian equilibrium) type models were examined. Each approach examined appeared to suffer from a number of conceptual weakneses, however, with the exception of the effective excess demand hypothesis, each clearly gave significant insights into the problem. Particular processes of price adjustment seemed more plausable as means of describing the behaviour of some markets than others. Reality is clearly a combination of the processes.

In chapter; 6 and 7 rather than address the general question, why do prices adjust and how?, attention was focused on the mechanisms by which the price of labour, the wage rate, might be determined. The argument that wages are determined by bargaining between a union and the employer was adopted and investigated in some detail. Section 6.1 developed a simple bargaining model of wage and employment determination and examined the comparative static properties of monopolist, monopsonist, Nash and Market Power solutions. Utilizing the approach developed in 6.1, in section 6.2 partial equilibrium analysis of the impact of disequilibrium regimes upon bargaining was carried out. Two scenario's which had the characteristic that the wage rate varied little but the level of employment considerably, were studied. The important element in these scenarios was the fix-price regime operating in the sector where unions members or the firms capital found alternative employment. In scenario 1 it was assumed that the sector in which the union members had their alternative employment possibilities was characterized by a repressed inflation type situation. Then a tightening of the constraints operating in that sector will produce offsetting effects upon wages and complementary effects upon employment. In scenario 2 it was assumed that the sector in which the firms capital could find alternative employment was characterized by Keynesian unemployment. Relaxation of the constraints in that sector was shown to produce offsetting effects both on employment and wages. This analysis was somewhat exploratory in nature and reproduced independently, through somewhat more extensively some results obtained by McDonald and Solow [1981]. In chapter 6 the effect of disequilibrium upon wage employment bargaining was examined. In chapter 7 the task of incorporating bargaining into a complete disequilibrium macroeconomic model was undertaken. In section 7.1 a simple single sector model was constructed and examined first under the hypothesis that the price of output is rigid and second that it adjusts to clear the product market. In both analysis the wage rate was determined by the Nash bargaining solution. When the output price was assumed rigid some very interesting results arose. It was found that five fix-price regimes could arise depending upon whether supply or demand was on the short side of the goods market, whether full employment or unemployment was contracted for in the bargain and whether or not labour hoarding resulted. I.e., was the agreed employment level higher or lower than that required to satisfy demand for output. The model had two exogenous parameters, government goods demand and the level of unemployment benefit, paid by the government. The magnitude of the exogenous parameters determined which rationing regime obtained. Two regimes with Keynesian features, two with neoclassical features and one repressed inflation type regime arose. The comparative static effects of changes in government expenditure and unemployment benefit upon the models endogenous parameters in each regime were investigated, and some interesting results were found to hold. On the Keynesian unemployment regime, (where the bargain gave unemployment and labour hoarding together with a shortfall in effective goods demand), an increase in government expenditure had no effect upon the wage rate. The price of labour, although endogenous, was found to be rigid. Upon the semi-neoclassical regime (where the bargain determined the wage rate and employment was determined by product demand via the production function) increases in government expenditure reduced the wage rate.

On the repressed inflation regime (full employment and a shortfall of goods supply) increases in unemployment benefit raised the wage rate despite the absence of unemployment. A full listing of comparative static results may be found in table (7.1.1).

Under the hypothesis that the product price always adjusts to clear the output market the simple single sector model produced less clear cut results, however it did appear probable that increases in government expenditure would raise output employment and wages but would have an ambiguous effect upon prices. An increase in unemployment benefit would probably raise output and employment, due to the extra demand generated, but had an ambiguous effect upon both prices and wages.

The simple single sector model developed in section 7.1 assumed that the wage employment bargain adjusted continuously. In section 7.2 a two sector model was developed with wage employment bargaining only in one sector. It was argued that in such a complex world continuous bargaining would be more costly and that bargains would be struck at discrete intervals and would remain in force for a specific period. It was assumed that both labour and capital was immobile between sectors in the short run and that all prices other than the wage detemined in the bargain were perfectly flexible.

It was argued that the wage employment bargain would be struck on the basis of the firm and unions expectations about the prices that would obtain over the period it was in force. This suggested two possible equilibrium concepts. A short-run equilibrium, established, given the bargain, by price adjustments upon other markets. The prices arising on other markets would differ from the price expectations upon which the bargain was struck. Consequently the bargain and equilibrium would change when the bargain was renegotiated. A medium-run equilibrium would arise when the expectations upon which it was based proved to be accurate and subsequent renegotiation would yield the same outcome.

One of the most interesting consequences of adopting this structure was that involuntary unemployment can arise in either production sector both in the short and medium run. Unemployment can arise in the unionized sector as a consequence of the bargaining process, where the union may be prepared to accept some unemployment in return for a higher wage. In the non-unionized sector unemployment may arise as a consequence of market failure, low levels of government expenditure together with a low wage employment bargain and a low level of unemployment benefit, may result in the price of the non-unionized sector output and hence its labour demand at each nominal wage being very low. The non-unionized sectors wage cannot go to zero as wages must exceed unemployment benefit to attract any labour.

It was found that in the short-run the government could cure the market failure problem by increasing expenditure on the output of <u>either</u> sector or by increasing unemployment benefit (given certain elasticity and substitution assumptions). The medium-run effects of government policy occurred when the bargain was restruck to incorporate the changes that occurred in prices. It was argued that if the government had pursued expansionary policies, raising expenditure and unemployment benefit such that output prices had risen, then the bargain could be effected in the following manner. A rise in the price of the good produced in the unionized sector and a rise in the unemployment benefit rate would tend to push the wage rate and level of employment in the unionized sector upwards. However, and perhaps somewhat counter intuitively, a rise in the price of the good produced in the non-unionized sector tends to reduce the wage and level of employment in the bargain. It was argued that this effect arose because an increase in this price reduced the value of unemployment benefit by more than that of the wage rate, consequently increasing the unions utility surplus. A reduction in wages and employment was then required to redistribute some of this utility gain to the firm.

The case where there was unemployment in the unionized sector, but full employment in the non-unionized sector was interesting. In the short-run expansionary government policy simply gave rise to inflation, with no real effects. In the medium run the effect upon the bargain was to probabily raise both output and employment in the unionized sector, particularly if the price of that sectors output had risen sharply. A little inflation, was a good thing.

It was argued that the existence of a unionized sector within a simple macroeconomic model introduced short-run rigidities, but in the medium run renegotiation of the wage employment aggreement took place in response to changes in the environment. Therefore government policy should be devised in the light of both its short and medium run consequences.

In section 7.3 a model very similar to the one examined in 7.2 was analyzed, however the major difference was that the fix-price method was adopted. This generated a large number of regime combinations depending upon whether demand or supply was on the short side of the two output and non-unionized labour markets, and whether full or unemployment with or without labour hoarding arose as a consequence of the bargaining solution. Sixteen possible constraint regimes could be generated, these were not all investigated thoroughly and attention was focused upon five examples which seemed the most interesting and plausable. Since the novel component of the model was the introduction of a unionized production sector analysis centered on how the existence of a wage employment bargain effected the comparative statics of government policy, and on how changes in this bargain affected the models other endogenous variables. The bargain effected the models comparative static properties in a number of ways. The fixed level of employment in the unionized sector implied that labour hoarding could exist, and if it did, then government purchases of that sectors output would have no multiplier effects. If the employment component of the bargain constrained output, then government purchases of the sectors output could be used to raise employment in the non-unionized sector. This was due to a tightening of the goods supply constraint for the unionized sectors output, causing consumers to switch expenditure to the non-unionized sectors output, giving a rise in employment.

The effects of changes in the wage employment bargain were typically dependent upon the regimes obtaining on the output markets. If the unionized sectors output market was supply constrained, an increase in employment and wages in that sector would have two offsetting effects upon employment in the non-unionized sector. Increased income would tend to raise output and employment, by raising demand. Increased availability of unionized sector output would cause consumers to switch expenditure away from the non-unionized sectors good, with a tendency to reduce output and employment. If however the unionized sectors output market was supply constrained at full employment output, any rise in the wage rate would increase demand employment and output in the non-unionized sector, provided that the market was demand constrained. However, if a situation of classical unemployment obtained in the non-unionized sector, then changes in the wage employment bargain would have no effect since the firm in the non-unionized sector would be on the short-side of both its input and output markets and would be unwilling to change its behaviour.

Further to its implications for the comparative static properties of the model, it was also argued that the wage employment bargain explained labour hoarding and consequently threw some light onto how the average productivity of labour varies.

In chapters 6 and 7 several of the implications of wage employment bargaining in disequilibrium were examined. Firstly the actual mechanics of various bargaining solutions were examined. Secondly, the way bargaining in one sector might be affected by constraint changes in others was analysed, and then in chapter 7 some simple single sector models with continuous efficient bargaining incorporated were developed and analysed. Finally in sections 7.2 and 7.3 a pair of two sector models were introduced in which a bargain was struck on the basis of current parameter values. The bargain was assumed to be renegotiated at discrete intervals generating an iterative process. A bargain is struck, then prices and quantities adjust. After the institutionally defined time period, the bargain is renegotiated on the basis of the new information.

The general approach of this thesis has been to explore a number of areas within the main themes of expectations and bargaining. The aim was not to construct a single consistant macroeconomic model incorporating all the thesis ideas, but rather to make a series of suggestive contributions upon various aspects of both problems. Further developments along some of the lines examined are possible, and in the next section, a few tentative suggestions are made.

8.2 Potential Further Developments

Neither of the problems addressed in this thesis has been exhaustively investigated either here or in the literature. There are many potential further developments. Disequilibrium or Neo-keynesian economics attempts to build viable macroeconomic models with an acceptable choice-theoretic microeconomic base. Two lines for further development immediately suggest themselves, improvements to the microeconomic basis and extensions of the macroeconomic models. There are two fundamental problems with disequilibrium economics, firstly we do not have a good microeconomic explanation of price adjustment (or lack of adjustment). Secondly, the macroeconomic models are too simplistic, and require at least the incorporation of investment and asset markets before they can be used to make useful policy prescriptions.

With specific reference to the contributions made in this thesis a number of possible refinements and extensions could be made. Most of the analysis could have been carried out in an open rather than closed economy context. However, reference to the survey of open Neo-keynesian models presented in the appendix suggests there are a number of alternative specifications which could be adopted. These would complicate the analysis considerably but would possible provide some interesting conclusions. The introduction of wage employment bargaining into an open economy framework might be very interesting. Chapters 2 and 3 examined expectations together with restrictions upon quantity adjustments in fix-price models. Extensions could be made to the models by investigating different forms of expectations adjustment, Bayesian learning may be easy to incorporate into Chapter 3's analysis. The behaviour of agents using the fix-price model to formulate their expectations might produce some interesting results. The work of Neary and Stiglitz (1981) suggests this.

In chapter 4 some very simple assumptions are made about firms inventory holding decisions, these could be modified to include a complete story of inventory holding costs and optimal inventories. The impact of constraint expectations upon target inventory stocks could provide some interesting extensions.

The introduction of a government issued bond as an alternative form of deficit financing, could be made in the models presented in the first three chapters, this would involve the specification of an asset market and bond demand functions. Further the assumptions made about profit throughout the thesis were deliberately simplistic, the 100% profits tax could be replaced by the assumption that firms retain some profits or issue equity. Profit retention or equity issue might be the way firms respond to constraint expectations.

In the two sector bargaining models, the specification of one sector as producing an intermediate or capital good for use in the other sector could lead to some interesting conclusions. A high wage low employment contract in the capital goods producing sector might create capacity constraints in the consumption goods sector producing a very undesirable equilibrium, with low employment and goods availability.

Further developments of the bargaining models might entail investigation of different forms of the unions utility function and different solutions to the bargaining problem (the Nash solution has been the mainstay in this thesis). Dynamization of the fix-price bargaining models is another potential development. If output prices at the end of each period adjusted according to some specified rule the evolution of the model over time might provide an interesting area for study. In the two sector models relaxation of the labour immobility assumption may allow the study of inter-sectoral labour mobility and the union non-union wage differential, we might anticipate an equalization of expected wage income across the two sectors.

Clearly then the work of formulating a well articulated comprehensive disequilibrium macroeconomic model has yet to be completed. It is hoped that this thesis has made some helpful contributions and suggestions towards that end.

APPENDIX

A Survey of Open Neo-Keynesian Models

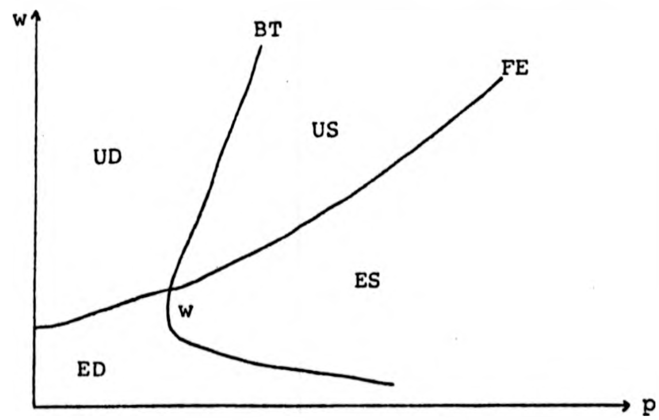
In the main body of this thesis attention has been focused upon the role of expectations and the introduction of wage bargaining in disequilibrium macroeconomics. The analyses were carried out in a closed economy context for reasons of tractability. Several of the analyses would be interesting in an open economy setting, most particularly wage bargaining in an open model. This however is beyond the scope of this thesis.

There have been a large number of recent contributions to the literature on open Neo-Keynesian Macroeconomics, here only a brief review of the ideas and results of the main papers will be presented. A very extensive bibliography of the area may be found in Owen [1981].

The starting points for this literature are the simple fix-price models of a closed economy developed by Malinvaud [1977] and Barro and Grossman [1976], and the analysis of Dixit [1976] which assumes some prices are fixed whilst others clear markets. The literature may be regarded as divided into two strands, those treatments following Malinvaud, Barro and Grossman who treat all prices as exogenous parameters and those that follow the Dixit line. Open models with all prices éxogenously fixed will be examined first.

Dixit's [1978] contribution considers an economy in which there are three goods: money, labour and a tradable commodity. The prices of money and labour are domestically fixed, the price of the tradable commodity is determined upon the world market. The country may buy or sell all of the tradable it wishes at the going world price which is simply the foreign price multiplied by the fixed exchange rate. Agents cannot encounter rationing upon the goods market, excess demand or supply manifests itself in a balance of trade deficit or surplus. Rationing can however arise on the labour market since labour is assumed internationally immobile. Utilising Clowers dual decision technique Dixit divides wage price space up into regions of unemployment and excess demand for labour, and superimposes upon this a trade balance line as reproduced in figure A1.

Figure A1



where FE is the full employment locus, and BT is the trade balance locus. The four regions are (UD) unemployment and a trade deficit,(US) unemployment and a trade surplus, (ED) full employment and a trade deficit, and (ES) full employment and a trade surplus. W is Walrasian equilibrium, a position of both internal and external balance.

Dixit notes that the BT locus must change slope as it cuts the FE locus since the form of the domestic demand functions changes.

With this apparatus it is shown that if the economy is initially in a position of Walrasian equilibrium then an increase in the money supply will give rise to a trade deficit and excess demand for labour. With completely rigid prices and exchange rate the trade deficit will persist until all the money supply increase has been used in the purchase of tradables when the economy will return to its original position. If the wage rate responds to the excess demand for labour the system will display convergent cycles between the UD and ED regions until the deficit disappears and equilibrium is re-established at w with the original money supply. If the exchange rate can adjust instantaneously then the money supply will remain at its new value, and a new Walrasian equilibrium will be established with relative prices unchanged but nominal prices higher.

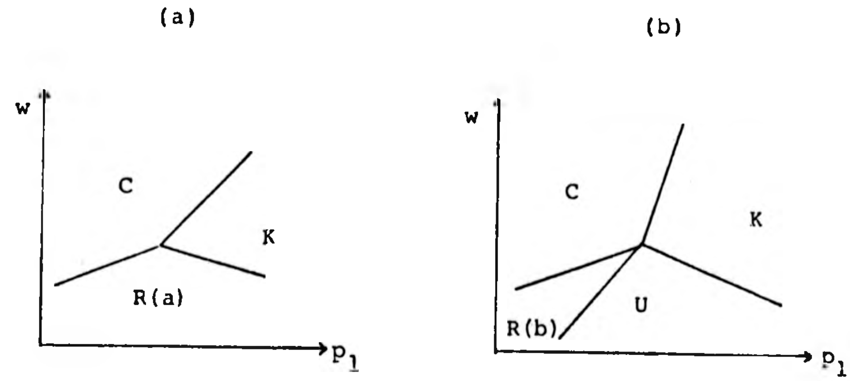
If the government attempts a fiscal expansion, this has no effect upon employment but reduces the amount of the domestically produced tradable good that is available for consumption at home or abroad and causes a ound for ound deterioration in the trade surplus. This is of course a result made famous by the New Cambridge school, the government simply spends reserves to buy goods from abroad.

Dixit also examines the effect of a rise in productivity, resulting from an increased marginal product of labour. This gives rise to an excess of tradables and hence a trade surplus together with excess demand for labour. This is very different from the effect a rise in productivity has in a closed economy, Malinvaud demonstrates that Keynesian unemployment will result.

These results are suggestive but as the author remarks the analysis is only a first approach to examining the balance of trade in an open Neo-Keynesian model. To obtain a more realistic treatment the analysis should recognise that not all goods are internationally traded, and that a country may be a large transactor on the markets for some tradables but small on others. It is thus necessary to introduce the distinction between traded and non-traded goods and to examine more closely the small country assumption.

Neary [1980] introduces a non-traded good into an analysis very similar to Dixit's. The small country assumption is made about the tradable, hence rationing may only arise on the non-tradable good and labour markets. Given that there are two production sectors the rationing scheme which distributes labour during periods of excess demand becomes important. Two scenarios are examined, one,where the traded goods sector gets priority upon the labour market, and is assumed to be never rationed, and two, the non-traded goods sector gets priority and cannot be rationed. Under these two scenarios the wage rate, non-traded good price space may be divided up as figure A2(a) and (b).

Figure A2



Traded goods sector gets priority on the labour market

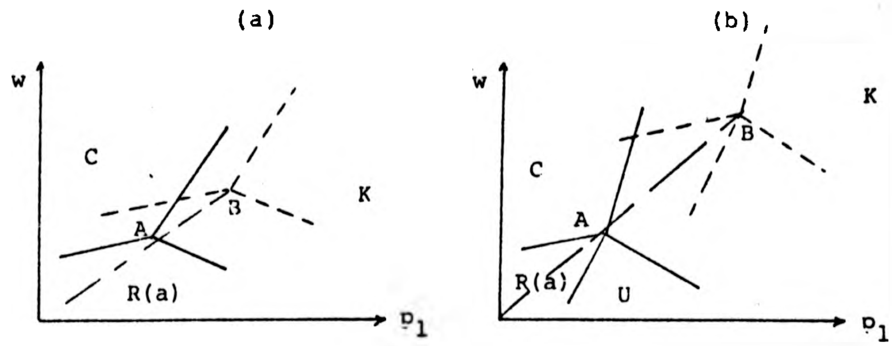
Non-Traded goods sector gets priority upon the labour market

The interpretation of figure A2 is as follows: the region indicated C in both panels displays excess supply of labour and excess demand for the non-traded good. The regions indicated K in both panels display an excess supply of the non-traded good and general excess supply of labour. The region R(a) is an area of excess demand for labour in the non-traded goods sector and excess demand for the non-traded good. R(b) is a region of excess demand for labour in the traded goods sector. The region U is an underconsumption region, its appearance is interesting since the model uses an atemporal production function. The underconsumption case arises when the traded goods sector is rationed for labour, households are unconstrained and the non-traded goods sector is rationed upon the goods market.

Having established the division of (w,p_1) space into the familiar regimes, Neary examines how changes in the money stock, government expenditure, the exchange rate and the state of technology will effect this division, and the comparative static effects changes in the exogenous variables will have upon the level of employment and the trade balance.

Monetary changes in the form of lump sum transfers (or taxes) to households cause a radial expansion (contraction) of the regimes in (w,p_1) space as figure A3 indicates.

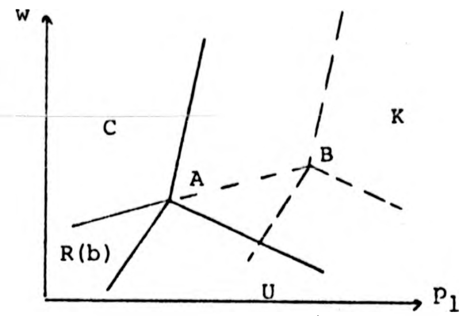
Figure A3



If the system was at a Walrasian equilibrium at A a monetary expansion gives repressed inflation. If it was at B a monetary contraction gives Keynesian unemployment.

Fiscal expansions financed by lump sum taxes effect the model differently depending upon which good the government purchases. Neary suggests that an increase in government purchases of the tradable has no effect upon employment but leads to a pari passu deterioration in the trade balance, which will only be the case however if the purchases are financed by printing money. If the means of finance is an increase in lump sum taxes then a Keynesian unemployment regime will arise as in B→A in figure A3. This suggests that the New Cambridge result first pointed out in this context by Dixit depends upon the type of financing adopted. If the government purchases non-traded goods and finances its expenditure with lump sum taxes the effect upon the division of wage, non-traded good price space will be as figure A4.

Figure A4



Thus the effect of the increase in government expenditure is to create excess demand for non-tradables.

If the assumption of gross substitutability is made then a devaluation has the same effect as a monetary expansion, as in figure A3. However it should be noted that gross substitutability is a sufficient but not necessary condition for a devaluation to have this effect. For a more detailed analysis providing the necessary conditions see Fender [1982].

Technical progress has differential effects in the two sectors. Starting at Walrasian equilibrium technical progress in the traded goods sector gives excess labour demand in the sector which does not have priority in its labour supply. In the non-traded goods sector, again starting at Walrasian equilibrium, a rise in productivity gives Keynesian unemployment.

The signs of the partial derivatives of changes in exogenous variables within a fix-price regime are summarised below.

Regime	Endogenous Variable	Exogenous Variable w	p_1	p_2	k_1	k_2	M	g_1	g_2
C	L	-	+	+	+*	+*	0	0	0
K	L	?	-	+'	-	+*	+	+	0
C	S	?	?	+	?	+	-	?	-
K	S	-'	?	?	+	+	-	-	-
R(a)	S	-'	-	?	-	+	-	-	-
R(b)	S	?	?	?	?	+	-	-	-
U	S	-'	?	?	+	+	-	-	-

where w wage rate

p_1 price of non-traded goods

p_2 price of traded goods (and exchange rate).

k_1 productivity parameter in non-traded goods sector

k_2 productivity parameter in traded goods sector

M Households money stock

g_1 government purchases of non-traded goods

g_2 government purchases of traded goods

L employment (total)

S trade balance

'indicates gross substitutability ensures the sign,

*indicates that result depends upon technical progress increasing labour demand at a given real wage.

Both Dixit and Neary consider one traded good in which the country is a small supplier or demander and is never rationed. Cuddington [1980] relaxes this assumption and examines an economy which produces two tradable goods and no non-tradables. The tradable goods are termed exportables and importables although both are produced domestically. The country is assumed to be a large supplier of the exportable and may be rationed upon the world market. The small country assumption is applied to the importable where the economy can always achieve its desired trade. Under the assumption that the domestic economy always gets priority of supply in exportables Cuddington examines the effects of fiscal, exchange rate and incomes control policies inside the classical Keynesian and Repressed Inflation regimes, where regimes are defined in terms of the exportable and aggregate labour markets. Since no constraints are encountered upon the importables market changes in government expenditure upon this good have a pari passu effect upon the balance of trade but have no internal effects. The other comparative static results obtained by Cuddington are summarised below.

Exogenous Variable

	e	g^E	p	w
Exportable output	0 + -	0 + 0	+ - +	- 0 +
Importable output	+ + -	0 0 0	0 0 -	- - +
Nominal G.N.P.	+ + +	0 + 0	+ ? ?	- - +
Exports	- + -	- 0 -	? - ?	? 0 ?
Imports	? ? ?	0 + 0	+ ? +	? + ?
Balance of Trade	+ ? +	- - -	+ ? +	- - +

where the first entry in each row refers to classical, the second Keynesian and the third repressed inflation regimes.

- e exchange rate (devaluation)
- g^E government purchases of the exportable
- p domestic price of the exportable
- w domestic wage rate

Although Cuddington's approach may be regarded as an improvement over Neary and Dixit in that it explicitly recognises the different characteristics of importables and exportables; it does not consider the additional complications arising from introducing a non-traded good. Both Steigum [1978] and Fender [1981] consider models which have three goods, an importable, exportable and a non-traded good.

Steigum considers a model in which there are two tradable commodities, an exportable good produced domestically and an importable input good ('raw materials') which is used in the production of both the exportable, and a non-tradable. Domestic firms can be constrained upon all markets, the country is assumed large on both the importables and exportables markets. There are clearly numerous constraint combinations in this model, Steigum examines four which are characterised by unemployment. A classical case arises when firms producing both the exportable and non-traded good are unconstrained upon output and labour markets. A regime termed Keynesian Unemployment with exogenous export occurs when producers of both exportables and non-tradables are demand constrained. Keynesian unemployment with endogenous export describes a situation where firms producing non-tradable goods are sales constrained whilst firms producing exportables can sell all they wish. Finally mixed type unemployment describes a situation in which firms producing exportables are demand constrained, in firms producing non-tradables are unconstrained. Little attention is paid to importables in the classifications, indeed the definition of the importable is strange. It is described as raw material yet it is substitutable for labour in the production processes of the two sectors, perhaps it should be described as flow of imported capital, this would raise considerable problems. Because of the manner in which it is introduced it is not very important for the comparative statics of the analysis whether the firms are constrained for the importable or not. Since Steigum's main concern is to evaluate the effectiveness of various policies in increasing employment in an open economy of this sort he reports only the comparative static effects of changes in exogenous parameters upon employment in the various regimes. These are summarised below.

Regime	Imports	Endogenous Variable	Exogenous Variable										
			g_1	g_2	M	w	p_1	p_2^*	e	v^*	$\bar{z}_1$	$\bar{z}_2$	a
C	EN	L	0	0	0	–	+	+	?	–			
	EX	L	0	0	0	–	+	+	+	0	+	+	
K.EX	EN	L	+	+	–	–	?	?	?	?			+
K.EN	EN	L	+	0	–	–	–	+	+	?			
	EX	L	+	0	–	–	–	+	+	–	–	+	
M	EN	L	+	+	–	?	?	?	?	?			+

where g_1, w, L, e, M, p_1 are as the previous case.

g_2 government purchases of exportables

p_2^* foreign price of exportables

v^* foreign price of importables

$\bar{z}_1$ and $\bar{z}_2$ are the constraints on importables to the non-traded and exportable producing sectors respectively.

a quantity of exports

EN endogenous

EX exogenous

Steigum stresses that one of the most interesting characteristics of these comparative static results is the difference between the two Keynesian regimes, when exports are exogenous most relative price changes have indeterminate effects. In particular, the analysis cannot say whether or not devaluation raises employment.

Fender [1981] considers an economy with several characteristics in common with Steigum and Neary's treatments. There are three consumption goods importables, exportables and non-tradables. Importables are not produced domestically and exportables are not consumed domestically. A large number of the results may be obtained by treating exportables and importables as one net traded good. This makes the model very similar to Neary's in all but one important respect, when there is excess demand for labour it is assumed that both domestic production sectors are constrained. Neary examines the case where one sector gets priority and is unrationed. Fender makes the small country assumption about both exportables and importables which allow regimes to be identified by the constraint combinations that arise in the labour and non-traded goods markets. Three regimes arise, Keynesian unemployment where both the labour and non-traded goods markets are demand determined. Classical unemployment where there is excess demand for the non-traded good and excess supply of labour. Thirdly, Repressed Inflation arises when there is excess demand on both the labour and non-traded goods markets. The comparative static properties of these regimes are as in Neary's treatment, however in the cases where effects are not unambiguously signed Fender gives the appropriate conditions.

One major drawback with each of the above approaches is that only one country is analysed, and the state of the world markets in which it trades is exogenously given. In treatments which allow rationing upon export markets it is desirable that the constraint regimes operating in other countries be specified and examined. Owen [1981] approaches this problem by examining a world in which there are two countries each of which specialises in the production of one consumption good. Workers in both countries are immobile and consume both goods. The condition of the world economy will be characterised by the state of the domestic labour markets in the two countries and the state of the aggregate markets for each of the two consumption goods. Since neither good is storable there are six regimes of interest in the two countries summarised below.

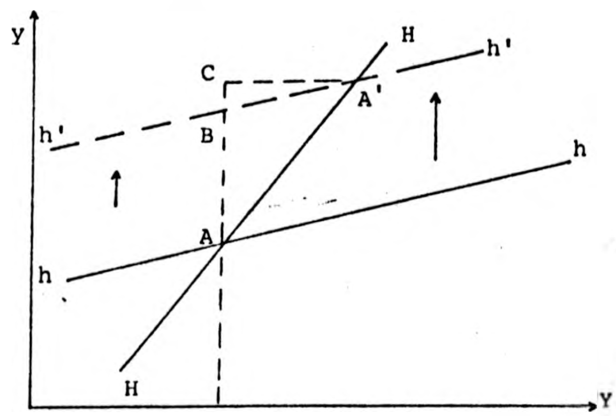
		Country 1		
		Keynesian Unemployment	Classical Unemployment	Repressed Inflation
Country 2	Keynesian Unemployment	1	4(b)	5(b)
	Classical Unemployment	4(a)	2	6(b)
	Repressed Inflation	5(a)	6(a)	3

The comparative static properties of the model depend upon which of the six potential regime combinations occurs. To devise the correct stabilization policies, policy makers need to know not only the fixed prices but also the quantity constraints which obtain in both countries. Single country analysis examined earlier suggest a general Keynesian type trade off between internal and external balance. Fiscal expansion gives increased employment but a balance of trade deterioration. Here if the government of country 1 increases its expenditure in regime 5(a) there can be no balance of trade deterioration.

This type of approach suggests that co-ordination of policy between the two countries may be required,for example if country 1 expands demand simultaneously with 2 contracting demand in regime 5(a) the net effect may be zero. In this analysis of a two country model the actual rationing schemes adopted are particularly important, one government may use quotas to prevent domestic expansionary policies spilling over into the other country. Clearly there are regimes where such applications, and adjustments of quota levels may be beneficial to both economies.

Dixit and Norman [1980, chapter 8] present an analysis of a two economy international equilibrium very similar to Owens, they study in detail only the case of Keynesian unemployment in both countries, Owens case 1. Using the appropriate effective demands equilibrium national income loci are defined for the domestic country (h,h) and the foreign country (H,H) as figure A5.

Figure A5



Dixit and Norman demonstrate that various policies can be examined by shifting the equilibrium loci. Figure A5 illustrates the effects of an increase in the domestic money supply giving higher income in both economies. In the domestic economy the impact effect of a rise in the money supply is A→B, the extra increase in income B→C arises as a feedback effect due to the stimulus provided to the foreign economy. Similar experiments can be carried out on the other regimes for other policy measures. The important implication here is the same as in Owen in that policies undertaken by one government effect both economies and may have beggar thy neighbour effects unless correctly co-ordinated.

There are some general conclusions and observations which arise from the literature on open economy fix-price models. Fiscal expansions were shown to have generally beneficial effects upon output and employment in the country undertaking them, but to cause a deterioration in the countries balance of payments position. The New Cambridge result appears in several single economy analyses, in that government purchases of tradables cause a deterioration in the balance of trade pari passu. Noticably this effect does not arise in the two economy models where even if government expenditure does have no direct effect upon the domestic economy it will still be stimulatory due to the international feedback effect. The major message for policy makers which arises from this literature is that a correct stabilization policy must be devised by recognising fixed prices, the regimes operating both domestically and abroad, and the size of the country relativeto the world markets in tradables.

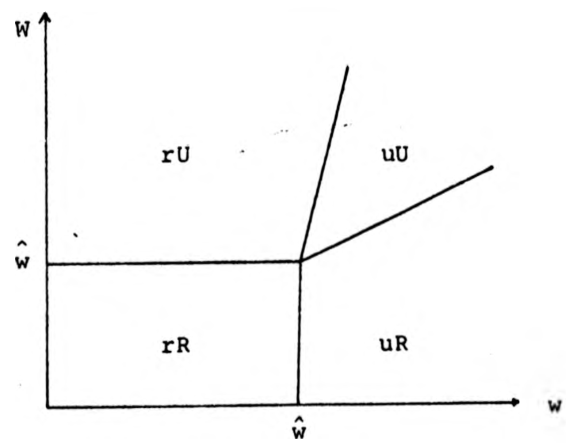
The second line of analysis of open disequilibrium macro models follow the initial contribution of Dixit [1976] who analysed a closed economy where input prices were exogenously fixed and output prices cleared the markets. Liviatan [1979] takes up this approach and examines a small trading economy which produces a tradable and a non-tradable good. Under a fixed exchange rate, and hence fixed world price of tradables, two possibilities are studied, one, where the price of non-tradables is fixed and the wage adjusts to clear the domestic labour market, and two, the price of non-tradables adjusts to clear the market and the wage rate is fixed. Liviatan concentrates his attention upon the effects that changes in the models exogenous parameters will have upon the balance of trade under these two scenarios.

In the case where the price of non-tradables is fixed there are four possibilities,either excess supply or demand for non-tradables,associated with either a balance of trade surplus or deficit. A monetary expansion will cause a deterioration in the balance of trade, this deterioration will be greater under a fixed price of non-tradables than it would be otherwise and will also be worse if they are in excess demand. Interestingly, Liviatan shows that an increase in the price of the non-tradable must improve the balance of trade unambiguously if the good is in excess demand. The idea of a case where wage rates are flexible and prices of non-tradable goods fixed is not particularly appealing, more interesting is Liviatan's second case.

In the fixed wage rate case Liviatan makes the somewhat strange assumption that if there is excess demand for labour it is rationed in the same proportions that it is allocated in at Walrasian equilibrium. With this allocation mechanism he then argues that a monetary expansion will cause a deterioration in the balance of trade since the price of non-traded goods will be bid up and the consumers will substitute tradables,the price of which is determined exogenously. A wage rate increase will also cause a balance of trade deterioration due to the increased price of non-tradables and the subsequent substitution effects.

Dixit and Norman [1980 chapter 8] consider a two country model in which there is only one homogeneous output good. The world market for the output good clears by price adjustment with the relative prices in the two countries depending upon the fixed exchange rate. Fixed nominal wages in the two countries allow labour to be in excess supply or demand in the two labour markets. Dixit and Norman show that for a given exchange rate and given money endowments the various labour market rationing regimes may be described as figure A6.

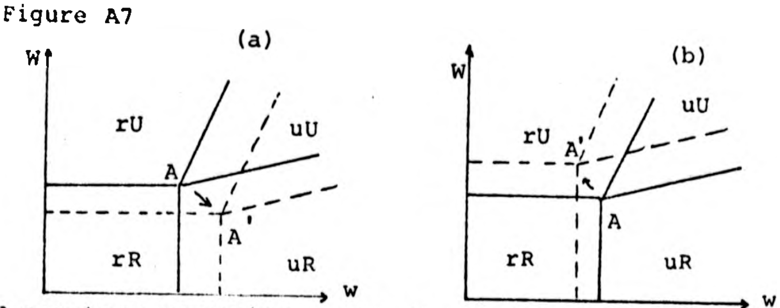
Figure A6



r,u and w refer to labour rationing, unemployment and the wage rate in the domestic economy. R, U and W indicate the same for the foreign economy.

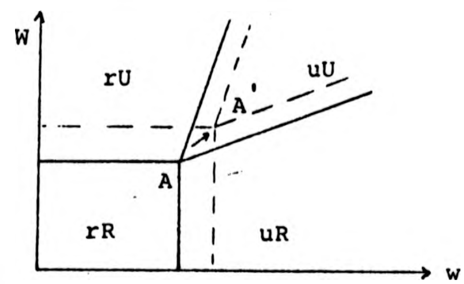
A devaluation by the domestic countries policy makers will shift the division of wage space as A7(a) and a devaluation by the foreign country will move the division as A7(b).

Figure A7



A monetary expansion causes the division to shift as described by figure A8.

Figure A8



The actual comparative static effects of a monetary expansion or devaluation will depend upon the unemployment regimes in the two countries. Dixit and Norman note that in a regime where there is unemployment in both economies a monetary expansion by one economy varies employment in both, as figure A8 suggests, but worsens the trade balance of the expanding country to the benefit of the other.

Despite its extreme simplicity this last approach does provide an intuitively appealing analysis of the interaction between two economies with domestic factor price rigidities.

Generally the literature upon open disequilibrium macroeconomic models provides a reasonably diverse menu. The analysis' have examined situations where some prices are fixed and others clear markets, and situations where all prices are fixed (all prices flexible gives essentially the monetary approach to the balance of payments). The distinctions between traded and non-traded goods have been examined as has the distinction between exportables and importables. The implications of an imported input good (raw material) have been analysed. The importance of both labour market and goods market rationing schemes have been stressed, and perhaps most importantly the implication of both small and large economy assumptions have been examined in the context of fixed prices.

In the absence of asset markets the literature can only address itself to problems of the current account of the balance of payments. The addition of asset markets and the consideration of the capital account is an avenue for further enquiry.

The implications for standard monetary and expenditure policy instruments of opening up the fix-price and partially fix-price models are diverse and depend upon the regimes and circumstances which characterise the economy in question.

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